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FOREWORD

Seismic qualification methods have been the subject of continuous discussion since seismic loads became the key part of design requirements for nuclear power plants. In the 1960's, the emphasis of seismic qualification was focused on laying the ground rules for achieving a consistent and conservative design basis. This effort continued in the 1970's, enhanced by the newly found theory of plate tectonics. By the late 1970's, significant accomplishments had been made in all areas of seismic design and qualification methods, culminating in the issuance of numerous Nuclear Regulatory Commission regulatory guides and nuclear industry standards. It is to be anticipated, however, that much of the earlier developments were centered in conservatism rather than accuracy and reliability. Although the intent was to insure the safety of nuclear power plant design, the more conservative prediction of seismic loads may not have improved the overall safety margin during normal operating conditions, especially for systems and components under high temperature environments. Therefore, it is necessary to doubly emphasize the need of developing realistic qualification methods, which would provide more reliable seismic loads leading to a balanced design.

This special publication contains papers presented in the Symposium on Seismic Analysis of Systems and Components, sponsored by the Operations, Applications, and Components Committee, at the 1983 Fourth National PVP Congress. It provides a glimpse of some of the more significant advancements in the area of seismic qualification with the emphasis on accuracy and reliability, not just conservatism.

For instance, it is found in one paper that the closely spaced modes requirement can be too conservative for a small piping in resonance with a large piping. A method was developed to reduce the subsystem response to a more realistic level. In another paper, the excessive conservatism of the envelope response spectrum analysis and the adequacy of the multiple support response spectra approach were demonstrated through a comparison with the time history analysis. The latter method is the subject of another paper that presented a large amount of test data in an attempt to quantify the advantage of the method. Also included in this volume are two papers dealing with piping damping values. This is a subject of particular importance, not only because it is a key factor in determining the correct piping response loads, but also because of the issues that have been raised concerning the existing regulatory positions. More test data sould no doubt contribute to an early satisfactory resolution of these issues. These and other papers included in this volume provide a significant contribution to the state of the art development, and toward a much more reliable, efficient, and realistic basis for seismic design.

On behalf of the ASME, the editors wish to express their sincere thanks to the authors and the reviewers for their effort in making this publication a reality.

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PIPING DYNAMIC ANALYSIS WITH SUBSYSTEM/SYSTEM INTERACTION

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ABSTRACT

In a piping analysis which consists of lines of different sizes, difficulties can develop when the two systems have similar modal response magnitudes and the frequencies are close to each other where closely spaced modes requirement is to be satisfied. When two modes are very close to each other and have similar magnitudes on modal coefficients, the use of a closely spaced modes formula would result in the absolute combination of the two modes, regardless of the signs of the two modal coefficients.

In this paper, the two modal coefficients for a two-degree-of-freedom system (representing the system and subsystem coupled model) are first studied to prove that they have opposite signs. Next, the response equation for the subsystem is studied to arrive at a most probable maximum response for the subsystem. A formula is then developed to obtain reduction coefficients when the response is computed using the closely spaced mode requirements. This formula can then be used by analysts to compute a more realistic subsystem response for modes which are in resonance with the main system and which have been found to have unrealistically large response loads.

NOMENCLATURE

c	Ratio of participation factor times mode shape (at mass 2) for masses 1 and 2.
K_1, K_2, K_c	Spring constants for masses 1 and 2, and between masses.
M_1, M_2	Masses for masses 1 and 2, respectively.
P_i	Participation factor for mode i.
$\ddot{q}_i$	Second derivative of the generalized coordinate for the ith mode.
R	Response reduction factor.
R_f, R_K, R_m	Ratio of frequencies, spring constants, and masses for the two-mass model.
$S_a(\pi)$	Acceleration response spectral value at the frequency π.
$\ddot{U}_g$	Acceleration input.
$\ddot{X}_1, \ddot{X}_2$	Absolute acceleration response for masses 1 and 2, respectively.
ω_1, ω_2	Natural frequency for masses 1 and 2, respectively, when decoupled.

ϕ_{1i}, ϕ_{2i}	Mode shape for masses 1 and 2, respectively, for mode i.
π_i	ith mode natural frequency for the coupled model.
β	Damping ratio.

INTRODUCTION

In a piping analysis which consists of lines of different sizes, difficulties can develop in two areas. First, when there is a large difference in piping sizes in the model, numerical instability could result for modes where modal effective mass ratios become very small. For such lines, a decoupled analysis would be appropriate. Or, if it is desirable, a coupled analysis can be made using a double precision version of the computer program. However, the latter effort is generally more time consuming and costly. Therefore, coupling of the system and subsystem is not recommended if and when the piping sizes are judged to have the potential of creating numerical difficulties.

