



FRANK S. BUDNICK

APPLIED MATHEMATICS

FOR BUSINESS,
ECONOMICS,
AND THE SOCIAL SCIENCES

FRANK S. BUDNICK

University of Rhode Island

APPLIED MATHEMATICS

**FOR BUSINESS, ECONOMICS,
AND THE SOCIAL SCIENCES**

McGRAW-HILL BOOK COMPANY

New York St. Louis San Francisco Auckland Bogotá
Düsseldorf Johannesburg London Madrid Mexico
 Montreal New Delhi Panama Paris
São Paulo Singapore Sydney Tokyo Toronto

CHAPTER PHOTO CREDITS

O. Chris Maynard—Magnum; 1. Werner Bischof—Magnum; 2. Mary Alice McAlpin—Nancy Palmer; 3. Burk Vzzle—Magnum; 4. American Museum of Natural History; 5. U.S. Department of Agriculture; 6. Bruce Thomas—Nancy Palmer; 7. Tektronix Corporation; 8. IBM Corporation; 9. Cornell Capa—Magnum; 10. Burk Vzzle—Magnum; 11. Harald Sund—Nancy Palmer; 12. Reflejo—Nancy Palmer; 13. Bruce Davidson—Magnum; 14. David A. Rahm—McGraw-Hill; 15. Bruce Davidson—Magnum; 16. Ken Kennedy—Nancy Palmer.

Library of Congress Cataloging in Publication Data

Budnick, Frank S

Applied mathematics for business, economics, and the social sciences.

Includes index.

1. Business mathematics. I. Title.

HF5691.B88 513'.93 78-31948

ISBN 0-07-008851-9

APPLIED MATHEMATICS FOR BUSINESS, ECONOMICS, AND THE SOCIAL SCIENCES

Copyright © 1979 by McGraw-Hill, Inc.

All rights reserved.

Printed in the United States of America.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher.

1234567890 DODO 7832109

This book was set in Bodoni by Progressive Typographers.

The editors were Charles E. Stewart and Stephen Wagley;

the designer was Nicholas Krenitsky;

the production supervisor was Dominick Petrellese.

The drawings were done by Fine Line Illustrations, Inc.

R. R. Donnelley & Sons Company was printer and binder.

PREFACE

INTRODUCTION Mathematics is an integral part of the education of students in business, economics, and the social sciences. There is increasingly a desire to improve the level of quantitative sophistication possessed by graduates in these types of programs. The objective is not to make mathematicians of these students, but to make them as comfortable as possible in an environment which increasingly makes use of quantitative analysis and the computer. Students are discovering that they must integrate mathematics, statistical analysis, and the computer in both required and elective courses within their programs. Furthermore, organizations are becoming more effective users of quantitative tools and the computer. Decision makers will be better equipped to operate within this type of environment if they are familiar with the more commonly used types of quantitative analyses and the technology of the computer. Such familiarity can assist them in being better “critics” and “users” of these tools, and hopefully, better decision makers.

DESIGN OF BOOK This book is an applied mathematics book for students in business, economics, and the social sciences. It provides a comprehensive treatment of selected topics in both finite mathematics and calculus. Although intended principally for students in business and economics, the book is appropriate for students in the social sci-

ences. Designed primarily for a two-term course, the book can be adapted easily for a one-term course. It is appropriate for use in both two-year schools and four-year schools, as well as at the “foundation” level for graduate programs which require some mathematics background. M.B.A. and M.P.A. programs are typical graduate programs having this type of requirement.

Suggested course structures follow. For a two-term course, Term 1 would cover Chapters 0 to 9, while Term 2 would cover Chapters 10 to 16.

The first term is primarily a course in finite mathematics. Using Chapters 2 to 6 and 8 as a suggested core, Chapters 0, 1, 7, and 9 may be included depending upon the needs of students and the interest of the instructor. Chapters 0 and 1 should be included when students require a review of algebra and an introduction to set theory. Chapter 7 may be covered where a more complete coverage of linear programming is desired. Chapter 9 will probably be required for students in business and economics.

The second term is primarily an applied calculus course. Complete coverage of Chapters 10 to 15 is recommended; however, certain material may be excluded depending upon the needs and abilities of students.

The following are some suggested course structures for one-term courses:

A. Emphasis on Calculus A one-term course with emphasis on calculus would have a suggested core consisting of Chapter 2, Sections 5.1 to 5.2, and Chapters 10 to 15. Where students require a review of algebra and an introduction to set theory, Chapters 0 and 1 should be included.

