ELSEVIER
ACADEMIC
PRESS

Fuzzy Logic in Geology

Edited by

Robert V. Demicco

George J. Klir

Foreword by Lotfi Zadeh

Fuzzy Logic in Geology

Edited by

Robert V. Demicco
and
George J. Klir
CENTER FOR INTELLIGENT SYSTEMS
BINGHAMTON UNIVERSITY (SUNY)
BINGHAMTON, NEW YORK, USA

ELSEVIER
ACADEMIC
PRESS

Amsterdam Boston Heidelberg London New York Oxford
Paris San Diego San Francisco Singapore Sydney Tokyo

This book is printed on acid-free paper. ♾

Academic Press
An imprint of Elsevier Science
525 B Street, Suite 1900, San Diego, California 92101-4495, USA
http://www.academicpress.com

Academic Press
84 Theobald's Road, London WC1X 8RR, UK
http://www.academicpress.com

Library of Congress Cataloging-in-Publication Data

ISBN 0-12-415146-9

PRINTED IN THE UNITED STATES OF AMERICA
03 04 05 06 07 08 9 8 7 6 5 4 3 2 1

Fuzzy Logic in Geology

Contributors

- **Andras Bardossy** (Chapter 6): Institute of Hydraulic Engineering, University of Stuttgart, Pfaffenwaldring 61, D-70550 Stuttgart, Germany [bardossy@europe.com].
- **Radim Bělohlávek** (Chapter 7): Department of Computer Science, Palacký University of Olomouc, Tomkova 40, CZ-77900 Olomouc, Czech Republic [belohlavek@inf.upol.cz].
- **Istvan Bogardi** (Chapter 6): Department of Civil Engineering, University of Nebraska at Lincoln, Lincoln, Nebraska 68588, USA [ibogardi@unl.edu].
- **Robert V. Demicco** (Chapters 1, 3, 4, and 5): Department of Geological & Environmental Studies and Center for Intelligent Systems, Binghamton University (SUNY), Binghamton, New York 13902, USA [demicco@binghamton.edu].
- **Lucien Duckstein** (Chapter 6): Ecole Nationale du Génie Rural des Eaux et des Forêts, 19 avenue du Maine, 75732 CEDEX 15, France [duckstein@engref.fr].
- **Chongfu Huang** (Chapter 8): Institute of Resources Science, Beijing Normal University, 19 Xinjiekouwai Street, Beijing 100875, China [hchongfu@bnu.edu.cn].
- **George J. Klir** (Chapters 1 and 2): Department of Systems Science & Industrial Engineering and Center for Intelligent Systems, Binghamton University (SUNY), Binghamton, New York 13902, USA [gklir@binghamton.edu].
- **Vilem Novák** (Chapter 10): Institute for Research and Applications of Fuzzy Modeling, University of Ostrava, 30. dubna 22, 70103 Ostrava, Czech Republic [Vilem.Novak@osu.cz].
- **Irina Perfilieva** (Chapter 9): Institute for Research and Applications of Fuzzy Modeling, University of Ostrava, 30. dubna 22, 70103 Ostrava, Czech Republic [Perfili@dynami.osu.cz].
- **Rita Pongracz** (Chapter 6): Department of Meteorology, Eotvos Lorand University, Pazmany P. setany1/A, H-1117 Budapest, Hungary [prita@caesar.elte.hu].

Foreword

In October 1999, at the invitation of my eminent friend, Professor George Klir, I visited the Binghamton campus of the State University of New York. In the course of my visit, I became aware of the fact that Professor Klir, a leading contributor to fuzzy logic and theories of uncertainty, was collaborating with Professor Robert Demicco, a leading contributor to geology and an expert on sedimentology, on an NSF-supported research project involving an exploration of possible applications of fuzzy logic to geology. What could be more obvious than suggesting to Professors Klir and Demicco to edit a book entitled "Fuzzy Logic in Geology." No such book was in existence at the time.