In this paper, special attention is given to the second area of difficulties when system and subsystem are coupled. This area of difficulties arises when the two systems have similar modal response magnitudes and the frequencies are close to each other where closely spaced modes requirement (Reference 1) is to be satisfied.

The three modal combination methods recommended in Reference 1 are in similar form compared with that presented in Reference 2 for closely spaced modes. However, one important difference exists. While Reference 2 allows the sign of each modal coefficient (participation factor times mode shape coefficient) be maintained, Reference 1 requires that all signs be positive. When two modes are very close to each other and have similar magnitudes on modal coefficients, the use of Reference 1 would result in the absolute combination of the two modes, regardless of the signs of the two modal coefficients. For the two frequencies which are very close to each other, the generalized coordinates (as represented by Duhamel's Integrals) combine to form a beat type of motion. Therefore, the signs and the magnitudes of the modal coefficients have a strong bearing on the final response value. This has been recognized by Reference 3 in its probabilistic estimation of the final response. Since the use of absolute signs in Reference 1 is only to increase conservatism and not based on any mathematical fact, it should be modified whenever justifiable.

In References 3 and 4, a justification is provided for subsystems with extremely small masses such that a reduction in response can be made from the use of Reference 1 methods. However, since subsystems with extremely small masses need not be coupled with the main system (or else numerical instability could occur), the recommendation in Reference 3 has only a limited practical value.

In this paper, the two modal coefficients for a two-degree-of-freedom system (representing the system and subsystem coupled model) are first studied to prove that they have opposite signs. Next, the response equation for the subsystem is studied following the same analogy of Reference 3 but without using the assumption of extremely small mass ratios for the subsystem and the main system. In this study, the nature of the modal coefficients for the model with closely spaced modes is utilized (including the fact that the two modal coefficients have opposite signs) to arrive at a most probable maximum response for the subsystem. A formula is then developed to obtain reduction coefficients when the response is computed using the closely spaced mode requirements set forth in Reference 1. This formula can then be used by analysts to compute a more realistic subsystem response for modes which are in resonance with the main system and which have been found to have unrealistically large response loads.

MODAL COEFFICIENT CHARACTERISTICS FOR A TWO MASS SYSTEM

In order to properly represent a subsystem and a system in its coupled state, a two-degree-of-freedom model shown in Figure 1 is constructed, where M_1 represents the large mass of the system and m_2 represents the small mass of the subsystem. Masses M_1 and m_2 are connected by a spring K_c.

In addition, each mass is supported by a spring (K_1 and K_2, respectively). Therefore, the model closely resembles the subsystem and system design where they are connected but also supported by independent supports.

The natural frequencies of the combined model can be written as the following:

$$\left(\frac{\pi_{1,2}}{\omega_1}\right)^2 = \frac{1}{2}\left\{(1 + R_K + R_f^2) \mp [(1 + R_K - R_f^2)^2 + 4 R_K^2/R_m]^{\frac{1}{2}}\right\} \quad (1)$$

where $\pi_{1,2}$ represents either first or second mode of the coupled model corresponding to the values calculated by taking either "-" "+" sign in the right hand side equation, respectively. In addition, R_K, R_f, and R_m are the stiffness ratio, frequency ratio, and the mass ratio represented by the following:

$$R_K = \frac{K_c}{K_1} \quad (2)$$

$$R_f = \omega_2/\omega_1 \quad (3)$$

and

$$R_m = m_2/M_1 \quad (4)$$

where

$$\omega_1 = (K_1/M_1)^{1/2} \quad (5)$$

$$\omega_2 = (K_2 + K_c)^{1/2}/m_2 \quad (6)$$

are the decoupled frequencies for mass 1 and mass 2, respectively.

The mode shape coefficients for the two masses have the following ratios (Reference 5):

$$\frac{\phi_{1i}}{\phi_{2i}} = R_K/(1 + R_K - \frac{\pi_i^2}{\omega_1^2}) \quad (7)$$

$$i = 1, 2$$

where subscript i represents the i th mode, and ϕ_1 and ϕ_2 are the mode shape coefficients for masses 1 and 2, respectively.

