APPLIED CALCULUS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

FOURTH EDITION

RAYMOND A. BARNETT
MICHAEL R. ZIEGLER

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LIFE SCIENCES, AND SOCIAL SCIENCES

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PREFACE

The fourth edition of *Applied Calculus for Business, Economics, Life Sciences, and Social Sciences* is designed for a one- or two-term course in calculus and for students who have had $1\frac{1}{2}-2$ years of high school algebra or the equivalent. It is one of six books in the authors' College Mathematics series (see page ii for a brief comparison of all six books).

This edition encompasses the experiences and recommendations of a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. The choice and independence of topics make the text readily adaptable to a variety of courses (see the chapter dependency chart on page xvii).

◆ PRINCIPAL CHANGES FROM THE THIRD EDITION

This edition has been divided into two parts, "Part One: Preliminaries" and "Part Two: Calculus." Many users have requested a more comprehensive **review of relevant topics from intermediate and college algebra** to be placed at the beginning of the book. Part One is our answer to this request. Chapters 1–3 comprise a completely rewritten and reorganized review of algebra topics. Depending on the background of a given class, this material can be totally omitted, referred to as needed, or selected topics can be covered in detail at the beginning or during the course. A few additional review topics are also included in Appendix A for convenient reference.

The following general improvements are found throughout the text: an increased emphasis on and use of **calculators; boxed definitions, results, and step-by-step processes;** and **schematic illustrations. Examples** have been improved, and many new examples have been added. **Exercise sets,** including applications, have been improved and expanded. All **exposition** has been carefully reviewed and fine-tuned or rewritten.

Specific improvements in Part Two are as follows:

1. The first four sections in the introductory **derivative** chapter of the last edition have been completely rewritten and condensed into three sections. The treatment has been both simplified and expanded. The new first section

provides an **intuitive geometric introduction to limits and continuity.** Because of numerous reviewer requests, **left- and right-hand limits** are again included. Because of its importance in solving optimization problems, **continuity on a closed interval** is now defined. **Increment notation** has been removed from this introductory chapter to keep the notation as simple as possible. Increments are now part of the section on **differentials** (Section 5-5). **Computation of difficult limits** has been reduced, and more emphasis has been placed on the **0/0 indeterminate form** because of its relevance to the definition of the derivative and its various interpretations (slope, instantaneous rate, marginal analysis, and so on). **Infinite limits** and **limits at infinity** receive more attention. **Vertical and horizontal asymptotes** are defined using these concepts.

2. The section on **curve sketching** (Section 5-3) now precedes the section on **optimization** (Section 5-4). **Solving inequalities using continuity** has become a subsection of the section on the first derivative and graphs (Section 5-1). This process of solving inequalities is fundamental to the production of **sign charts** throughout the chapter. The **curve sketching strategy** (Section 5-3) has been modified into a form that should make it easier for students to use and remember [Step 1: Use $f(x)$; Step 2: Use $f'(x)$; Step 3: Use $f''(x)$; Step 4: Graph f].
3. To satisfy many reviewer's requests, **L'Hôpital's rule** has been moved forward to Chapter 6 and the treatment has been expanded to include **one-sided limits, infinite limits, and limits at infinity.**
4. In Chapters 7 and 8, the material on integration has been extensively revised. More attention is given to **substitution techniques.** The **definite integral** is now introduced as a **limit of a Riemann sum,** using **summation notation and formulas.** The approach now parallels the development of the derivative. That is, first simple definite integrals are evaluated by the definition to provide a firm understanding of the concept, then the **fundamental theorem** is introduced to simplify evaluation. **Area between the graphs of two functions** has been moved to Chapter 8, and the discussion has been expanded. In the applications of integration in Section 8-2, a Riemann sum approach is used to develop the formula for the **future value of a continuous income stream,** instead of the present value.
5. In Chapter 9, on multivariable calculus, the important applications of **Cobb–Douglas production functions** are given more attention and now include a discussion of the **marginal productivity of money.** At the request of many reviewers, our discussion of the **least-squares method** now includes two equivalent methods of determining the least-squares line: solving the system of **normal equations** or using standard formulas.
6. The section on **interpolating polynomials** (Section 12-2) has been simplified and now uses the **divided difference table** to find these polynomials.
7. A new section on the important **Poisson distribution** and its applications has been added to Chapter 13, on probability and calculus.