I was delighted when Professors Klir and Demicco accepted my suggestion. And, needless to say, I am gratified that the book has become a reality. But, what is really important is that Professors Klir and Demicco, the contributors and the publisher, Academic Press, have produced a book that is superlative in all respects.

As the editors state in the preface, *Fuzzy Logic in Geology* is intended to serve three principal purposes: (1) to examine what has been done in this field; (2) to explore new directions; and (3) to expand the use of fuzzy logic in geology and related fields through exposition of new tools.

To say that *Fuzzy Logic in Geology* achieves its aims with distinction is an understatement. The excellence of organization, the wealth of new material, the profusion of applications, and the high expository skill of contributors, including Professors Klir and Demicco, combine to make the book an invaluable reference and an important source of new ideas. There is no doubt that *Fuzzy Logic in Geology* will be viewed as a landmark in its field.

In the preface, Professors Klir and Demicco note that applications of fuzzy logic in science are far less visible than in engineering and, especially, in the realm of consumer products. Is there an explanation?

In science, there is a deep-seated tradition of striving for the ultimate in rigor and precision. Although fuzzy logic is a mathematically based theory, as is seen in Chapter 2, there is a misperception, reflecting the connotation of its label, that fuzzy logic is imprecise and not well-founded. In fact, fuzzy logic may be viewed as an attempt to deal precisely with imprecision, just as probability theory may be viewed as an attempt to deal precisely with uncertainty.

A related point is that in many of its applications, a concept which plays a key role is that of a linguistic variable, that is, a variable where values are words rather than numbers. Words are less precise than numbers. That is why the use of linguistic variables in fuzzy logic drew critical comments from some of the leading members of the scientific establishment. As an illustration, when I gave my first lecture on linguistic variables in 1972, Professor Rudolf Kalman, a brilliant scientist/engineer, had this to say:

> I would like to comment briefly on Professor Zadeh's presentation. His proposals could be severely, ferociously, even brutally criticized from a technical point of view. This would be out of place here. But a blunt question remains: Is Professor Zadeh presenting important ideas or is he indulging in wishful thinking? No doubt Professor Zadeh's enthusiasm for fuzziness has been reinforced by the prevailing climate in the US—one of unprecedented permissiveness. 'Fuzzification' is a kind of scientific permissiveness; it tends to result in socially appealing slogans unaccompanied by the discipline of hard scientific work and patient observation.

In a similar vein, a colleague of mine at UCB and a friend, Professor William Kahan, wrote:

> Fuzzy theory is wrong, wrong, and pernicious. I cannot think of any problem that could not be solved better by ordinary logic. ... What Zadeh is saying is the same sort of things as, 'Technology got us into this mess and now it can't get us out'. Well, technology did not get us into this mess. Greed and weakness and ambivalence got us into this mess. What we need is more logical thinking, not less. The danger of fuzzy theory is that it will encourage the sort of imprecise thinking that has brought us so much trouble.

What Professors Kalman, Kahan, and other prominent members of the scientific establishment did not realize is that mathematically based use of words enhances the ability of scientific theories to deal with real-world problems. In particular, in both science and engineering, the use of words makes it possible to exploit the tolerance for imprecision to achieve tractability, robustness, simplicity and low cost of solution. The use of linguistic variable is the basis for the calculus of fuzzy if-then rules—a calculus which plays a key role in many of the applications of fuzzy logic—including its applications in geology.

During the past few years, the use of words in fuzzy logic has evolved into methodology labeled *computing with words and perceptions* (CWP)—a methodology which casts a new light on fuzzy logic and may lead to a radical enlargement of the role of natural languages in science and engineering.

Computing with words and perceptions is inspired by the remarkable human capability to perform a wide variety of physical and mental tasks, e.g., driving a car in city traffic or playing tennis, without any measurements and any computations. In performing such tasks, humans employ perceptions—perceptions of distance, speed, direction, intent, likelihood, and other attributes of physical and mental objects.

