

Block Theory and Its Application to Rock Engineering

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To

Dr. John Wade Bray
Imperial College, London

and

Dr. Kiang, Tse-Hang
University of Peking

Preface

As the authors began their collaboration, which happened by the lucky accident of a well-timed international visit, it rapidly became obvious that the unfolding story would require more than the limited space and scope of journal articles. Although the succession of development and applications could be stated in a long series of articles, it would be the rare reader who could make his or her way through all in the proper order. Therefore, the authors decided to write a book. This work was intended, right from the beginning, to point toward the practical applications of interest to tunnelers, miners, and foundation engineers. Yet, because the threads of the developments are new, it seemed important to show the theoretical foundation for important steps, lest there be disbelievers. In the end, the methods are so easily applied, and the conclusions so simple in their statement, that one could entertain doubts about the universality, completeness, and rigor of the underpinnings if the complete proofs of theorems and propositions were not included. It is not necessary to follow all the steps of all the proofs in order to use the methods of block theory, but they are there, in appendices at the end of most chapters, and in some cases, within the chapters themselves when the material is especially fundamental.

The applications of block theory in planning and design of surface and underground excavations and in support of foundations are illustrated in a large number of examples. Computer programs (available from the authors*) can be used by the reader to duplicate these illustrations. Alternatively, the examples

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can be worked using manual stereographic projections. It should be understood that the theory is independent of the methods of its application. This book is not about stereographic projection or computation as such, but about the geometric facts of intersecting discontinuities penetrating a three-dimensional solid. The advantage of stereographic projection is its ease and economy. The distinct advantage of computational solution is automatic operation, which allows its incorporation in larger enterprises.

The authors have developed a number of computer programs for use with this material, but limitations of space forbade their inclusion here. Interested readers are encouraged to write for further information. (See footnote on previous page.)

The notation employed here may be somewhat new to many readers, since topology and set theory are not standard components of courses in engineering mathematics. However, the level of mathematical skills actually required to understand the material is not advanced, and the book presents all the background required to understand all the developments. Readers should be familiar with matrix notation and vector operations. The mathematical basis of stereographic projection is presented more completely here than in any other book known to the authors. However, some familiarity with simple steps in applying the stereographic projection, as presented in some of the references cited, will certainly speed comprehension.

On first learning of the ideas of block theory, after hearing an introductory lecture, a valued colleague suggested that we were then "at the tip of an iceberg." We have since exposed considerably more material but have hardly begun to exhaust the possibilities. We hope that some of you who read this material will discover yet new directions and possibilities in your particular specialties.

Many colleagues have assisted in the development of this book. We are especially grateful to Dr. Bernard Amadei, Dr. Daniel Salcedo, William Boyle, and Lap-Yan Chan. Partial support was provided by the California Institute for Mining and Mineral Resources Research, Douglas Fuerstenau, Director, and by the National Science Foundation, Civil and Environmental Technology Program, Charles Babendreier, Program Officer. A grant from Horst Eubacher and Associates was also appreciated. The authors wish to thank Ms. Marcia Golner for devoted typing of a laborious manuscript.

Richard E. Goodman
Gen-hua Shi

Notation and Abbreviations

\mathbf{A}	A vector	$U(\hat{n}, Q)$	The upper half-space whose boundary is normal to \hat{n} and passes through Q
\hat{a}	A unit vector	\emptyset	The empty set
$A \cap B$	The intersection of A and B	JP	Joint pyramid, a closed set
$A \cup B$	The union of A and B	EP	Excavation pyramid, a closed set
$A \in B$	A is an element of set B	BP	Block pyramid
or $B \ni A$		SP	Space pyramid (= \sim EP), an open set
$A \notin B$	A is not an element of B	$A \neq B$	A is not equal to B
or $B \not\ni A$		$A \parallel B$	A is parallel to B
$A \subset B$	A is a subset of B ; A is contained in B	$A \parallel\!\!\parallel B$	A is parallel and in the same direction as B
$A \not\subset B$	A is not a subset of B	α, β	Dip and dip direction of a plane
$A \supset B$	B is a subset of A ; A contains B	$(A)^T$	The matrix transpose of (A)
$A \not\supset B$	B is not a subset of A	(x, y, z)	Coordinates of the tip of a vector whose tail is at $(0, 0, 0)$
$\sim B$	The complement of B —the set of elements not included in B		
$U(\hat{w})$	The upper half-space of \hat{w} , the boundary of which passes through the origin		
$L(\hat{w})$	The lower half-space of \hat{w} , the boundary of which passes through the origin		