The participation factors can be written in the form as follows:

$$P_i = \frac{M_1 \phi_{1i} + m_2 \phi_{2i}}{M_1 \phi_{1i}^2 + m_2 \phi_{2i}^2}$$

$$= \frac{1}{\phi_{2i}} \left(\frac{\phi_{1i}}{\phi_{2i}} + R_m\right)/[\left(\frac{\phi_{1i}}{\phi_{2i}}\right)^2 + R_m] \quad (8)$$

Or, one may write

$$P_i \phi_{2i} = \left(\frac{\phi_{1i}}{\phi_{2i}} + R_m\right)/[\left(\frac{\phi_{1i}}{\phi_{2i}}\right)^2 + R_m] \quad (9)$$

Finally, the acceleration response ($\ddot{x}$) for both masses can be written in the following form:

$$\ddot{x}_j = P_1 \phi_{j1} \ddot{q}_1 + P_2 \phi_{j2} \ddot{q}_2 , \; j = 1, 2 \quad (10)$$

Here subscript j represents the mass number 1 or 2, q_1 and q_2 are the generalized coordinates for modes 1 and 2, respectively, and overdot shows that it is a time derivative.

It has been shown in Reference 4 that the response of the large mass system (M_1) is usually small and bounded. It is only when dealing with the subsystem response for the small mass m_2 that the response becomes in question. Hence, this paper will concentrate on studying the response of m_2 which is

$$\ddot{x}_2 = P_1\,\phi_{21}\,\ddot{q}_1 + P_2\,\phi_{22}\,\ddot{q}_2 \qquad (11)$$

It is to be noted here since the two modes have very close frequencies, the response indicated by the generalized coordinates q_1 and q_2 will have similar magnitudes and probably even with the same signs. Therefore, to study Eq. (11), it is essential that the sign relationship be known for $P_1\,\phi_{21}$ and $P_2\,\phi_{22}$. To do so, one may write

$$\frac{P_1\,\phi_{21}}{P_2\,\phi_{22}} = \frac{\dfrac{\phi_{11}}{\phi_{21}} + R_m}{\left(\dfrac{\phi_{11}}{\phi_{21}}\right)^2 + R_m} \qquad \frac{\dfrac{\phi_{12}}{\phi_{22}} + R_m}{\left(\dfrac{\phi_{12}}{\phi_{22}}\right)^2 + R_m}$$

$$= \frac{\dfrac{\phi_{11}}{\phi_{21}} + R_m}{\dfrac{\phi_{12}}{\phi_{22}} + R_m} \cdot \frac{\left(\dfrac{\phi_{12}}{\phi_{22}}\right)^2 + R_m}{\left(\dfrac{\phi_{11}}{\phi_{21}}\right)^2 + R_m} \qquad (12)$$

Therefore, to identify the sign relationship of $P_1\,\phi_{21}$ and $P_2\,\phi_{22}$, one has only to study the signs of

$$\frac{\phi_{11}}{\phi_{21}} + R_m \text{ and } \frac{\phi_{12}}{\phi_{22}} + R_m$$

To do so, the denominator of the right hand side of Eq. (7) may be manipulated with the help of Eq. (1) to yield the following equation:

$$1 + R_K - \frac{\pi_1^2}{\omega_1^2} = \frac{1}{2}\left\{(1 + R_K - R_f^2) + [(1 + R_K - R_f^2)^2 + 4R_K^2/R_m]^{\frac{1}{2}}\right\} \qquad (13)$$

Hence, for $R_f \leq 1$,

$$1 + R_K - \frac{\pi_1^2}{\omega_1^2} > 0 \qquad (14)$$

Also, for $R_f > 1$ and $R_K > 0$,

$$[(1 + R_K - R_f^2)^2 + 4R_K^2/R_m]^{\frac{1}{2}} > (1 + R_K - R_f^2) \qquad (15)$$

This implies that Eq. (14) is valid also for $R_f < 1$.

Consequently,

$$\frac{\phi_{11}}{\phi_{21}} + R_m > 0 \tag{16}$$

Similarly, the use of Eqs. (1) and (7) results in an equation comparable to Eq. (13) but with a different sign in front of the bracket "[ ]" term. By comparing terms in the equation, it is obvious that

$$1 + R_K - \frac{\pi_2^2}{\omega_1^2} = \frac{1}{2} \; (1 + R_K - R_f^2) - [(1 + R_K - R_f^2)^2 + 4R_K^2/R_m]^{\frac{1}{2}} < 0 \tag{17}$$

That is, for all R_f (whether it is larger than, equal to, or less than zero), Eq. (17) is a negative quantity.

Consequently one concludes that

$$\frac{\phi_{12}}{\phi_{22}} + R_m < 0 \tag{18}$$

Finally, by substituting Eqs. (16) and (18) into Eq. (12), one arrives at the following conclusion:

$$\frac{P_1 \, \phi_{21}}{P_2 \, \phi_{22}} < 0 \tag{19}$$

In other words, $P_1 \, \phi_{21}$ and $P_2 \, \phi_{22}$ will always have a different sign from each other. This is an important conclusion, one that should have a direct bearing in the study of the combined response motions.

