RECENT ADVANCES IN STATISTICAL PHYSICS

Proceedings of the International Bose Symposium on Statistical Physics

Calcutta, India, 28–31 Dec 1981

Edited by **B Datta**
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PREFACE

In 1982, the academic members of the Satyendra Nath Bose Institute of Physical Sciences of the Calcutta University were eager to celebrate the Sixteenth Anniversary of Bose Statistics and Ninetyeth Birth Anniversary of National Professor S.N. Bose, F.R.S. in 1984-85 through an International Symposium of 'Statistical Physics'. Due to lack of co-operation - nay opposition of the University administration, no progress in the organisational work was possible. The Calcutta Mathematical Society, of which Professor Bose was a former president and Honorary Member, was approached for collaboration. The Society came forward to take the lead in the matter. At its initiative, an organising committee with members from different Institutions with which Prof. Bose had his connection in his life time and a technical advisory committee with eminent scientists from all over the world were formed. Due to the said initial difficulties and uncertainties and due to a serious car-accident of the President of the Society, Working Chairman of organising committee, and one of the main organisers in August, 1984 the symposium could not be organised as initially contemplated.

The responses to the circulars for the symposium were good. Many eminent scientists expressed their willingness to participate. As necessary intimations could not be conveyed in due time, many of them could not personally participate in the symposium. Some kindly sent the manuscripts of their lectures to be delivered in it. In the proceedings, besides the papers presented personally, those invited papers, taken as read, have been included. The organisers are extremely sorry for delay in publication.

Hearty thanks and deep gratitude are due to International Centre for Theoretical Physics, Trieste, Italy and its Director, Professor A. Salam for collaborations and necessary supports. Thanks are due to Indian National Science Academy (New Delhi), Satyendra Nath Bose

Institute of Physical Sciences (Calcutta University), the Asiatic Society (Calcutta), Burdwan University (Burdwan), Bose Institute (Calcutta), Saha Institute of Nuclear Physics (Calcutta), Birla Institute of Technological Museum (Calcutta) for collaborations and to the Department of Science and Technology, the Government of India for financial support. Thanks are also to members of Technical Advisory Committee and/or organising Committees, to members of the office of Calcutta Mathematical Society and of the Satyendra Nath Bose Institute of Physical Sciences.

Many thanks are due to Messers World Scientific Publishing Co. Pte. Ltd., Singapore, for kindly publishing this proceedings.

EDITORS' NOTE

The diversity of the topics treated in this book reflects intense activities which reign in Statistical Physics in recent years. The presentation of a wide and varied spectrum of topics by leading experts in their respective fields contains very useful materials which depend to a large extent on their freshness in the frontiers of science. A brief resume of its contents is given in the next paragraph.