There is an enormous literature on perceptions, spanning psychology, philosophy, linguistics, and other fields. But what has not been in existence is a theory in which perceptions can be operated on as objects of computation. Fuzzy logic provides a basis for such a theory—a theory which is referred to as the *computational theory of perceptions* (CTP).

In the computational theory of perceptions, perceptions are dealt with not as patterns of brain activity, but through their descriptions in a natural language. In this sense, a natural language may be viewed as a system for describing perceptions. Thus, if classical, bivalent logic is viewed as the logic of measurements, then fuzzy logic may be viewed as the logic of perceptions.

Although the methodology of computing with words and perceptions is not treated explicitly in the book, the basic ideas which underlie it are in evidence throughout. Furthermore, *Fuzzy Logic in Geology* ventures beyond well-established techniques and presents authoritative expositions of methods which lie on the frontiers of fuzzy logic. In this respect, particularly worthy of note are the chapters on formal concept analysis (R. Bělohlávek), F-transformation (I. Perfilieva), and linguistic theory (V. Novák).

In sum, *Fuzzy Logic in Geology* is a true role model. It is a high quality work which opens the door to application of new methods and new viewpoints to a variety of basic problems in geology, geophysics, and related fields. It is well-organized and reader-friendly. The editors, the contributors, and the publisher deserve our thanks and accolades.

Lotfi A. Zadeh
May 13, 2003
Berkeley, CA

Preface

This book has three purposes. Its first purpose is to demonstrate that fuzzy logic opens a radically new way to represent geological knowledge and to deal with geological problems, and that this new approach has been surprisingly successful in many areas of geology. This book's second purpose is to help geologists understand the main facets of fuzzy logic and the role of these facets in geology. The final purpose of this book is to make researchers in fuzzy logic aware of the emerging opportunities for the application of their expertise in geology.

This book is a chimera in that it is oriented not only at theoreticians, practitioners, and teachers of geology, but also at members of the fuzzy-set community. For geologists, the book contains a specialized tutorial on fuzzy logic (Chapter 2), a basic introduction to the application of fuzzy logic to model geological situations (Chapter 3), an overview of currently known applications of fuzzy logic in geology (Chapter 4), and six additional chapters with more extensive examples of applications of fuzzy logic to problems in a broad range of geological disciplines. For fuzzy logicians, the book is an overview of areas of geology in which fuzzy logic is already well established or is promising. Thus, our overall aim in preparing this book is to provide a useful link between the two communities and further stimulate interdisciplinary research.

The book is a product of a close cooperation between the editors and the several contributing authors. The authors were commissioned to write chapters on specific topics. Great care has been taken to assure that the mathematical terminology and notation are uniform throughout the book. Moreover, care was also taken to assure that the structure of individual chapters and the style of referencing were consistent throughout. Furthermore, authors were requested to focus on clarity of presentation, adding summaries of technical content where appropriate. All these features make the book attractive and appropriate as a text for graduate courses and seminars.

The book is written, by and large, in a narrative style, with the exception of a few sections in Chapters 7 and 9. These chapters are dependent on fairly complex mathematical preliminaries. It is far more efficient to introduce these preliminaries in a more formal style, typical of mathematical literature, using numbered definitions, lemmas, theorems, and examples. Although this formal presentation in Chapters 7 and 9 is essential for understanding operational details of the described methods, it is not necessary for a conceptual understanding of the methods and their geological applications. In fact, these chapters are structured conceptually. With this structure, the reader may still get the gist of the chapter without studying the details of the formal presentation.

The idea of preparing a book on fuzzy logic in geology was suggested to the editors by Lotfi Zadeh, the founder of fuzzy logic, during his visit to Binghamton University in October 1999. Our opinion then, and now, is that it was a good idea. While fuzzy logic is now well established as an important tool in engineering, its applications in science are far less developed. Nevertheless, the utility of fuzzy logic in various areas of science has been increasingly recognized since at least the mid 1990s. A good example is in chemistry, where the role of fuzzy logic is examined in the excellent book *Fuzzy Logic in Chemistry*, edited by Dennis H. Rouvray and published by Academic Press in 1997. It thus seemed natural to propose this book, which examines the role of fuzzy logic in geology, to Academic Press, with an eye toward obtaining a synergistic effect. We hope that this book will not only serve its purpose well, but that it will stimulate publication of other books exploring the role of fuzzy logic in other areas of natural sciences such as biology and physics as well as in the social sciences such as geography and economics.