B. Emphasis on Finite Mathematics A one-term course with emphasis on finite mathematics would select from among Chapters 0 to 9 and 16. Chapters may be included or excluded for the same reasons mentioned in the discussion of Term 1 of the two-term structure.

C. Combination of Finite Mathematics and Calculus A one-term course emphasizing both areas might have a variety of structures. A suggestion is that this type of course have a core which includes Chapters 2 to 5, 10 to 13, and 15. To this core may be added other chapters which expand the coverage of either area or both.

Specific features of this book are

1 A level of presentation which carefully develops and reinforces topics.

2 A style which appeals to the intuition of students and provides a great deal of visual reinforcement (almost 250 figures).

3 An applied orientation which motivates students and provides a sense of purpose for studying mathematics (see table on pages xv to xviii indicating number and breadth of applications).

4 An approach which first develops the mathematical concept and then reinforces with applications.

5 An approach which minimizes the use of rigorous mathematical proofs. Proofs are included at the end of selected chapters for interested persons.

6 Special aids which address the most universal shortcoming of students entering this type of course: weak algebra skills. These aids include an optional chapter (Chapter 0) which reviews key algebra principles. A chapter pretest allows the student and instructor to identify areas requiring special attention. In addition, “Algebra Flashbacks” are used throughout the book to assist the student in the recall of key rules or concepts. The flashback usually consists of a restatement of a rule or concept with a reference to the appropriate section in Chapter 0.

7 Notes to students which provide them with special insights.

8 “Points for Thought and Discussion” which allow students to pause for a moment and reconsider a concept or example from a different perspective. Their purpose is to reinforce and extend the student’s understanding.

9 A multitude of other learning aids including

almost 450 solved examples

a wealth of exercises (1761)

chapter tests

chapter objectives

end of chapter checklists

lists of key terms and concepts

summary lists of important formulas

10 An instructors manual which contains

answers for all exercises and tests

suggestions for different course structures

prototype examples for new applications

transparency masters for selected figures

a bank of questions for constructing quizzes and tests

Although applications are presented throughout the book, Chapters 5 and 13 are devoted entirely to applications. The intent is that instructors cover as many applications in these chapters as they feel appropriate for their students. Chapter 14 (Classical Optimizations: Functions of Several Variables) is optional and not a prerequisite for Chapter 15 (Integral Calculus). Chapter 9 (Mathematics of Finance) has no prerequisite chapter. Except for the last part of Sec. 16.5, Chapter 16 (Introduction to Probability Theory) has no prerequisite chapters.

Some exercises in the book are considered to be of a high level of difficulty. These are preceded by an asterisk (*).

ACKNOWLEDGMENTS I wish to express my sincere appreciation to the many persons who have contributed either directly or indi-

rectly to this project. These include: my students who endured the class testing of the manuscript and who provided valuable feedback and suggestions for improvements; Richard R. Weeks and Warren F. Rogers, both of the University of Rhode Island, who provided me with administrative support and allowed minimal distractions during the course of the project.

I wish to thank Professor Howard T. Bell, Shippensburg State College; Professor Robert I. Canavan, Monmouth College; Professor H. Howard Frisinger, Colorado State University; Professor Edward L. Keller, California State University; Dr. Marvin Rothstein, University of Connecticut; Professor Charles Sinclair, Portland State University; Professor Martin K. Starr, Columbia University; Professor Dale E. Walston, University of Texas; and Professor Robert A. Yawin, Springfield Technical Community College for reviewing the manuscript at various stages along the way. *Their comments proved extremely helpful in rewriting.*

A very special thanks goes to Professor Susan E. Potter, Rhode Island Junior College, who stayed with the project from start to finish and helped me clean up those initial “rough” drafts of chapters. Also, to Professor Terry D. Shaw, University of Texas at Austin, who provided excellent feedback for rewriting the calculus chapters.

I want to thank the people at McGraw-Hill with whom I worked directly. These persons include Donald E. Chatham, Charles E. Stewart, Stephen Wagley, and Nicholas Krenitsky. They provided the kind of support that an author truly appreciates.

I also wish to thank: Françoise Boulanger, Mary Howard, Mary Tafari, Cathy Hebert, and Joseph Slott for their assistance in developing problems and solution sets; Sue E. Rubinsky, Jean Parrish, and Edith Williams for typing the manuscript and the Instructor’s Manual; and Diane Marcotte for her assistance in preparing copies of chapters for classroom testing.

I also wish to make a special acknowledgment to Dr. Rudolph P. Lamone, University of Maryland, who has had an important influence on my career.

I also wish to thank my parents, Mr. and Mrs. Willard L. Budnick, for their continued support and encouragement during this endeavor as well as all others.