Among the transport properties of neutrino systems the phenomenon of volume viscosity being a relativistic effect assumes an interesting role. S.R. de Groot (The Netherlands) computes the volume viscosity as a function of the neutrino mass and the temperature and discusses in detail its role in the way of expanding or contracting of neutrino clouds in the universe. In the discussion of some critical quantities in non-equilibrium phase transitions, F. Schlogl (F.R.G.) shows that the second cumulant, the bit-number variance, can be treated as a generalization of specific heat for nonequilbria. The author demonstrates some interesting features in regard to the behaviour of the bit-number variance by means of comparison of some special examples on nonequilibrium phase transitions. S. Lengyel and I. Gyarmati (Hungary) present an informative and in-depth report on studies concerning the constistency (or inconsistency?) between thermodynamics and chemical kinetics, suggesting in a way to search for alternative definition of thermodynamic force of chemical reactions in place of the usual ones. S. Lengyel (Hungary) discusses critically under broad spectrum the problem of extension of the original Onsager-Casimir theory and concludes that the most general thermodynamics of irreversible processes seem, at present, to be the theory of internal variables which, in principle, corresponds to the very general and orginal Onsager-Casimir theory. V. Majernik (Czechoslovakia) discusses in minute details the conceptualisation of organization in physics and biophysics and investigates it from the standpoint of modern general systems of theory where the organization appears as the unifying concept for a large class of different complex systems. In his fascinating article Majernik shows that the general systems theory is the suitable framework for the description of physical statistical ensembles representing probabilistic systems with Shannon's organization as well as the living object representing goal-directed complex system with the functional organization. N.D. Sengupta (India) gives an outline of a programme for obtaining the distribution of a system of particles in thermodynamic equilibrium with the assumptions that (i) in the equilibrium state the entropy is an additive functional of N_r or P_{nr} as the case may be in classical or quantum systems and (ii) the distribution persists, as long as the thermodynamic equilibrium is maintained. B.K. Datta, R. Datta and S. Bandyopadhyay (India) obtain relativistic thermodynamic formulation via entropy flow vector with all thermodynamic quantities defined covariantly. A.C. Biswas and C.S. Warke (India) derive a microscopic theory for propagation of solitons in superfluid ^4He films at temperature $T = 0^oK$. P. Ghosh (India) discusses about the possibility of a "perpetually oscillating universe" pointing out the logical possibility for the validity of the second law of thermodynamics for such a model. A. Jamiolkowski (Poland) discusses the minimal number of observables for reconstructibility of N-level quantum systems governed by Gaussian emigroups. Interpreting quantum mechanical fluctuations in the same way as those from statistical physics, S. Dumitru (Romania) demonstrates that all the main assertions of "conventional interpretation of uncertainity relations" are affected in an unsurmountable way by a whole class of authentic shortcomings and hence should be abandoned as an incorrect conception. M. Dutta (India) sketches an essential statistical model for investigating the behaviours of thermodynamic systems and discusses the problems of phase transitions in this model. In another paper Dutta discusses methods that are quite suitable for the investigations of thermodynamics behaviour for an ensemble of interacting particles based separately on cell-model and on a lattice-model.

Furthermore, it is explicitly pointed out that the applications of the results of this modified version in strong electrolytes in solutions not only yield better agreement with experimental values, but also show good agreement within wider range of concentrations which cannot be exhibited by the usual models.

EDITORS

CONTENTS

SYSTEMS OF NEUTRINOS WITH MASS

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Abstract:

From the formalism of relativistic kinetic theory and the weak interaction Lagrangian the volume viscosity of a massive neutrino system is derived. Its value is calculated as a function of the neutrino mass and the temperature. Its role in the way of expanding or contracting of neutrino clouds in the universe is discussed.

1 - Introduction

The transport properties of a diluted fluid system may be found from an appropriate statistical non-equilibrium method and the knowledge of the interaction between the particles[8)]. For neutrino systems the former is relativistic kinetic theory and the latter the weak force. Amongst the transport properties, the phenomenon of volume viscosity is interesting since it is a relativistic effect[4)]. For massless particles it vanishes. The neutrino was supposed to be massless until recently when it was suggested by various experimentalists to possess mass[11,12)]. The volume viscosity was calculated[13)]. This text contains the general outline, the results and moreover a discussion of its importance in the description of expansion (or contraction) of neutrino clouds in the universe. The details of the calculation are given in sections 8 through 11 and an appendix, see also notes c. and d.

2 - The transport equation

The generalization of the kinetic transport equation to relativity was achieved by Lichnerowicz and Marrot[1]. This was done in a classical framework. The quantum-mechanical treatment was studied by van Weert, de Boer and Siskens[2]. (The non-relativistic quantum-mechanical theory was developed by Waldmann and Snider[3].)

The distribution function (the classical one or the quantum-mechanical Wigner function) $f(x,p)$ depends on the time-space coordinates $x = x^{\mu}$ with $\mu = 0,1,2,3$ with $\mu = 0$ the temporal component ct (c the speed of light and t the time) and for $\mu = 1,2,3$ the spacial vector $\vec{x}$ with Cartesian components x, y and z. The energy-momentum four-vector $p = p^{\mu}$ has the $\mu = 0$ component p^0 (with cp^0 the energy) and for $\mu = 1, 2, 3$ the momentum $\vec{p}$. The zero-component is not independent since one has the equality $p^0 = \sqrt{p^2+m^2c^2}$ with m the particle mass. For neutrinos one needs no spin indices since the spin is anti-parallel to the motion for neutrinos and parallel for the anti-neutrinos.