R	Radius of the reference sphere	$\mathbf{A} \cdot \mathbf{B}$	Scalar product of \mathbf{A} and \mathbf{B} (dot product)
r	Radius of a circle on the stereographic projection	W_i	Wall i
\mathbf{r}	Resultant applied force	E_{ij}	The interior edge formed by the intersection of W_i and W_j
n	The number of joint sets	C_{ijk}	The interior corner formed by the intersection of W_i , W_j , and W_k
\hat{v}	The unit normal vector pointed into a rock block	E_{ij}	An exterior edge at the intersection of two chambers
\hat{w}	The unit normal vector pointed into space	C_{ijk}	An exterior corner at the intersection of two chambers
\hat{n}_i	Upward unit normal to plane i		
$\mathbf{A} \times \mathbf{B}$	Cross product of \mathbf{A} and \mathbf{B}		

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chapter 1

Introduction

Rock is one of the oldest yet one of the least well understood construction materials. Although ancient civilizations demonstrated remarkable ability in cutting and erecting stone monuments, excavating tunnels, erecting fortifications, and creating sculptures in rocks, it is doubtful that these architects and builders ever created analytical procedures to govern their engineering activities. In an age of technological advance, with precise and powerful analytical tools at our fingertips, it is reasonable to hope for a more fundamental basis for rock engineering. Yet we must cope with hard facts that make our task complex. First, rock as an engineering material is variable and all-encompassing since we find, within the earth, rock materials possessing all classes of mechanical behavior. Second, rocks differ significantly from most other materials with which we build in possessing numerous flaws and weaknesses that together tend to interrupt the continuity of material and divide it into domains of different types. Common rock displays so many planes of weakness as to be essentially a collection of separate blocks tightly fitted in a three-dimensional mosaic. We call such material “discontinuous rock.”

An example of how the network of discontinuities in rock affects engineering performance of excavations is shown in Fig. 1.1, a photograph of the spillway for old Don Pedro Dam, California. The concrete training wall of the spillway can be seen in the upper right-hand corner of the figure. This structure was designed to conduct water along a single bedding plane down the limb of a roughly cylindrical fold in quartzite. In this ingenious way the water would flow along a natural surface all the way to the river. Unfortunately, crossing joints carried water down to a parallel surface forming the bottom of the folded layer.

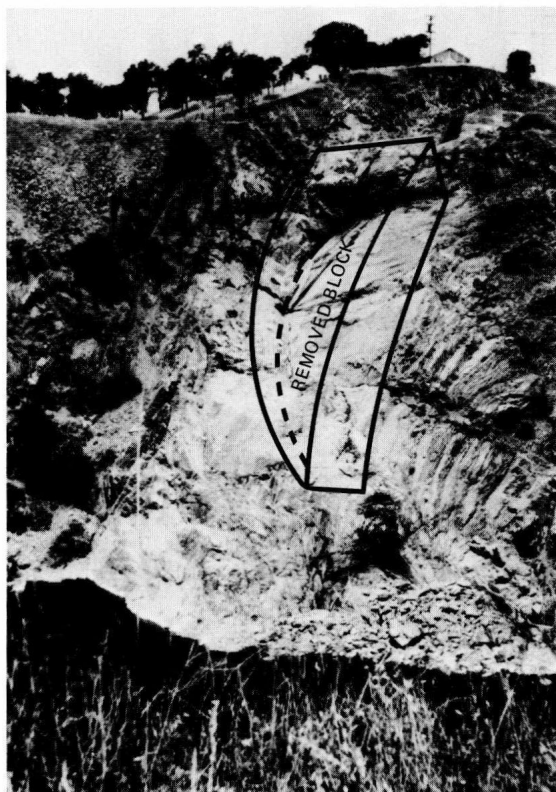


Figure 1.1 Rock block eroded from spillway of old Don Pedro Dam, California.