Robert V. Demicco and George J. Klir
Binghamton, New York
December 2002

Glossary of Symbols

General Symbols

$\{x, y, \ldots\}$	Set of elements $x, y, \ldots$
$\{x \mid \mathrm{p}\{x\}\}$	Set determined by property p
$\langle x_1, x_2, \ldots, x_n\rangle$	n-tuple
$[x_{ij}]$	Matrix
$[x_1, x_2, \ldots, x_n]$	Vector
$[a, b]$	Closed interval of real numbers between a and b
$[a, b), (b, a]$	Interval of real numbers closed in a and open in b
(a, b)	Open interval of real numbers
$A, B, C \ldots.$	Arbitrary sets (crisp or fuzzy)
$x \in A$	Element x belongs to crisp set A
$A(x)$ or $\mu_A(X)$	Membership grade of x in fuzzy set A
${}^{\alpha}A$	α-cut of fuzzy set A
${}^{\alpha+}A$	Strong α-cut of fuzzy set A
$A = B$	Set equality
$A \neq B$	Set inequality
$A - B$	Set difference
$A \subseteq B$	Set inclusion
$A \subset B$	Proper set inclusion ($A \subseteq B$ and $A \neq B$)
$\mathrm{SUB}(A, B)$	Degree of subsethood of A in B
$\mathcal{P}(X)$	Set of all crisp subsets of X (power set)
$\mathcal{F}(X)$	Set of all standard fuzzy subsets of X (fuzzy power set)
$\|A\|$	Cardinality of crisp or fuzzy set A (sigma count)
h_A	Height of fuzzy set A
$\overline{A}$	Complement of set A
$A \cap B$	Set intersection
$A \cup B$	Set union
$A \times B$	Cartesian product of sets A and B
A^2	Cartesian product $A \times A$
$f: X \to Y$	Function from X to Y
f^{-1}	Inverse of function f
$R \circ Q$	Standard composition of fuzzy relations R and Q

$R * Q$	Join of fuzzy relations R and Q
R^{-1}	Inverse of a binary fuzzy relation
$<$	Less than
$\leq$	Less than or equal to (also used for a partial ordering)
$x \mid y$	x given y
$x \Rightarrow y$	x implies y
$x \Leftrightarrow y$	x if and only if y
$\sum$	Summation
Π	Product
$\max(a_1, a_2, \ldots, a_n)$	Maximum of $(a_1, a_2, \ldots, a_n)$
$\min(a_1, a_2, \ldots, a_n)$	Minimum of $(a_1, a_2, \ldots, a_n)$
$\mathbb{N}$	Set of positive integers (natural numbers)
$\mathbb{N}_n$	Set $\{1, 2, \ldots, n\}$
$\mathbb{R}$	Set of all real numbers

Special Symbols

$\mathcal{B}(X, Y, I)$	The set of all fuzzy concepts in a given context $\langle X, Y, I \rangle$
c	Fuzzy complement
$d(A)$	Defuzzified value of fuzzy set A
E	Similarity relation (fuzzy equivalence)
h	Averaging operation
h_p	Generalized means
i	Fuzzy intersection or t-norm
$i_{\min}$	Drastic fuzzy intersection
i_w	Fuzzy intersection of Yager class
$\mathcal{J}$	Fuzzy implication operator
L	Set of truth degrees
$\mathbf{L}$	Complete residuated lattice
L^X	The set of all fuzzy sets in X with truth values in L
m	Fuzzy modifier
Nec_E	Necessity measure corresponding to Pos_E
p_A	Fuzzy propositional form and truth assignment
p	Fuzzy probability qualifier
Pos_E	Possibility measure associated with a proposition "ν is E"
$S(Q, R)$	Solution set of fuzzy relation equation $R \circ Q = R$
T	Fuzzy truth qualifier
$\mathcal{X}, \mathcal{Y}$	Variables
$\langle X, Z, I \rangle$	Fuzzy context
u	Fuzzy union or t-conorm
$u_{\max}$	Drastic fuzzy union