◆ IMPORTANT FEATURES

Emphasis and Style

The text is **written for student comprehension.** Great care has been taken to write a book that is mathematically correct and accessible to students. Emphasis is on computational skills, ideas, and problem-solving rather than mathematical theory. Most derivations and proofs are omitted except where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

Examples and Matched Problems

Over 400 completely worked examples are included. Each example is followed by a similar problem for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

Exercise Sets

The book contains over 4,300 problems. Each exercise set is designed so that an average or below-average student will experience success and a very capable student will be challenged. Exercise sets are mostly divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) levels.

Applications

Enough applications are included to convince even the most skeptical student that mathematics is really useful. The majority of the applications are included at the end of exercise sets and are generally divided into business and economics, life science, and social science groupings. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems taken from professional journals and books. No specialized experience is required to solve any of the applications.

◆ STUDENT AIDS

1. **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-2 and 1-3).
2. **Annotation** of examples and developments, in color type, is found throughout the text to help students through critical stages (see Sections 1-2 and 1-3).
3. **Functional use of color** improves the clarity of many illustrations, graphs, and developments, and guides students through certain critical steps (see Sections 1-2 and 1-3).
4. **Boldface type** is used to introduce new terms and highlight important comments.
5. **Screened boxes** are used to highlight important definitions, theorems, results, and step-by-step processes.

6. **Answers** to odd-numbered problems are included in the back of the book.
7. **Chapter review** sections include a review of all important terms and symbols and a comprehensive review exercise. Answers to all review exercises are included in the back of the book.
8. A student's **solution manual** is available at a nominal cost through a book store. The manual includes detailed solutions to all odd-numbered problems and all review exercises.
9. A manual for an **Interactive Computer Applications Package (ICAP)** by Carolyn L. Meitler is available at a nominal cost through a book store. The manual contains instructions, examples, and exercises that demonstrate the use of the programs on the *ICAP for Applied Calculus* disks. The disks containing the programs are distributed free of charge to institutions using this book. No previous computing experience is necessary to use this package.
10. A **Supplemental Applications and Topics** manual by Jon E. Baum is available at a nominal cost through a book store. Part I of the manual expands the application exercises in the text and reinforces the important role of the mathematics presented in the text. These exercises provide the student with a richer and more varied experience in solving real-world problems. Part II of the manual presents some applications that are not covered in the text, including transportation problems, assignment problems, sensitivity analysis, and a variety of finance topics. After completing the prerequisite material in the text, students interested in these more specialized topics will realize substantial benefits by studying this portion of the manual.

◆ INSTRUCTOR AIDS

See page xvi for detailed information regarding examination copy requests and orders for the instructor aids described below.

1. A unique **computer-generated random test system** is available to instructors without cost. The test system utilizes an IBM-PC, XT, or AT Personal Computer® and will produce high-quality output on an IBM-compatible dot-matrix printer or on a Hewlett-Packard Laserjet II®-compatible laser printer. The test system has been greatly expanded and now contains over 400 different problem algorithms directly related to material in the text. These carefully constructed algorithms use random number generators to produce different, yet equivalent, versions of each of these problems. The test system is available now in both **free-response and multiple-choice editions.** An almost unlimited number of quizzes, review exercises, chapter tests, mid-terms, and final examinations, each different from the other, can be generated quickly and easily. At the same time, the system will produce answer keys and student work sheets, if desired. Upon request, the publisher will supply institutions using this textbook with **DellenTest III (IBM**

Free-Response Edition or Multiple-Choice Edition) on 5.25 inch floppy disks. **IBM Edition User Notes** and **Annotated Problem Printouts** are included with the disks. The notes provide step-by-step instructions for using the testing system and a complete description of the options in this menu-driven program. The annotated printouts identify by chapter and number each question the system is capable of generating, and also correlate each question with the prerequisite section from the text. When used in conjunction with the user notes, the annotated printouts enable instructors to select any combination of questions for an examination.