The system of cross joints and bedding surfaces above and below the uppermost quartzite bed delimited a physical rock block, hundreds of cubic meters in volume, which was lifted and removed from its position by the running water. Consequently, instead of a smooth water course, the picture shows a cliff below the edge of the training wall. Further spills are now able to quarry the rock even more effectively, and if unchecked, could destroy the training wall and work backward to release the entire contents of the reservoir. This will not happen because the dam has been replaced by a larger structure downstream and the place is now submerged. But it serves to illustrate how the movement of a key joint block can create a worsening situation.

In discontinuous rock, we assume that the motions of points within individual blocks are derived mainly from rigid body motions of the block system. This is an acceptable assumption in hard rocks such as granite, quartzite, gneiss, limestone, slate, and other rock types, where the rock material is often considerably stronger than good concrete. There are softer rocks, however, in which soil-like deformations of the blocks are at least as important as inter-block translations and rotations. This is true, for example, in Tertiary age sandstones, claystones, tuffs, in chalk or very young limestone, and in decomposed

or altered rocks of any kind, in which relatively large rock deformation accompanied by new crack growth cannot be ignored even if the rock mass is highly fractured. As in all systems that have ideal end members, the usual rock mass encountered in excavation work lies somewhere between the extremes considered. To those who live and work in the real world, we say only that the methods described in this book apply to an idealized mode, whose validity will always have to be tested in practice. On the other hand, we would not burden you with reading this unless our experiences demonstrated a wide applicability for this theory.

The types of discontinuities that chop up the rock mass into blocks depend on the scale of interest. The theory of blocks can be applied to explain fabrics observed in rock specimens, in which case the controlling planes of discontinuity are *fissures* and *microfaults*. In detailed rock excavation, to shape the interior of an underground gallery, for example, individual *joints* parallel to the bedding planes, or from sets in other spatial attitudes, may control the outcome of the work. In constructing a large underground room in a mine or for a hydroelectric power project, important through-going joints, sheared zones, disturbed zones along *contacts* of different rock types, and *faults* will bound the individual blocks of the rock mass. Finally, the structural geologist addresses systems of blocks kilometers on edge formed by systems of major and minor faults and major formational contacts. Most of the examples in this book and the main experience of the authors concern excavations in which it is the joint system that decides the rock blocks. This implies that the most immediate applications of this theory will be in rock engineering for underground and surface space, transportation routes, hydroelectric power, water supply, and mining.

EXCAVATIONS

An excavation is a new space created by removing earth or soil from its natural place. The objective may be to use the excavated material for fill or in manufacture, or to gain space at or below the surface. Occasionally, by advanced planning, both objectives are combined in one project: for example, when a quarry is intended to be converted into a park or development after the product has been removed to a predefined contour. In any event, the space created by removal of rock has to behave according to the design, or the project or excavation will founder in delay and excessive cost. In civil engineering work, the design invariably requires that the excavated space remain stable during the process of excavation, and usually for the lifetime of the engineering work, which may be on the order of a hundred years. Examples of permanent civil engineering excavations are underground space complexes for industrial use or storage; chambers for hydroelectric turbines and transformers; tunnels for roads, water supply, and water power; shafts and tunnels for subterranean sewer systems; open cuts for highways, railroads, pipelines, and other transportation routes; open cuts for spillways of reservoirs; and open cuts to site surface structures.

Temporary excavations that will eventually be backfilled with earth or concrete, or allowed to collapse, provide surface cuts for foundations of buildings and dams and for quarries attached to specific construction projects. In all these excavations the loosening of rock blocks along the contour of the excavation is to be avoided to achieve safe, sound conditions for the users of the space.

In the mining field it is sometimes preferred to create intentionally unstable excavations so that the costs of rock breakage will be minimized. The idea will be to initiate self-destruction of the rock through a properly executed initial excavation that triggers the caving process. There are also many uses for permanent underground mine openings: for example, as shafts to gain access to a long-life mine, for hoist machinery, and for underground crushers. Moreover, even though mines that exhaust the ore in a few tens of years can one day be abandoned, the requirements for safety usually dictate that they be excavated as if they were to be stable, permanent openings.