DYNAMIC RESPONSE OF THE TWO MASS SYSTEM WITH CLOSELY SPACED MODES

The dynamic response of the two mass system discussed in Chapter 2 can be written as the following

$$\ddot{x}_2 \; (t) = \sum_{i=1}^{2} \ddot{q}_i \; \phi_{i2} \; P_i \tag{20}$$

where $\ddot{q}_i$ is the second derivative of the generalized coordinate q_i with i = 1, 2 representing the first and the second mode, respectively. Only response of the second (lighter) mass is represented in Eq. (20) since the response in first (heavier) mass can be shown to have a response value substantially less than mass 2, and is, therefore, less of an interest.

The second derivative of the generalized coordinate can be written in the following form

$$\ddot{q}_i = \pi_i \int_0^t \ddot{u}_q \; (\tau) \; e^{-\beta \, \pi_i \, (t - \tau)} \sin \pi_i \; (t - \tau) \; d\tau \tag{21}$$

where π_i is defined in Eq. (1) and β is the modal damping and has been assumed the same for both modes. This is a very realistic assumption in view that the two modes are closely spaced where a Rayleigh damping approach would imply essentially the same damping values for both modes. Secondly, the two modes would exhibit approximately the same total energy (e.g., the larger mass with smaller vibration response and the smaller mass with the larger vibration response). Therefore, the two damping values can be assumed the same without loosing generality.

Also in Eq. (21), $\ddot{u}_g$ is the acceleration base input. It is assumed the same for both supports. For supports with different inputs, the treatment would be more involved. However, should the support inputs be drastically different, the subsystem/system response could be alleviated by using the multiple support spectra approach. Only when the support inputs are similar would large response be anticipated for a secondary system. Therefore, the assumption of an uniform base input is considered to be representative.

Combining Eqs. (19), (20), and (21), one may write

$$\ddot{x}_2 = P_2\, \phi_{22}\, (-c\, \ddot{q}_1 + \ddot{q}_2) \tag{22}$$

where c is defined as

$$c = P_1\, \phi_{21} / (P_2\, \phi_{22}) \tag{23}$$

and is a positive quantity.

It is obvious that from Eq. (22), $\ddot{x}_2$ is entirely dependent on the magnitudes of $\ddot{q}_1$ and $\ddot{q}_2$ and the time when the two terms are combined.

To study Eq. (22), it is important to point out that c can be shown to be greater than 1; and the value is close to 1 when the two modal frequencies are within the resonance range (within 10 percent of each other), If, however, c is substantially larger than 1 (a possibility for very small stiffness ratio R_K), then even when $\ddot{q}_1$ and $\ddot{q}_2$ have a similar magnitude, the two terms in Eq. (22) will not be of the same magnitude. In such a case, it is of a lesser concern how the two terms should be properly combined, since the first term will most likely govern.

When c is close to 1, Eq. (22) can be rewritten as the following

$$\ddot{x}_2 = P_2\, \phi_{22}\, [(-\ddot{q}_1 + \ddot{q}_2) - (c - 1)\, \ddot{q}_1] \tag{24}$$

It is obvious that c is assumed to be larger than or equal to 1. Should this assumption be invalid, then the multiplier $P_2\, \phi_{22}$ should be modified to $P_1\, \phi_{21}$ so that the revised c will be larger than 1.

As a conservative measure, Eq. (24) may then be treated in the following manner:

$$\left|\ddot{x}_2\right|_{max} \le P_2\, \phi_{22}\, \left[\, \left|\ddot{q}_1 - \ddot{q}_2\right|_{max} + (c - 1)\left|\ddot{q}_1\right|_{max} \right] \tag{25}$$

where $|\cdot|$ represents the absolute value.

The overconservatism of Eq. (25) depends primarily on the value of c.

The first part of Eq. (25) can be expanded as:

$$\ddot{q}_1 - \ddot{q}_2 = \int_0^t \ddot{u}_g\, (\tau)\, [\pi_1\, e^{-\beta\, \pi_1\, (t - \tau)} \sin \pi_1\, (t - \tau)$$

$$-\pi_2\, e^{-\beta\, \pi_2 (t - \tau)} \sin \pi_2\, (t - \tau)]\, d\tau \tag{26}$$

To further develop Eq. (26), some approximation has to be made.