The theory has as starting points: i: covariance, ii: diluted systems in which only binary collisions have to be taken into account, iii: "molecular chaos" (the "Stosszahlansatz" of Boltzmann): no correlations between particles before their collision. (A weaker hypothesis: no correlations in the infinite past is due to Bogolyubovand used in reference 2), iv: smooth distribution functions, i.e. with negligible variations over molecular interaction times and distances. The ensuing classical transport equation is

$$p^{\mu}\,\partial_{\mu}\,f(x,p) = \int \frac{d^3p_1}{p_1^0}\,\frac{d^3p'}{p'^0}\,\frac{d^3p_1'}{p_1'^0}$$

$$\left[f(x,p')f(x,p_1')W(p',p_1'|p,p_1) - f(x,p)f(x,p_1)W(p,p_1|p',p_1')\right] \qquad (1)$$

with $\partial_{\mu} = \partial/\partial x^{\mu}$. . The left-hand side is referred to as the "streaming term", whereas the right-hand side is called the "collision term". The form of the latter is a consequence of the hypothesis of molecular chaos: for collisions (p,p_1) to $(p'.p_1')$ one loses particles with momentum $\vec{p}$: this gives rise to the term with the minus sign proportional to $f(x,p)f(x,p_1)$ and the transition rate $W(p,p_1|p',p_1')$. For the inverse collision (p',p_1') to (p,p_1) one gains particles with $\vec{p}$: this leads to the first term at the right-hand side. The transition rate is according to collision theory

$$W(p,p_1|p',p_1') = c^{-2}(2\pi)^{10}\hbar^{8}\delta^{(4)}(p+p_1-p'-p_1')|\langle p',p_1'|L_I(0)|p,p_1\rangle|^2 \qquad (2)$$

where the delta function reflects energy-momentum conservation ($p^{\mu}+p_1^{\mu} = p'^{\mu}+p_1'^{\mu}$; $\mu = 0,1,2,3$) and where the matrix element of the interaction Lagrangian $L_I(x)$ appears. The latter will be specified for the physical system studied in section 5. (The right-hand side of (2) is sometimes written as the delta function multiplied by $s\sigma(s,\theta)$ where s is $(p+p_1)^2$, the quantity θ the angle between $\vec{p}$ and $\vec{p}'$ and σ is called the cross-section). One may note that from (2) follows the equality $W(p,p_1|p',p_1')$ and $W(p',p_1'|p,p_1)$, called the property of "detailed balance". (A weaker condition is "bilateral normalization" which reads that the integral over p' and p_1' over W for a collision in one direction is equal to the integral for the oppsoite collision. This property is generally valid: it reflects the unitarity of the scattering matrix).

3 - The linearized transport equation

The general form of the transport equation is difficult for further use because of its highly non-linear character. However, in practice a linearized version leads to the desired physical results: the linear laws which express that the various irreversible fluxes (heat flow, viscous flow, etc) are proportional to gradients of state variables (temperature, hydrodynamic velocity, etc.). The proportionality coefficients are the transport coefficients (heat conductivity, the viscosities, etc.), which depend upon the particle interactions and the state variables. Of the latter the cvolume viscosity, discovered by Israel and Kelly[4)], is interesting since it is a purely relativistic property. (As we shall see, it occurs in the linear law which expresses that the trace of the viscous pressure tensor is proportional to the four-divergence of the hydrodynamic velocity.) Weinberg, Murphy, Belinskii, Khalatnikov, Caderni, Fabbri, Siskens and van den Horn have considered astrophysical and cosmological implications of the phenomenon of volume viscosity.