And last, but certainly not least, I want to thank my family—Jane, Chris, Scott, and (newly arrived) Kerry—for their patience, understanding, encouragement, and love. They allowed me the necessary solitude of “my lonely writer’s garret,” but they regularly liberated me whenever I needed reminding that I am (first and foremost) a father and a husband.

Frank S. Budnick

CONTENTS

Applications	xv
Preface	xix
CHAPTER 0 A REVIEW OF ALGEBRA (OPTIONAL)	1
0.1 THE REAL NUMBER SYSTEM	3
Real numbers Rules of signs Absolute value Addition of real numbers Subtraction of real numbers Multiplication and division of real numbers	
0.2 POLYNOMIALS	11
Positive integer exponents Polynomial expressions Addition and subtraction of polynomials Multiplication of polynomials Division of polynomials	
0.3 FACTORING	20
Monomial factors Factoring quadratic polynomials Other special forms	
0.4 FRACTIONS	26
Some basic properties Addition and subtraction of fractions Multiplication and division	

0.5 EXPONENTS AND RADICALS	32
Fractional exponents Radicals Radicals and fractional exponents	
0.6 EQUATIONS	36
Equations and their properties Solving first-degree equations Solving second-degree equations	
CHAPTER 1 SET THEORY	45
1.1 SETS DEFINED	45
Sets Venn diagram representation	
1.2 SET OPERATIONS	52
Set equality Union of sets Intersection of sets	
1.3 SAMPLE APPLICATIONS	59
1.4 SUMMARY	65
CHAPTER 2 MATHEMATICAL FUNCTIONS	69
2.1 THE CARTESIAN PRODUCT AND RELATIONS	69
Cartesian product Relations Graphing relations	
2.2 FUNCTIONS	79
Functions defined The nature and notation of functions Characterizing mathematical functions Composite functions	
2.3 GRAPHICAL REPRESENTATION OF FUNCTIONS	89
“A picture is worth. . .” Graphing functions in two dimensions A graphical characteristic of functions	
2.4 SUMMARY	96
CHAPTER 3 LINEAR EQUATIONS	99
3.1 CHARACTERISTICS OF LINEAR EQUATIONS	100
General form Representation using linear equations Generalizing for n -variable linear equations	
3.2 GRAPHICAL CHARACTERISTICS	106
Graphing two-variable equations Intercepts The equation $x = k$ The equation $y = k$ Slope	
3.3 SLOPE-INTERCEPT FORM	116
From a different vantage point Interpreting the slope and y -intercept	
3.4 DETERMINING THE EQUATION OF A STRAIGHT LINE	119
Slope and intercept Slope and one point Two points	

3.5 LINEAR EQUATIONS INVOLVING MORE THAN TWO VARIABLES	125
Equations involving three variables Equations involving more than three variables	
3.6 ADDITIONAL APPLICATIONS	131
CHAPTER 4 SYSTEMS OF LINEAR EQUATIONS	139
4.1 INTRODUCTION	139
Systems of equations Solution sets	
4.2 TWO-VARIABLE SYSTEMS OF EQUATIONS	141
Graphical analysis Graphical solutions The elimination procedure $(m \times 2)$ systems	
4.3 THREE-VARIABLE SYSTEMS	149
Graphical analysis Elimination procedure for (3×3) systems Fewer than three equations More than three equations n -variable systems	
4.4 GAUSS-JORDAN PROCEDURE	156
The general idea The method	
4.5 SUMMARY	161
CHAPTER 5 APPLICATIONS OF LINEAR FUNCTIONS AND SYSTEMS OF EQUATIONS	165
5.1 LINEAR FUNCTIONS	165
General form and assumptions Linear cost functions Linear revenue functions Linear profit functions	
5.2 OTHER EXAMPLES OF LINEAR FUNCTIONS	172
5.3 BREAK-EVEN MODELS	177
Assumptions Break-even analysis	
5.4 OTHER APPLICATIONS	184
CHAPTER 6 LINEAR PROGRAMMING: FORMULATION AND GRAPHICAL SOLUTIONS	197
6.1 LINEAR INEQUALITIES	198
The nature of inequalities The algebra of linear inequalities Solution sets for linear inequalities The graphics of linear inequalities Systems of linear inequalities	
6.2 LINEAR PROGRAMMING	208
Introduction A scenario Structural constraints and nonnegativity constraints	