u_w	Fuzzy union of Yager class
W	Set of possible worlds
X	Universal set (universe of discourse)
$\emptyset$	Empty set
$\otimes$	Operation on L corresponding to conjunction (t-norm)
$\rightarrow$	Operation on L corresponding to implication
$\wedge$	Classical operation of conjunction or minimum operation
$\vee$	Classical operation of disjunction or maximum operation

Contents

Chapter 1 Introduction

Robert V. Demicco and George J. Klir

Traditionally, science, engineering, and mathematics showed virtually no interest in studying *uncertainty*. It was considered undesirable and the ideal was to eliminate it. In fact, eliminating uncertainty from science was viewed as one manifestation of progress. This attitude towards uncertainty, prevalent prior to the 20th century, was seriously challenged by some developments in the first half of that century. Among them were the emergence of statistical mechanics, Heisenberg's uncertainty principle in quantum mechanics, and Gödel's theorems that established an inherent uncertainty in formal mathematical systems. In spite of these developments, the traditional attitude towards uncertainty changed too little and too slowly during the first half of the century. While uncertainty became recognized as useful, or even essential, in statistical mechanics and in some other areas (such as the actuarial profession or the design of large-scale telephone exchanges), it was for a long time tacitly assumed that *probability theory* was capable of capturing the full scope of uncertainty.

The presumed equality between uncertainty and probability was challenged only in the second half of the 20th century. The challenge came from two important generalizations in mathematics. The first one was the generalization of classical measure theory [Halmos, 1950] to the *theory of monotone measures*, which was first suggested by Choquet [1953] in his theory of capacities. The second one was the generalization of classical set theory to *fuzzy set theory*, which was introduced by Zadeh [1965]. In the theory of monotone measures, the additivity requirement of classical measures is replaced with a weaker requirement of monotonicity with respect to set inclusion. In fuzzy set theory, the requirement of sharp boundaries of classical sets is abandoned. That is, the membership of an object in a fuzzy set is not a matter of either affirmation or denial, as it is in the case of any classical set, but it is in general a matter of degree.

For historical reasons of little significance, monotone measures are often referred to in the literature as *fuzzy measures* [Wang & Klir, 1992]. This name is somewhat confusing since no fuzzy sets are involved in the definition of monotone measures. However, monotone measures can be fuzzified (i.e., defined on fuzzy sets), which results in a more general class of monotone measures—*fuzzy monotone measures* [Wang & Klir, 1992, Appendix E].

ISBN: 0-12-415146-9

As is well known, probability theory is based on classical measure theory which, in turn, is based on classical set theory [Halmos, 1950]. When classical measures are replaced with monotone measures of some type and classical sets are replaced with fuzzy sets of some type, a framework is obtained for formalizing some new types of uncertainty, distinct from probability. This indicates that the two generalizations have opened a vast territory for formalizing uncertainty. At this time, only a rather small part of this territory has been adequately explored [Klir & Wierman, 1999; Klir, 2002].

Liberating uncertainty from its narrow confines of probability theory opens new, more expressive ways of representing scientific knowledge. As is increasingly recognized, scientific knowledge is organized, by and large, in terms of systems of various types (or categories in the sense of mathematical theory of categories) [Klir & Rozehnal, 1996; Klir & Elias, 2003]. In general, systems are viewed as relations between states of some variables. They are constructed for various purposes (prediction, retrodiction, prescription, diagnosis, control, etc.). In each system, its relations are utilized, in a given purposeful way, for determining unknown states of some variables on the basis of known states of some other variables. Systems in which the unknown states are determined uniquely are called *deterministic*; all other systems are called *nondeterministic*.