First, the integrand in Eq. (26) represents two similar (but with opposite phase) modulating sine waves; and π_1 and π_2 are very close to each other (within, say, $\pm$ 10 percent using the closely spaced rule). The magnitude of the integrand will be a maximum if it is assumed that the multiplier π_1 and π_2 is equal to π where

$$\pi^2 = \frac{1}{2}\, (\pi_1^2 + \pi_2^2) \tag{27}$$

As seen from Eq. (1), this requires that the second term (bracketed [ ] term) be dropped for π_1^2 and π_2^2. This in fact makes π_1^2 somewhat larger and π_2^2 somewhat smaller. Hence, the integrand will result in a larger value than indicated by Eq. (26).

Secondly, recognizing that the exponential terms form the envelopes of the modulating sine waves, these envelopes are slowly varying time functions. Therefore, the π_1 and π_2 in the exponential functions can be replaced by the averaging frequency showing in Eq. (27) without significant effects.

Consequently, Eq. (26) may be modified as follows:

$$\ddot{q}_1 - \ddot{q}_2 = \int_0^t \ddot{u}_g(\tau)\, \pi\, e^{-\beta \pi (t - \tau)} [\sin \pi_1 (t - \tau) - \sin \pi_2 (t - \tau)]\, d\tau \tag{28}$$

Eq. (28) may be rearranged to yield

$$\ddot{q}_1 - \ddot{q}_2 = 2\pi \int_0^t \ddot{u}_g(\tau)\, e^{-\beta \pi (t - \tau)} \cos \pi (t - \tau) \sin \eta (t - \tau)\, d\tau \tag{29}$$

where

$$\eta = \frac{\pi_2 - \pi_1}{2} \tag{30}$$

The integrand in Eq. (29) represents a modulating sine beat with a beat frequency of η and the motion modulated by the exponential function. Since η is a small value, the beat motion has a rather long period. When the two frequencies differ by less than 10 percent of each other, the beat frequency η is approximately 5 percent of the basic frequency of π. That is, there will be twenty basic frequency waves occurring before the occurrence of one full beat frequency.

Eq. (29) can be expanded as the following:

$$\ddot{q}_1 - \ddot{q}_2 = 2\pi \int_0^t \ddot{u}_g(\tau)\, e^{-\beta \pi (t - \tau)} \cos \pi (t - \tau) [\sin \eta t \cos \eta \tau - \cos \eta t \sin \eta \tau]\, d\tau \tag{31}$$

Eq. (31) may be further simplified by using the following approximation:

$$\cos \pi (t - \tau) \cos \eta \tau$$

$$= \frac{1}{2} [\cos (\pi t - (\pi - \eta) \tau) + \cos (\pi t - (\pi + \eta) \tau)]$$

$$\simeq \cos \pi (t - \tau) \tag{32}$$

for small η when compared with π. This assumption is valid for closely spaced modes since η represents the difference of the two frequencies (which is within plus or minus 10 percent of each other) divided by 2 (Eq. (30)). Therefore, η is likely to be within 5 percent of π. Also, the two terms in the first part of Eq. (32) consist of η with opposite signs. That is, one term will have neglected η on the up side then the other would have neglected η on the down side. They tend to compensate each other. Therefore, Eq. (32) is a fairly realistic approximation.

Similarly,

$$\cos \pi (t - \tau) \sin \eta \tau$$

$$= \frac{1}{2} [\sin (\omega t - (\omega - \eta) \tau)$$

$$- \sin (\omega t - (\omega + \eta) \tau)] \quad (33)$$

$$\simeq 0$$

for small η when compared with π.

Both Eqs. (32) and (33) can be confirmed by simply integrating the terms over the region of 0 to t for the variable τ. When η is assumed small compared with π, the approximate results in both equations will be equal to the exact solutions.

Substitution of Eqs. (32) and (33) into Eq. (31) results in the following equation:

$$\ddot{q}_1 - \ddot{q}_2 = 2\pi \sin \eta t \int_0^t \ddot{u}_g (\tau) e^{-\beta \pi (t - \tau)} \cos \pi (t - \tau) d\tau \quad (34)$$

which can be expanded to yield

$$\ddot{q}_1 - \ddot{q}_2 = 2\pi \sin \eta t e^{-\beta \pi t} \int_0^t \ddot{u}_g (\tau) e^{\beta \pi \tau}$$

$$(\cos \pi t \cos \pi \tau + \sin \pi t \sin \pi \tau) d\tau \quad (35)$$

Let

$$A = \int_0^t \ddot{u}_g (\tau) e^{\beta \pi \tau} \cos \pi \tau d\tau \quad (36)$$

and

$$B = \int_0^t \ddot{u}_g (\tau) e^{\beta \pi \tau} \sin \pi \tau d\tau \quad (37)$$