6.3	SOME APPLICATIONS OF LINEAR PROGRAMMING	210
6.4	GRAPHICAL SOLUTIONS Area of feasible solutions Incorporating the objective function Corner point solutions Alternative optimal solutions No feasible solution	216
6.5	SUMMARY	224
CHAPTER 7 THE SIMPLEX METHOD		229
7.1	NONGRAPHICAL SOLUTIONS Overview of the simplex procedure Requirements of the simplex Basic feasible solutions	229
7.2	MAXIMIZATION PROBLEMS The algebra of the simplex method Adding the objective function Summary of the simplex procedure	236
7.3	OTHER PROBLEM STRUCTURES <i>Artificial variables</i> <i>Minimization problems</i>	246
7.4	SUMMARY	249
CHAPTER 8 MATRIX ALGEBRA		253
8.1	INTRODUCTION TO MATRICES What is a matrix? Purpose of studying matrix algebra	253
8.2	SPECIAL TYPES OF MATRICES Vectors Square matrices Identity matrix Transpose of a matrix	255
8.3	MATRIX OPERATIONS Matrix addition and subtraction Scalar multiplication The inner product Matrix multiplication	257
8.4	MATRIX REPRESENTATION OF EQUATIONS <i>Representation of an equation</i> <i>Representation of systems of equations</i>	266
8.5	THE DETERMINANT The determinant of a (1×1) matrix The determinant of a (2×2) matrix The determinant of a (3×3) matrix The method of cofactors	268
8.6	THE INVERSE OF A MATRIX <i>Determining the inverse</i> <i>Finding the inverse using cofactors (optional)</i> The inverse and systems of equations	274
8.7	APPLICATIONS OF MATRIX ALGEBRA	282

CHAPTER 9 MATHEMATICS OF FINANCE	290
9.1 INTEREST AND ITS COMPUTATION <i>Simple interest Compound interest Compound interest</i>	292
9.2 SINGLE-PAYMENT COMPUTATIONS <i>Compound amount Present value Other applications of the compound-amount formula Effective interest rates</i>	295
9.3 ANNUITIES AND THEIR FUTURE VALUE <i>The sum of an annuity Determining the size of an annuity</i>	305
9.4 ANNUITIES AND THEIR PRESENT VALUE <i>The present value of an annuity Determining the size of an annuity Mortgages</i>	311
9.5 SUMMARY	317
INTEREST TABLES	322
 CHAPTER 10 NONLINEAR FUNCTIONS	 329
10.1 WHEN LINEAR FUNCTIONS ARE INAPPROPRIATE	330
10.2 QUADRATIC FUNCTIONS AND THEIR CHARACTERISTICS <i>Mathematical form Graphical representation Special insights to sketching quadratic functions Determining the equation of quadratic functions Polynomial functions</i>	332
10.3 QUADRATIC FUNCTIONS: APPLICATIONS	340
10.4 EXPONENTIAL FUNCTIONS <i>Characteristics of exponential functions Base-e exponential functions</i>	347
10.5 APPLICATIONS OF EXPONENTIAL FUNCTIONS	353
10.6 LOGARITHMIC FUNCTIONS <i>Logarithms Properties of logarithms Logarithmic functions</i>	359
 CHAPTER 11 DIFFERENTIATION	 369
11.1 INTRODUCTION	369
11.2 LIMITS AND CONTINUITY <i>Limits of functions Some properties of limits Limits and asymptotes Continuity</i>	370
11.3 AVERAGE RATE OF CHANGE <i>Average rate of change and slope</i>	382

11.4	THE DERIVATIVE	387
	Instantaneous rate of change The limit approach to finding the derivative Instantaneous rate of change revisited	
11.5	DIFFERENTIATION	397
	Rules of differentiation Instantaneous rate of change—again	
11.6	HIGHER-ORDER DERIVATIVES	406
	The second derivative Third- and higher-order derivatives	
11.7	DIFFERENTIATION OF SPECIAL FUNCTIONAL FORMS	409
	Chain rule Other derivatives	
11.8	SUMMARY	411
	APPENDIX Proofs of Selected Rules of Differentiation	416
CHAPTER 12	CLASSICAL OPTIMIZATION: METHODOLOGY	421
12.1	DERIVATIVES: ADDITIONAL INTERPRETATIONS	422
	The first derivative The second derivative Concavity and inflection points	
12.2	IDENTIFICATION OF MAXIMA AND MINIMA	429
	Relative maxima and minima Stationary points The first-derivative test The second-derivative test When the second-derivative test fails Higher-order derivative test	
12.3	RESTRICTED-DOMAIN CONSIDERATIONS	442
	When the domain is restricted	
12.4	CURVE SKETCHING	445
	Key data points Ultimate direction	
12.5	SUMMARY	450
CHAPTER 13	CLASSICAL OPTIMIZATION: APPLICATIONS	453
13.1	REVENUE, COST, AND PROFIT APPLICATIONS	453
	Revenue applications Cost applications Profit applications	
13.2	MARGINAL APPROACH TO PROFIT MAXIMIZATION	465
	Marginal revenue Marginal cost Marginal profit analysis	
13.3	ADDITIONAL APPLICATIONS	472
13.4	SUMMARY	483