2. An **instructor's test battery** is also available to instructors without cost. This printed and bound manual, organized by chapter, contains three equivalent versions (with answers) of over 400 different problems.
3. An **instructor's resource manual** provides over 175 transparency and handout masters, a detailed discussion of chapter and topic dependencies, a comparison of this edition with the previous edition, and a detailed topic chart for comparing this book with other books in the authors' College Mathematics series.
4. An **instructor's answer manual** containing all the answers not included in the text is available to instructors without charge.
5. A student's **solution manual** (see Student Aids) is available to instructors without charge from the publisher.
6. An **Interactive Computer Applications Package (ICAP)** by Carolyn L. Meitler (see Student Aids) is available to instructors without charge from the publisher. The programs in this package are available on disks for APPLE II® and IBM-PC® computers. Included on these disks are programs related to limit estimation, function graphing, numerical integration, least-squares approximations, root approximation, interpolating polynomials, and differential equations. The publisher will supply these disks without charge to institutions using this book.
7. A **Supplemental Applications and Topics** manual by Jon E. Baum (see Student Aids) is available to instructors without charge from the publisher. Instructors can use Part I of this manual to supplement the exercise sets in the text, providing students with additional experience in solving applications utilizing the mathematics presented in the text. Part II of the manual can be used to provide coverage of applications not covered in the text, such as transportation problems, assignment problems, sensitivity analysis, and a variety of finance topics, either as part of the syllabus for a course or as subjects for independent study.
8. **Z-graph,** a HyperCard© graphing stack for the APPLE Macintosh® computer, allows a user to graph most of the mathematical functions likely to be encountered, quickly, accurately, and with considerable control over axes, scales, graph size, and labeling. In addition to graphing functions, this program will perform a variety of mathematical operations related to numerical integration, root approximation, interpolating polynomials, least-square polynomials, and approximate solutions of differential equations. Instruc-

tors will find this program useful for preparing examination material, transparency masters, and handouts. The publisher will supply this program free of charge to instructors using this book, and the program may be freely distributed to students.

◆ ERROR CHECK

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error-free. For any errors remaining, the authors would be grateful if they were sent to: Dellen Publishing Company, 400 Pacific Avenue, San Francisco, CA 94133.

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R. A. Barnett
M. R. Ziegler

Ordering Information

When requesting examination copies or placing orders for this text or any of the related supplementary materials listed below, please refer to the corresponding ISBN numbers.