Eq. (35) becomes

$$\ddot{q}_1 - \ddot{q}_2 = 2\pi \sin \eta t e^{-\beta \pi t} (A \cos \pi t + B \sin \pi t) \quad (38)$$

Using the same approach as used in Reference 6 (p. 393), Eq. (38) can be shown to be

$$\ddot{q}_1 - \ddot{q}_2 = 2\pi \sin \eta t e^{-\beta \pi t} (A^2 + B^2)^{\frac{1}{2}} \cos (\pi t - \alpha) \quad (39)$$

where

$$\alpha = \tan^{-1} \frac{B}{A} \quad (40)$$

As has been illustrated in Reference 6, the term $(A^2 + B^2)^{1/2}$ is nearly a constant after the initial rise time; and it relates to the sum of the potential and kinetic energy in the following manner

$$m\varepsilon = V + KE = \frac{m}{2}(A^2 + B^2) \tag{41}$$

where m is the mass, ε is the energy for the unit mass, and V and KE symbolize the potential and kinetic energy, respectively.

Therefore, Eq. (39) can be related to the total energy of a unit mass with a frequency π in the following manner:

$$\ddot{q}_1 - \ddot{q}_2 = 2\ (2\varepsilon)^{\frac{1}{2}}\ \pi \sin \eta t\ e^{-\beta \pi t} \cos(\pi t - \alpha) \tag{42}$$

The absolute maximum value of Eq. (42) can then be written as

$$\left|\ddot{q}_1 - \ddot{q}_2\right|_{max} \le 2\pi\ (2\varepsilon)^{\frac{1}{2}} \left|\sin \eta t\ e^{-\beta \pi t} \cos(\pi t - \alpha)\right|_{max}$$

$$\le 2\pi\ (2\varepsilon)^{\frac{1}{2}} \left|\sin \eta t\ e^{-\beta \pi t}\right|_{max} \left|\cos(\pi t - \alpha)\right|_{max}$$

$$\le 2\pi\ (2\varepsilon)^{\frac{1}{2}} \left|\sin \eta t\ e^{-\beta \pi t}\right|_{max} \tag{43}$$

As developed in Reference 6, $(2\varepsilon)^{1/2}\pi$ is simply the acceleration response spectrum value $S_a(\pi)$ at the frequency π.

To evaluate the absolute maximum of

$\sin \eta t\ e^{-\beta \pi t}$

one may take the partial derivative of the product with respect to t and then set the solution to zero and solve for t.

This results in the time (t*), where the product will be a maximum

$$t^* = \frac{1}{\eta} \tan^{-1}\left(\frac{\eta}{\beta \pi}\right) \tag{44}$$

Substitution of Eq. (44) into Eq. (43) yields

$$\left|\ddot{q}_1 - \ddot{q}_2\right|_{max} \le 2\ S_a(\pi) \sin \eta t^*\ e^{-\beta \pi t^*} \tag{45}$$

where

$$\sin \eta t^* = \frac{\eta}{\beta \pi} \Big/ \left[1 + \left(\frac{\eta}{\beta \pi}\right)^2\right]^{\frac{1}{2}} \tag{46}$$

Eq. (45) is not that much different from the equation derived in Reference 3. However, none of the assumptions made in Reference 3 were utilized herein. Therefore Eq. (45) can be used without any restrictions on mass ratio, frequency ratio, response time and earthquake durations.

Finally, by substituting Eq. (45) into Eq. (25) one arrives at the absolute maximum response for the secondary mass which can be written as

$$x_{2\,max} \leq P_2\,\phi_{22}\,\left\{2\,S_a(\pi)\,\sin\eta\,t^*\,e^{-\beta\,\pi\,t^*} + (c-1)\,S_a(\pi_1)\right\}$$

$$\simeq P_2\,\phi_{22}\,\left\{2\,\sin\eta\,t^*\,e^{-\beta\,\pi\,t^*} + (c-1)\right\}\,S_a(\pi) \qquad (47)$$

for π_1 close to π.

Eq. (47) should be used for two closely spaced modes. Conversely, if it can be shown that for the two closely spaced modes, $P_1\,\phi_{12}$ and $P_2\,\phi_{22}$ have the opposite sings at nodes with the predominant motions, Eq. (48) can also be used to supplement the formulas recommended for closely spaced modes in Reference 1.