CHAPTER 14 CLASSICAL OPTIMIZATION: FUNCTIONS OF SEVERAL VARIABLES	489
14.1 GRAPHICAL REPRESENTATION OF BIVARIATE FUNCTIONS	490
Graphical representation Sketching bivariate functions	
14.2 PARTIAL DERIVATIVES	494
Derivatives of bivariate functions Interpreting partial derivatives Second derivatives	
14.3 OPTIMIZATION OF BIVARIATE FUNCTIONS	501
Stationary points Distinguishing among stationary points	
14.4 APPLICATIONS OF BIVARIATE OPTIMIZATION	508
CHAPTER 15 OVERVIEW OF INTEGRAL CALCULUS	521
15.1 ANTIDERIVATIVES	522
The antiderivative concept Revenue and cost functions	
15.2 RULES OF INTEGRATION	526
Integration Rules of integration	
15.3 DEFINITE INTEGRALS	535
The definite integral Evaluating definite integrals Definite integrals and areas Finding areas between curves	
15.4 APPLICATIONS OF INTEGRAL CALCULUS	548
15.5 WHEN OUR RULES OF INTEGRATION FALL (OPTIONAL)	557
Integration by parts Tables of integrals	
15.6 SUMMARY	561
CHAPTER 16 INTRODUCTION TO PROBABILITY THEORY	567
16.1 PERMUTATIONS AND COMBINATIONS	568
Permutations Combinations	
16.2 BASIC PROBABILITY CONCEPTS	571
Probabilities and odds Some rules of probability Statistical independence Statistical dependence	
16.3 MATHEMATICAL EXPECTATION	584
Random variables Probability distributions Mean and standard deviation Expected monetary value	
16.4 THE BINOMIAL PROBABILITY DISTRIBUTION	592
Bernoulli process The binomial distribution Mean and standard deviation of binomial distribution	

16.5	CONTINUOUS PROBABILITY DISTRIBUTIONS <i>The normal probability distribution</i> Integral calculus and probability	598
16.6	SUMMARY	608
	APPENDIX A TABLES	615
	APPENDIX B SUMMATION NOTATION	617
	SELECTED BIBLIOGRAPHY	618
	SELECTED ANSWERS	619
	INDEX	643

APPLICATIONS*

ACCOUNTING

Straight Line Depreciation	172
Straight Line Depreciation with Salvage Value	173
Depreciation	123
Auditing	583
Salvage Value	591
Internal Revenue	594

CRIMINAL JUSTICE SYSTEM/POLITICAL SCIENCE

Court Scheduling	132
Crime Deterrence	173
Police Patrol Allocation	366, 479
Voter Crossover	60
Victimization Survey	67
Criminal Justice System	64

ECONOMICS

Income Shifts	190
Linear Demand Functions	174
Linear Supply Functions	175

* Explicitly labeled in the text.

APPLICATIONS		
	Quadratic Demand Functions	343
	Quadratic Supply Functions	342
	Supply-Demand Equilibrium	184, 343
	Supply Shifts	191
	Law of Diminishing Returns	330
	Quadratic Revenue Functions	340
	Marginal Revenue	446, 524
	Marginal Cost	467, 525
	Marginal Profit	468
	Revenue	548
	Average Cost per Unit	458
	Consumer's Surplus	553
	Producer's Surplus	557
	 EDUCATION	
	College Admissions	282
	College Enrollments	302
	Grade Inflation	177
	Education Survey	64
	 EMERGENCY RESPONSE SYSTEMS	
	Emergency Airlift	130, 187
	Emergency Response: Location Model	345, 474
	Fire Protection	319
	 ENERGY	
	Energy Conservation	63, 555
	Solar Energy	460
	Nuclear Power	552
	Oil Consumption	564
	 FINANCE	
	Break-even Models	177
	Compound Interest: Continuous Compounding	354
	Investment Portfolio	131, 187
	Present Value: Continuous Compounding	365
	Retirement Planning	314
	Fund Raising	405, 550
	 HEALTH CARE DELIVERY	
	Cancer Research	61
	Hospital Administration	288
	Immunization	598
	Vitamin C Research	59
	Epidemic Control	405, 556
	Blood Bank Management	555
	 MANAGEMENT (NONPROFIT SECTOR)	
	Beach Management	463
	Import Tax Management	463