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Computer-generated random test system for Applied Calculus, Fourth Edition:	
DellenTest III Disks (IBM Free-Response Edition)	0-02-306404-8
DellenTest III Disks (IBM Multiple-Choice Edition)	0-02-306405-6
(*IBM Edition User Notes* and *Annotated Problem Printouts* are included with each edition of the disks.)	
Instructor's Answer Manual to accompany Applied Calculus, Fourth Edition	0-02-306402-1
Instructor's Resource Manual to accompany Applied Calculus, Fourth Edition	0-02-306403-X
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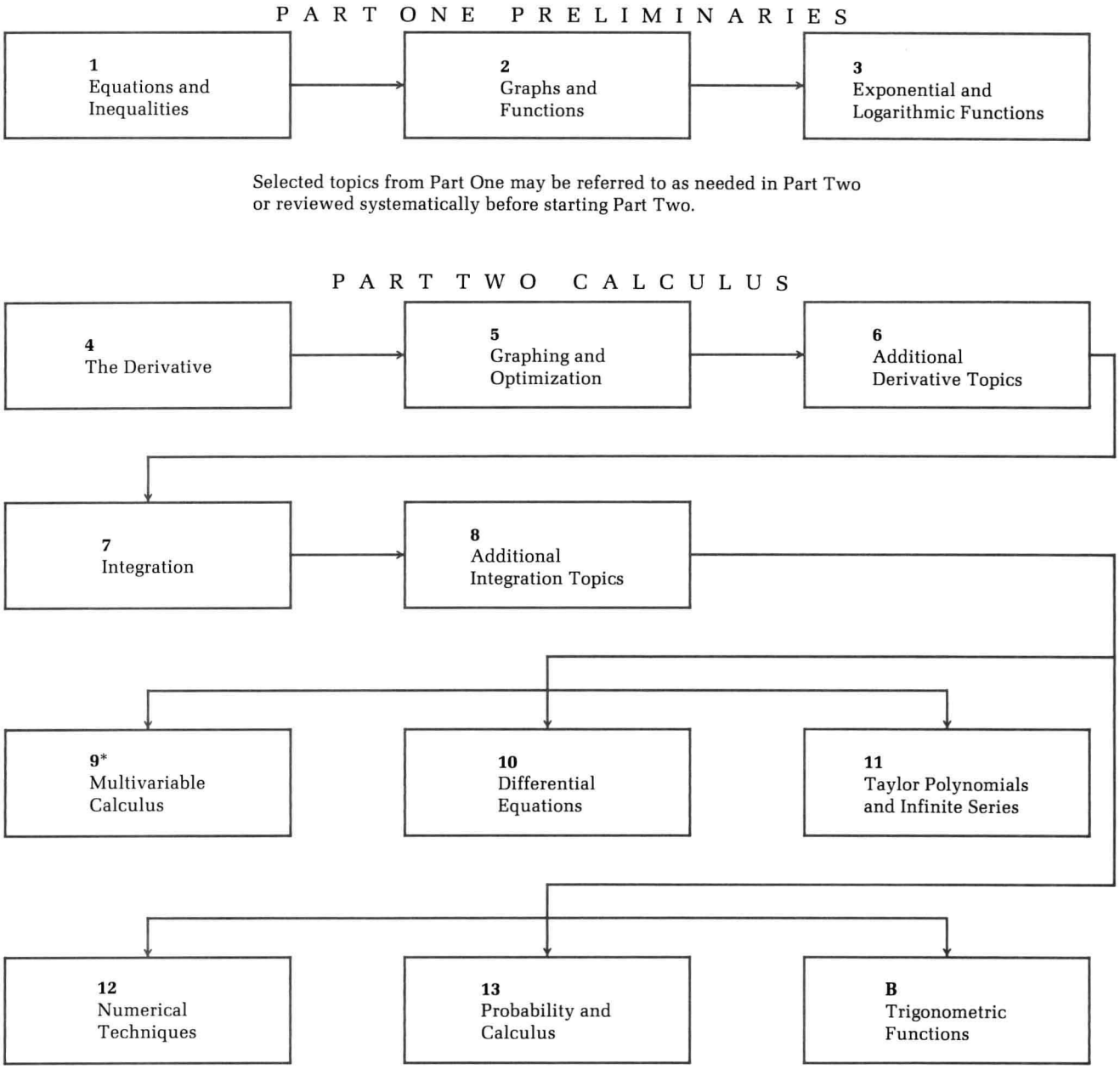
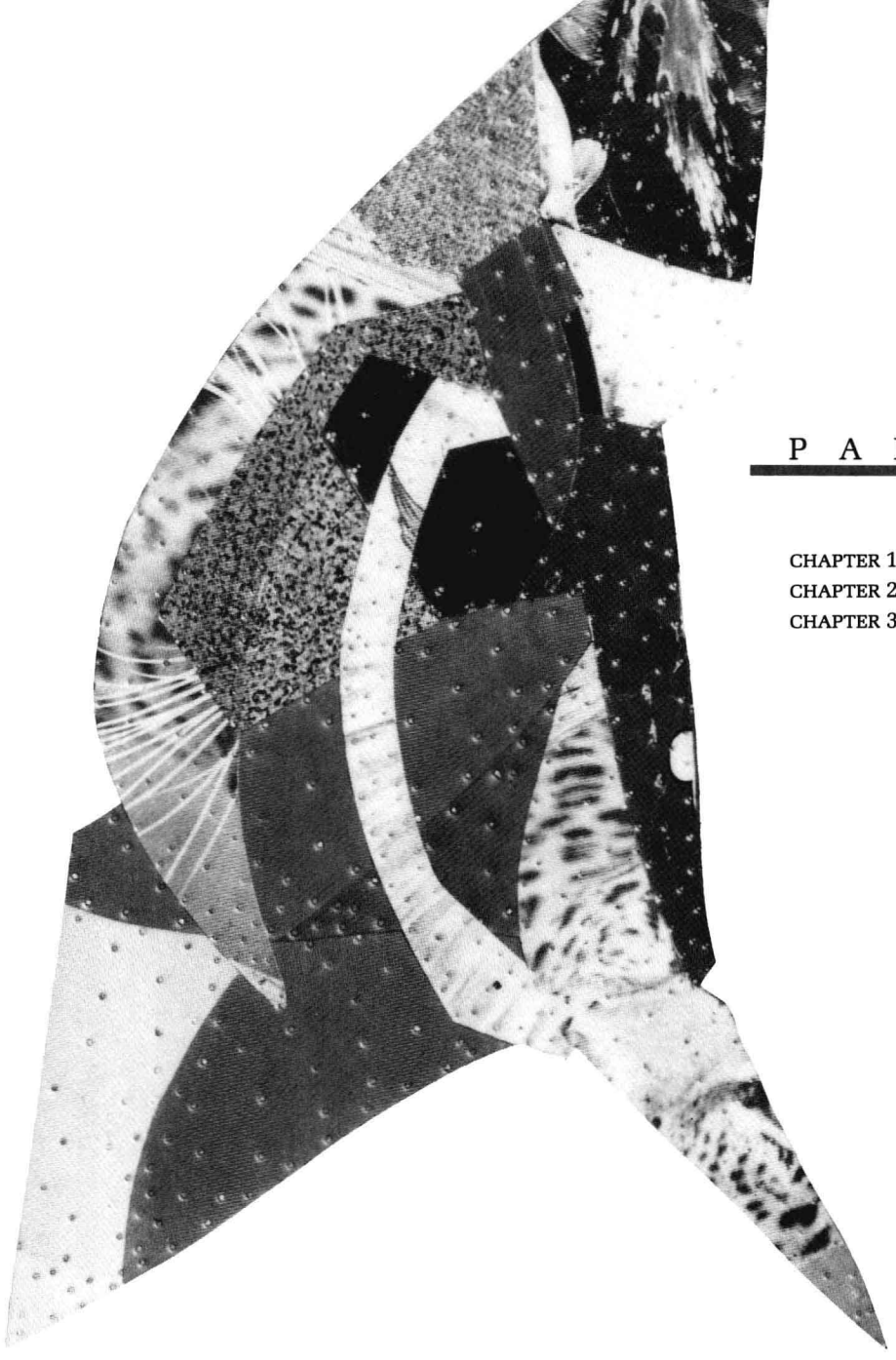


* Chapters 9—13 and Appendix B are relatively independent and may be covered in any order after Chapter 8 has been completed. See the *Instructor's Resource Manual* for a more detailed discussion of the relationships among the topics in these chapters.

CONTENTS



PART ONE

Preliminaries