In view that Eq. (47) has been derived at using generally conservative assumptions, it most likely results in an upper bound estimate for the secondary mass response. To arrive at a possible reduction from the closely spaced mode formulation when Eq. (47) is not used, it is acceptable then to assume that the two modes coincide, i.e., the ϵ's in the cross product terms of the closely spaced modes formulas are 1. With such an assumption, the final combination becomes an absolute sum of the two modes. Hence, the absolute maximum response of the secondary mass becomes

$$|x_2^*|_{max} = \left(|P_1\,\phi_{12}| + |P_2\,\phi_{22}|\right)\,S_a(\pi)$$

$$= |P_2\,\phi_{22}|\,(c+1)\,S_a(\pi) \qquad (48)$$

The ratio of the absolute sum resulted from the closely spaced modes formulations and the more realistic solution recommended in Eq. (47) becomdes then a reduction factor R, which is

$$R = \frac{|\ddot{x}^*|_{max}}{|\ddot{x}_2|_{max}} = \frac{(c+1)}{2\,\sin\eta\,t^*\,e^{-\beta\,\pi\,t^*} + (c-1)} \qquad (49)$$

Eq. (49) contains η and π which are functions of R_K, R_f, and R_m. It can be simplified by recognizing that

$$\eta = \frac{1}{2}\,(\pi_2 - \pi_1) = \frac{1}{4}\,(\pi_2^{\,2} - \pi_1^{\,2})\,/\,\pi \qquad (50)$$

Using Eq. (1) and making the assumption that the average frequency (π) of the coupled model differs only by a negligible amount from the main system frequency ω_1, one concludes that

$$\frac{\eta}{\beta\,\pi} = \frac{1}{4\,\beta}\,[(1 + R_K - R_f^{\,2})^2 + 4\,R_K^{\,2}/\,R_m]^{\frac{1}{2}} = \lambda \qquad (51)$$

Hence,

$$\beta\,\pi\,t^* = \beta\,\pi\,\frac{1}{\eta}\,\tan^{-1}\left(\frac{\eta}{\beta\,\pi}\right) = \frac{\tan^{-1}\lambda}{\lambda} \qquad (52)$$

Also, from Eq. (46), one has

$$\sin\eta\,t^* = \lambda/[1 + \lambda^2]^{\frac{1}{2}} \qquad (53)$$

Consequently, Eq. (49) becomes

$$R = \frac{(c + 1)\,(1 + \lambda^2)^{\frac{1}{2}}}{2\,\lambda\, e^{-\tan^{-1}\lambda/\lambda} + (c - 1)\,(1 + \lambda^2)^{\frac{1}{2}}} \qquad (54)$$

Eq. (54) can be used to reduce the responses computed for the secondary mass when under subsystem/system interaction mode. That is, when the secondary system has a frequency close to one of the main system frequency and the secondary system response has been observed to be greatly affected by the main system.

Finally, it is to be noted that, when c is assumed to be equal to 1 (for such subsystem/system mass ratios to be extremely small), Eq. (54) reduces to the form exactly the same as recommended in Reference 3. However, the derivation in Reference 3, in addition to the requirement of very small mass ratio, also requires that the duration of the earthquake be short and the secondary mass response be occurring after the earthquake motion has ended. Since the only assumption used in the derivation of Eq. (54) is that the frequencies are close (within $\pm$ 10 percent), Eq. (54) can be applicable to any subsystem/system interaction analysis with closely spaced modes without any further limitations as required in Reference 3.

NUMERICAL EXAMPLES AND CONCLUDING REMARKS

Eq. (54) has presented the reduction factor for the secondary system response when computed using the absolute sum rule. However, since absolute sum is only a result of two modes being extremely close to each other, a proper reduction factor applicable for closely spaced modes remains to be derived. To do so, one may review the following inequalities

$$|\ddot{x}_2^{\,*}|_{max} \geq |\overline{\ddot{x}_2}|_{max} \geq |\overline{\overline{\ddot{x}_2}}|_{max} \qquad (55)$$

where $|\ddot{x}_2^{\,*}|_{max}$ is the absolute sum solution.

Presented in Eq. (48), $|\overline{\ddot{x}_2}|_{max}$ indicates the closely spaced modes solution using a closely spaced mode formula, and $|\overline{\overline{\ddot{x}_2}}|_{max}$ is the square root sum of the squares solution for the modes.