The theory developed in this book applies to all the types of excavations mentioned above. In the case of cuts and underground openings that are intended as permanent features, block theory will permit the design of excavation shape, size, and orientation, and a system of internal supports (if necessary) that provides stability at minimum cost. In the case of mining openings that are intended to cave, block theory will help choose initial undercuts to maximize the chance of natural rock caving. This theory relates to the movement of joint blocks that are liberated by the artificial surfaces to be excavated. This is the correct approach wherever the mode of failure of the excavation involves the movement of rock blocks. To appreciate the place of this approach in the total scheme of things, it is useful to examine briefly all the important failure modes that can actually occur in an excavation.

MODES OF FAILURE

Failures of underground excavations are uncommon, fortunately, but all underground excavations undergo localized “overbreak” as the desired dimensions of the opening readjust themselves to geological realities. Surface excavations are less stable than underground openings and experience failures more frequently. But rock falls in the confined space of an underground chamber or tunnel are apt to be more troublesome than even large rock movements into surface cuts. It is the flow of stresses tangentially around the underground opening that makes it relatively safer than its surface counterpart, which lacks a completely encircling contour to confine rock movements driven by gravity. In very deep mines, or excavations into weak rock, these stresses may themselves initiate new cracking, or even violent rock bursts. More often, however, the tangential stress around the opening tends to hold the potentially moving rock blocks in place, thereby acting as a stabilizing factor. In fact, it is usually the absence of continuous compression around the opening, rather than its presence in elevated magnitudes, that permits serious overbreak and failure. For example,

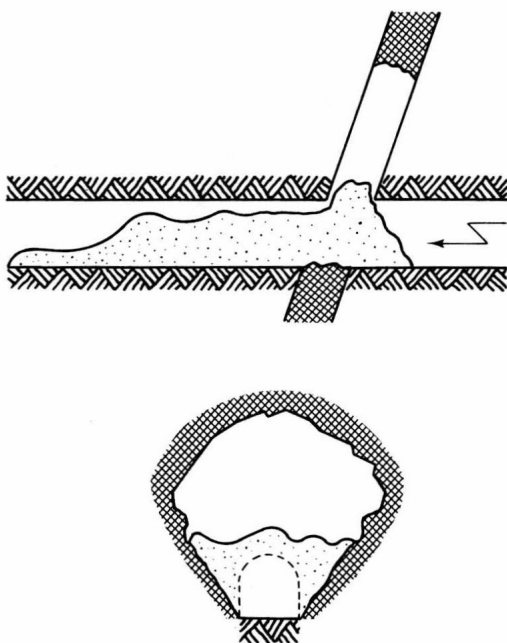


Figure 1.2 Rock fall from a Norwegian tunnel. (From Brekke, 1972.)

consider the slide of some 200 m^3 of rock, including blocks several cubic meters in volume, that fell from the roof of a Norwegian tunnel in 1960 (Fig. 1.2). The slide initiated from a tabular mass of altered metamorphic rocks between two seams of swelling clay whose squeeze into the tunnel left the rock free of normal stress and therefore unsupported along its sides.

One form of “failure” of both surface and underground excavations that is not addressed by this book is destruction of the surface by erosion from water, or gravity alone. This is likely to be critical when running water is allowed to travel along the surface of altered or weathered rocks, or poorly cemented sediments. Figure 1.3(a) depicts the formation of deep gullies by the action of rainwater charging down a face in friable, tuffaceous volcanic rocks. Flatter slopes are more seriously affected than very steep slopes. Loosening and ravelling of shales in underground galleries, with damage to support systems, is frequently experienced in these weaker rocks, or in harder rocks laced by seams or affected by hydrothermal alteration or weathering. As was seen in Fig. 1.2, such behavior can eventually undermine contiguous rock blocks to permit larger cave-ins.

Another rock failure mechanism not examined in this work is rock “squeeze” depicted in Fig. 1.3(b). The inward movement of the walls, roof, and floor, due to creep, slow crack growth, or slowly increasing load on the rock, can crush the support and lining structures under the worst conditions. Similar results accompany the swelling of active clay minerals in certain claystones, shales, and other rocks with disseminated clays (usually smectites). Occasionally, problems of active weathering, for example of sulfide-bearing rocks, can cause related problems.

In layered rocks, in relatively weak rocks at moderate or even shallow depth