To obtain the linearized theory we use the relativistic version of the Chapman-Enskog procedure (cf. refs. 5,6,7 and 8). To this end we write first the identity

$$\partial_\mu \equiv c^{-2} U_\mu D + \nabla_\mu \qquad (3)$$

with U_μ the hydrodynamic velocity (for which we take Eckart's definition as the particle flow, normalized to $c^2 = U_\mu U^\mu$) and the definitions of the convective time derivative D and the gradient ∇_μ as

$$D := U^\nu \partial_\nu \ , \ \nabla_\mu := \Delta_{\mu\nu}\partial^\nu \ (\Delta_{\mu\nu} := g_{\mu\nu} - c^{-2} U_\mu U_\nu) \qquad (4)$$

with $g_{\mu\nu}$ the metric tensor diag.(1,-1,-1,-1). With (3) one may write the transport equation (1) as

$$c^{-2}p^{\mu}U_{\mu}Df = -\ p^{\mu}\nabla_{\mu}f + C[f,f] \tag{5}$$

of which the last term is the right-hand side of (1) (the collision term). Division by mcf yields terms with the dimension of a reciprocal length. In fact the first term at the right-hand side of (5) becomes the reciprocal of the spatial non-uniformities of the system and the second the reciprocal mean free path. Since the mean free path is much smaller than the spatial non-uniformity lengths, the first term at the right-hand side of (5) is much smaller than the second term. We shall develop the equation, and the distribution function, in powers of the mean free path divided by the spatial non-uniformity. Then in (5) the second term at the right-hand side is called to be of order zero and the first term at the right-hand side of order one. We also write

$$f(x,p) = f^{(0)}(x,p) + f^{(1)}(x,p) + \dots \tag{6}$$

with $f^{(i)}$ of order i. The linearized theory will now consist in considering only quantities (and equations) or order zero and one (the latter will lead to the linear laws referred to in the beginning of this section). We choose for the zero order distribution function

$$f^{(0)}(x,p) = \frac{1}{(2\pi\hbar)^3}\exp\frac{\mu(x) - p^{\mu}U_{\mu}(x)}{kT(x)} \tag{7}$$

with μ the Gibbs potential, T the temperature, $2\pi\hbar$ a constant making $(2\pi\hbar)^3 f$ dimensionless and k Boltzmann's constant. The function (7) makes the zero order collision term vanish. Now for the first order equation, i.e. the equation for $f^{(1)}(x,p)$, which we shall write as $f^{(0)}(x,p)\phi(x,p)$, we obtain from (5) with the use of (6), (7) and the conservation laws for energy-momentum:

$$QX - (p^{\mu}U_{\mu} - h)p_{\nu}\, X_q^{\nu} + p^{\mu}p^{\nu b}\, \overset{\circ}{X}_{\mu\nu} = kT\; L[\phi] \tag{8}$$

with the quantity Q from

$$\hat{Q} := c^2 Q/(kT)^2 := (\tfrac{4}{3} - \gamma)\tau^2 + [(\gamma - 1)\hat{h} - \gamma]\tau - \tfrac{1}{3}\,z^2 \quad , \tag{9}$$

where γ is the specific heat at constant pressure divided by the specific heat at constant volume (so: $\gamma = k/c_v + 1$) and

$$\tau := p^{\mu}U_{\mu}/kT \quad , \quad \hat{h} := h/kT \quad , \quad z := mc^2/kT \quad , \tag{10}$$

with h the enthalpy per particle. Furthermore one has three "Thermodynamic forces":

$$X := -\nabla^{\mu}U_{\mu} \tag{11}$$

for expansion or contraction, X_q^{μ} a temperature gradient (and a relativistic term with the pressure gradient) and $\overset{\circ}{X}_{\mu\nu}$ the spatial part of the traceless velocity gradient tensor. Finally in the second member of (8) appears the "linearized collision operator":

$$L[\phi] := \tfrac{1}{2}\int \frac{d^3p_1}{p_1^0}\frac{d^3p'}{p'^0}\frac{d^3p_1'}{p_1'^0}\, f_1^{(0)}(\phi + \phi_1 - \phi' - \phi_1')W(p',p_1'|p,p_1) \; . \tag{12}$$

Equation (8) is indeed the equation of first order distribution quantity $\phi = f^{(1)}/f^{(0)}$. It will be solved and employed to find the linear laws and transport quantities. (The indices 1 and the accents refer to dependencies on four-momenta with these indices and accents).