By definition, each nondeterministic system involves uncertainty of some type. This uncertainty pertains to the purpose for which the system was constructed. It is thus natural to distinguish between predictive uncertainty, retrodictive uncertainty, diagnostic uncertainty, etc. In each nondeterministic system, the relevant uncertainty must be properly incorporated into the description of the system in some formalized language. To understand the full scope of uncertainty is thus essential for dealing with nondeterministic systems.

When constructing a system for some given purpose, our ultimate goal is to obtain a system that is as useful as possible for this purpose. This means, in turn, to construct a system with a proper blend of the three most fundamental characteristics of systems: *credibility, complexity*, and *uncertainty*. Ideally, we would like to obtain a system with high credibility, low complexity, and low uncertainty. Unfortunately, these three criteria conflict with one another. To achieve high usefulness of the system, we need to find the right trade-off among them.

The relationship between credibility, complexity and uncertainty is quite intricate and is not fully understood yet. However, it is already well established that uncertainty has a pivotal role in any efforts to maximize the usefulness of constructed systems. Although usually undesirable in systems when considered alone, uncertainty becomes very valuable when considered in connection with credibility and complexity of systems. A slight increase in relevant uncertainty may often significantly reduce complexity and, at the same time, increase credibility of the system. Uncertainty is thus an important commodity in the knowledge business, a commodity that can be traded for gains in the other essential characteristics of systems by which we represent knowledge. Because of this important role, uncertainty is no longer viewed in science and engineering as an unavoidable plague, but rather as an important resource that allow us to deal effectively with problems involving very complex systems.

It is our contention that monotone measures and fuzzy sets (as well as the various uncertainty theories opened by these two profound generalizations in mathematics) are highly relevant to geology, and that their utility in geology should be seriously studied in the years ahead. The aim of this book is to demonstrate this point by focusing on the role of fuzzy set theory, and especially the associated fuzzy logic, in geology.

The term "fuzzy logic" has in fact two distinct meanings. In a *narrow sense*, it is viewed as a generalization of classical multivalued logics. It is concerned with the development of *syntactic aspects* (based on the notion of proof) and *semantic aspects* (based on the notion of truth) of a relevant logic calculus. In order to be acceptable, the calculus must be *sound* (provability implies truth) and *complete* (truth implies provability). These issues have successfully been addressed for fuzzy logic in the narrow sense by Hájek [1998].

In a *broad sense*, fuzzy logic is viewed as a system of concepts, principles, and methods for dealing with modes of reasoning that are approximate rather than exact. The two meanings are connected since the very purpose of research on fuzzy logic in the narrow sense is to provide fuzzy logic in the broad sense with sound foundations. In this book, we are concerned only with fuzzy logic in the broad sense, which is surveyed in Chapter 2, and its role in geology, which is the subject of Chapters 3–10.

From the standpoint of science, as it is still predominantly understood, the ideas of a fuzzy set and a fuzzy proposition are extremely radical. When accepted, one has to give up classical bivalent logic, generally presumed to be the principal pillar of science. Instead, we obtain a logic in which propositions are not required to be either true or false, but may be true or false to different degrees. As a consequence, some laws of bivalent logic no longer hold, such as the law of excluded middle or the law of contradiction. At first sight, this seems to be at odds with the very purpose of science. However, this is not the case. There are at least the following four reasons why allowing membership degrees in sets and degrees of truth in propositions in fact enhances scientific methodology quite considerably:

1. Fuzzy sets and fuzzy propositions possess far greater capabilities than their classical counterparts to capture irreducible measurement uncertainties in their various manifestations. As a consequence, their use improves the *bridge between mathematical models and the associated physical reality* considerably. It is paradoxical that, in the face of the inevitable measurement errors, fuzzy data are always more accurate than their crisp (i.e., nonfuzzy) counterparts. Crisp data of each variable are based on a partition of the state set of the variable. The coarseness of this partition is determined by the resolution power of the measuring instrument employed. Measurements falling into the same block of the partition are not distinguished in crisp data, regardless of their position within the block. Thus, for