NONEQUILIBRIUM PHENOMENA II
FROM STOCHASTICS TO HYDRODYNAMICS

EDITORS

J. L. LEBOWITZ

Rutgers University, New Brunswick

E. W. MONTROLL†

University of Maryland, Maryland

1984

NORTH-HOLLAND PHYSICS PUBLISHING
AMSTERDAM · OXFORD · NEW YORK · TOKYO

ISBN: 0 444 86806 2

Published by:

North-Holland Physics Publishing

a division of

Elsevier Science Publishers B.V.
P.O. Box 103
1000 AC Amsterdam
The Netherlands

Sole distributors for the U.S.A. and Canada:

Elsevier Science Publishing Company, Inc.

52 Vanderbilt Avenue
New York, N.Y. 10017
U.S.A.

Library of Congress Cataloging in Publication Data
Main entry under title:

Nonequilibrium phenomena II.

(Studies in statistical mechanics ; v. 11)
Bibliography: p.
Includes index.
1. Stochastic processes. 2. Many-body problem.
3. Hydrodynamics. 4. Montroll, E. W. I. Lebowitz, Joel Louis, 1930- . II. Montroll, E. W. III. Series.
QC175.S77 vol. 11 530.1'3s [530.1'592] 84-5940
[QC20.7.S8]
ISBN 0-444-86806-2 (U.S.)

PRINTED IN THE NETHERLANDS

NONEQUILIBRIUM PHENOMENA II
FROM STOCHASTICS TO HYDRODYNAMICS

STUDIES IN STATISTICAL MECHANICS

VOLUME XI

EDITORS:

E. W. MONTROLL†

University of Maryland, Maryland

J. L. LEBOWITZ

Rutgers University, New Brunswick

NORTH-HOLLAND PHYSICS PUBLISHING
AMSTERDAM · OXFORD · NEW YORK · TOKYO

Dedicated to the memory of

Elliot W. Montroll

Elliott Montroll (May 4, 1916–December 3, 1983) was one of a rare breed, a very intelligent, happy man. He knew and understood very well the failings of humanity, yet loved people anyway. I never heard Elliott complain. When he found that he had cancer, he consulted the best doctors, did some research on the illness, stayed optimistic, and continued to work and enjoy living. He died of a heart attack in the midst of his family whom he loved so much. It was a much too early end to a life of which it can truly be said (quoting Mark Twain): "We should live in such a way that when we die, even the undertaker will be sorry."

I join his family, friends, colleagues, and students in mourning his death. He was a great friend, a great scientist, and a noble man. We shall miss him sorely.

December 1983 — Joel L. Lebowitz

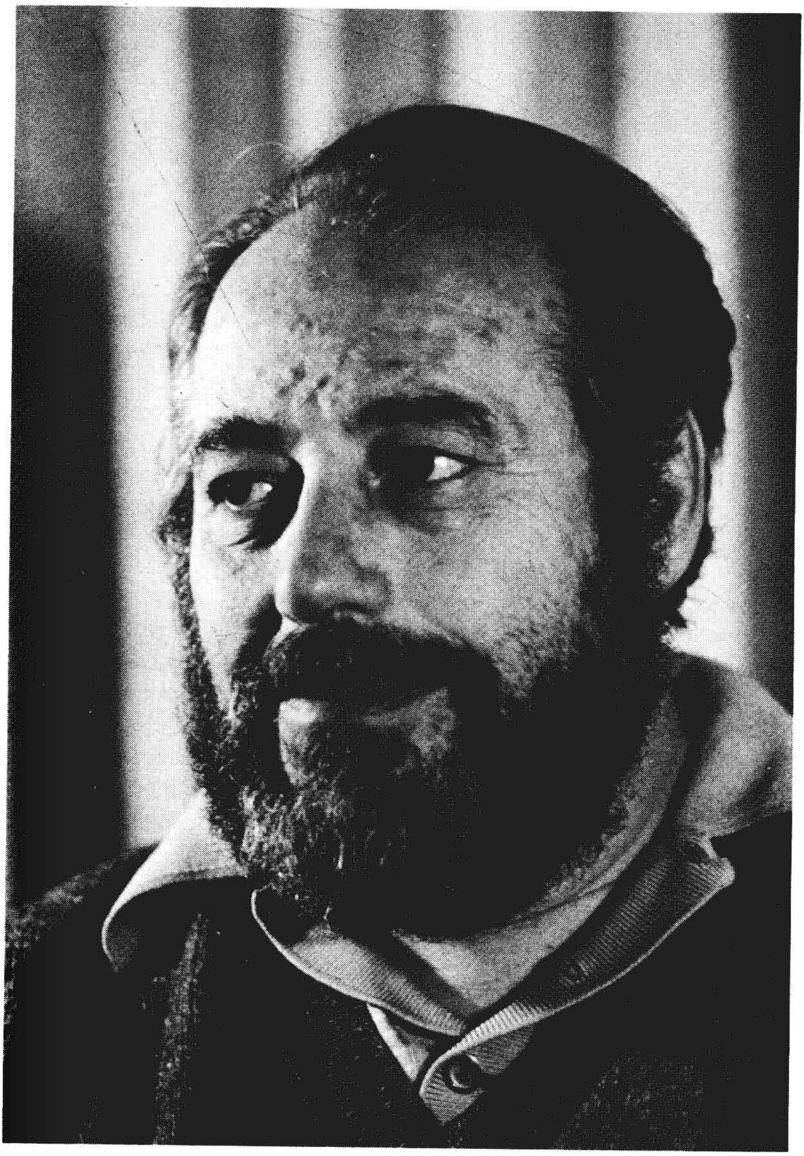

photo: John Ward

Elliott W. MONTROLL

(1916–1983)

PREFACE

The objective of statistical mechanics is to explain and predict the properties of macroscopic matter from the properties of its microscopic constituents. The subject is traditionally divided into an equilibrium and a nonequilibrium part.