Obviously, to insure a conservative solution for all cases of closely spaced modes, it is necessary that the reduction factor be the smallest. That is, when $|\overline{\overline{\ddot{x}_2}}|_{max}$ is divided by the reduction factor R, the result should be greater than the true solution $|\ddot{x}_2|_{max}$ (computed from Eq. (47)). This implies that for closely spaced modes, the reduction factor should be computed using Eq. (48) and divided by the square root sum of the squares solution. That is,

$$R = \frac{|\overline{\overline{\ddot{x}_2}}|_{max}}{|\ddot{x}_2|_{max}}$$

$$= \frac{(c^2 + 1)^{\frac{1}{2}}\,(1 + \lambda^2)^{\frac{1}{2}}}{2\lambda e^{-\tan^{-1}\lambda/\lambda} + (c - 1)\,(1 + \lambda^2)^{1/2}} \qquad (56)$$

where it has been assumed that the closely spaced modes have response values essentially the same as at the average frequency π. This is a realistic assumption in view that the two modes are no more than 10 percent apart and the use of square root sum of the squares rule yields smaller reduction factor than using the cross coupling term (ϵ) in the numerator of Eq. (56).

It should be pointed out here that Eq. (56) could be too conservative to certain closely spaced modes, especially if the two modes are quite close. In such a case, the closely spaced modes formula should then be used to replace the square root sum of the squares formula in deriving Eq. (56). However, since such derivation depends on each individual case of frequency, damping, and earthquake motion duration, its application is left to the user when he so desires.

To show how Eq. (54) can be used in the actual application, Figures 2 and 3 have plotted the reduction factor versus λ. Figure 2 is for the case where the two frequencies are very close where closely spaced modes solution is approaching that of absolute sum value. Whereas Figure 3 is for the case where the frequencies are not so close such that closely spaced modes solution is approaching the square root sum of the squares case. A comparison of Figures 2 and 4 shows that there is a large difference between the two cases. For instance, for a λ of 0.1 and when the modal response ratio (c) is 1.0, the response reduction for the resonance case is about 26, and it is only 20 for the closely spaced modes case. Therefore, it may be desirable to compute R using the closely spaced formula instead of the square root sum of the squares rule. Nevertheless, both Figures 2 and 3 show dramatic response reduction even for a large c, say 2, except for a λ greater than 3. It should be noted here that when c is large, it signals that the two modes have substantial difference in the modal response magnitudes. Hence one mode will have more a pronounced effect on the final response than the other mode. It is, therefore, reasonable to expect that the reduction factor will be smaller. Also, when λ is larger than 3, Figure 3 shows that the reduction factor may be less than 1, if the curves are allowed to be continued. This implies that the use of square root sum of the squares formula may not be conservative. The use of closely spaced modes formula without any response reduction may indeed be necessary in such a case.

Figures 2 and 3 are based on a damping value of 2 percent. Since piping systems could have damping values ranging from 0.5 percent to 2 percent for OBE, depending on pipe sizes, it may be desirable to evaluate the effect of damping on the response reduction factor. However, a glance at Figure 4 shows that damping has only a very minimal effect on the response reduction factor.

In Figures 2, 3 and 4, λ is used as the abscissa. Since λ is a function of mass ratio, stiffness ratio and frequency ratio, the influence by these factors has to be studied. For this purpose, Figure 5 has been plotted which shows λ/λ^* versus a, where λ^* is the λ value computed for $R_f = 1.0$ and $R_k = R_f^2 R_m$, λ is the value computed for a given R_m with $R_f = 0.9$ and ratio $R_k = a R_f^2 R_m$. Figure 5 shows that λ value is essentially constant for small mass ratios at and below 0.002. This indicates that stiffness ratio plays only a minor role for very small mass ratios. Also, the biggest influence of the stiffness ratio seems to come from when a is larger than 0.4 for mass ratios at and below 0.02.

A second study of λ/λ^* versus a is presented in Figure 6. In this figure, the λ/λ^* is seen essentially unchanged for R_f at and below 0.8 for a mass ratio of 0.01. Again, similar to Figure 5, the ratio changes more rapidly when a is at and above 0.4 for R_f to be as high as 0.9.

Figures 5 and 6 illustrate that when a is less than 0.4 with R_f less than 0.9 and R_m less than 0.02, λ can be treated the same as λ^* (i.e., at resonance). Under these conditions, the response reduction factor becomes large even when the subsystem and the system are not within the resonant range. This shows that the coupling of the two systems may not be necessary since a large reduction factor would reduce the coupled response to a low level. Conversely, when a is above 0.4, Figures 5 and 6 both show large increases in the ratio of λ/λ^*. A larger ratio of λ/λ^* means that the response reduction becomes smaller, as evidenced in Figures 2 and 3. This means that whatever response is computed in the coupled model, it could not be reduced substantially, and the response may be real. Therefore, coupled analysis could be a necessity when a is larger than 0.4.

In order to compare the effect of different response reduction factors (e.g., Eqs. (54) and (56)), Table 1 has been prepared. In this table, the two