The equilibrium properties of macroscopic systems, with a given microscopic Hamiltonian, can be obtained as suitable averages in well-defined Gibbs ensembles. While this only solves the problem in principle—the evaluation of integrals involving approximately 10^{23} variables is not simple—it does provide a starting point for both qualitative understanding and quantitative approximations to equilibrium behavior. In addition many interesting equilibrium phenomena, such as phase transitions, can be studied explicitly in simplified lattice model systems.

Our understanding of nonequilibrium phenomena is much less satisfactory at the present time. We do not have any general prescription for choosing appropriate ensembles for nonequilibrium systems. Instead we have to use a mélange of kinetic theories, some more reliable than others, to deal with the great variety of nonequilibrium phenomena occurring in different systems. These range from radiation transport in stars to metabolic transport across living cell membranes.

This is the second volume in the *Studies* devoted to a comprehensive review of nonequilibrium phenomena. It consists of two major original articles bearing on this subject: *A Survey on the Hydrodynamical Behavior of Many-Particle Systems* by A. De Masi, N. Ianiro, A. Pellegrinotti, and E. Presutti, and *On the Wonderful World of Random Walks* by Elliott W. Montroll and Michael F. Shlesinger.

The authors of the first article consider here one of the fundamental problems of nonequilibrium statistical mechanics: how does the microscopic dynamics of atoms and molecules lead to equations, such as Navier–Stokes, etc., which describe the macroscopic behavior? While we are still very far from being able to answer this question for real fluids, however, much progress has been made in recent years in understanding similar phenomena in simpler model systems. In these models the microscopic dynamics are simplified—typically they consist of correlated "stochastic" jumps of the particles on a lattice. Despite these simplifications, the models are far from trivial. In fact, their behavior depends strongly on the interaction between the particles—in just the same way as one expects it to occur in real systems. It is the interactions which give rise to local equilibrium states characterized by the conserved quantities which then satisfy hydrodynamic-type equations.

The second article by Elliott Montroll (co-editor of this series who recently died) and Michael Shlesinger is of a very different character. It is an account told informally, with charm and wit, of the historical development of our ideas about probability and stochasticity. The authors' obvious joy and technical expertise in the subject make this an article which will be treasured by all workers in the field (and others, too). It offers a rich source of insights both on the history and current status of the subject.

CONTENTS

CHAPTER 1

On the Wonderful World of Random Walks

Elliott W. MONTROLL† and Michael F. SHLESINGER*

Institute for Physical Science and Technology
University of Maryland
College Park, Maryland 20742
U.S.A.

La Jolla Institute
Post Office Box 1434
La Jolla, California 92038
U.S.A.

†Deceased.
**Present address: Office of Naval Research–Code 412, 800 N. Quincy St., Arlington, Virginia 22217, U.S.A.*

Nonequilibrium Phenomena II
From Stochastics to Hydrodynamics
Eds. J.L. Lebowitz and E.W. Montroll†

Contents

Introduction

Over the years two styles have emerged for the investigation of the temporal behavior of physical systems. The first exploits dynamical equations such as Newton's equations of motion, Schrödinger's equation of quantum theory, Maxwell's equations, or the Einstein field equation. These have been fantastically successful for the description of the behavior of relatively simple systems. However, as complexity arises to haunt us, the connection between basic dynamical equations and calculated results fades through the uncontrolled approximations and ad hoc assumptions that are sometimes needed to make required calculations possible.

The second style – the statistical style – is frequently called the application of the theory of stochastic processes, and becomes evermore fashionable for the investigation of complex systems. This is the style we stress in this review and have presented at several 60th birthday celebrations (M. Lax, P. Meijer, K. Shuler, and N. van Kampen) honoring those who have mastered this second style. We dedicate this manuscript to future masters as they attain their 60th birthday. Some episodes in the historical development of probability theory are first recounted; then, after reviewing several standard results we introduce new directions in the expansion of the style that we expect to be applicable to the understanding of features of our physical world that might display special clustering effects or temporal sequences of intermittencies followed by bursts of events. Most of our analysis involves fractals and is cast in the context of random walks. Finally, we shall show how a mathematician, if he were sufficiently clever, could proceed systematically from dynamical equations to stochastic processes, thus coupling the two styles.

In preparation we note that the classical distribution functions most commonly met in the analysis of physical processes are Gaussian or have exponential tails. However, in recent years long inverse power tails have become more evident. Such distributions may have no second or higher integer moments. To obtain an appreciation of the basic difference between the Gauss distribution and a distribution with a long tail compare the distribution of heights with the distribution of annual incomes for American adult males. An average individual who seeks a friend twice his height would fail. On the other hand, one who has an average income will have no trouble discovering a richer person with twice his income and that richer person may, with a little diligence, locate a third party with twice his income, etc. The income distribution in its upper range has a Pareto inverse power tail that has no second or higher moments. A considerable amount of Part II of this report concerns such distributions and the dynamical models that lead to them.

Those who are impatient to discover our views on these processes should skip to Part II. Those who are curious about the interesting personalities who were our stochastic forbearers should find some entertainment and surprises in Part I.