

# Separation of Molecules, Macromolecules and Particles

Principles, Phenomena and Processes

KAMALESH K. SIRKAR



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# Separation of Molecules, Macromolecules and Particles

Providing chemical engineering undergraduate and graduate students with a basic understanding of how the separation of a mixture of molecules, macromolecules or particles is achieved, this textbook is a comprehensive introduction to the engineering science of separation.

- Students learn how to apply their knowledge to determine the separation achieved in a given device or process.
- Real-world examples are taken from biotechnology, chemical, food, petrochemical, pharmaceutical and pollution control industries.
- Worked examples, elementary separator designs and chapter-end problems give students a practical understanding of separation.

The textbook systematically develops different separation processes by considering the forces causing the separation, and how this separation is influenced by the patterns of bulk flow in the separation device. Readers will be able to take this knowledge and apply it to their own future studies and research in separation and purification.

**Kamalesh K. Sirkar** is a Distinguished Professor of Chemical Engineering and the Foundation Professor of Membrane Separations at New Jersey Institute of Technology (NJIT). His research areas are membranes and novel membrane based processes.

"The first comprehensive book that takes the fundamentals of separation on a molecular level as the starting point! The benefit of this approach is that it gives you a thorough insight in the mechanisms of separation, regardless of which separation is considered. This makes it remarkably easy to understand any separation process, and not only the classical ones. This textbook finally brings the walls down that divide separation processes in classical and non-classical."

*Bart Van der Bruggen, University of Leuven, Belgium*

"This strong text organizes separation processes as batch vs continuous and as staged vs differential. It sensibly includes coupled separation and chemical reaction. Supported by strong examples and problems, this non-conventional organization reinforces the more conventional picture of unit operations."

*Ed Cussler, University of Minnesota*

"This book fills the need by providing a very comprehensive approach to separation phenomena for both traditional and emerging fields. It is effectively organized and presents separations in a unique manner. This book presents the principles of a wide spectrum of separations from classical distillation to modern field-induced methods in a unifying way. This is an excellent book for academic use and as a professional resource."

*C. Stewart Slater, Rowan University*

"This book is an excellent resource for the topic of chemical separations. The text starts by using examples to clarify concepts. Then throughout the text, examples from many different technology areas and separation approaches are given. The book is framed around various fundamental approaches to chemical separations. This allows one to use this knowledge for both current and future needs."

*Richard D. Noble, University of Colorado*

"This book provides a unique and in depth coverage of separation processes. It is an essential reference for the practicing engineer. Unlike more conventional textbooks that focus on rate and equilibrium based separations, Prof Sirkar focuses on how a given separation takes place and how this is used in practical separation devices. Thus the book is not limited by application e.g. chemical or petrochemical separations.

"As chemical engineering becomes increasingly multidisciplinary, where the basic principles of separations are applied to new frontier areas, the book will become an essential guide for practitioners as well as students.

"The unique layout of the text book allows the instructor to tailor the content covered to a particular course. Undergraduate courses will benefit from the comprehensive and systematic coverage of the basics of separation processes. Whether the focus of a graduate course is traditional chemical separations, bioseparations, or separation processes for production of renewable resources the book is an essential text."

*Ranil Wickramasinghe, University of Arkansas*

"This advanced textbook provides students and professionals with a unique and thought-provoking approach to learning separation principles and processes. Prof. Sirkar has leveraged his years of experience as a separation scientist and membrane separation specialist, to provide the reader with a clearly written textbook full of multiple examples pulled from all applications of separations, including contemporary bioseparations. Compared to other separations textbooks, Prof. Sirkar's textbook is holistically different in its approach to teaching separations, yet provides the reader with a rich learning experience. Chemical engineering students and practicing professionals will find much to learn by reading this textbook."

*Daniel Lepek, The Cooper Union*

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# Preface

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This is an introductory textbook for studying separation. Primarily, this book covers the separation of mixtures of molecules; in addition, it provides a significant treatment of particle separation methods. Separation of macromolecules has also received some attention. The treatment and coverage of topics are suitable for chemical engineering students at undergraduate and graduate levels. There is enough material here to cover a variety of introductory courses on separation processes at different levels.

This book is focused on developing a basic understanding of how separation takes place, and of how the resulting separation phenomenon is utilized in a separation device. The role of various forces driving molecules or particles from a feed mixture into separate phases/fractions/regions is basic to such an approach to studying separation. The separation achieved is then amplified in an open separator via different patterns of bulk-phase velocities vis-à-vis the direction(s) of the force(s). The forces are generated by chemical potential gradient, electrical field, rotational motion, gravity, magnetic field, etc. The resulting separation is studied under three broad categories of separation processes.

Separation processes driven by a negative chemical potential gradient are generally multiphase systems and are treated under the broad category of phase equilibrium driven processes. External force driven processes populate the second category, and include those operating under an electrical field, rotational motion, magnetic field or gravity; thermal diffusion processes are also briefly included here. The third category of membrane based processes studied is driven generally by a negative chemical potential gradient; however, electrical force is also relevant for some processes. The treatment of any external force driven processes will cover both separation of molecules and particle separations.

These physical separation methods are often reinforced by chemical reactions, which are usually reversible. An elementary treatment of the role of chemical reactions in enhancing separation across a broad spectrum of phase equilibrium driven processes and membrane based processes has been included. The level of treatment in this book assumes familiarity with elementary principles of chemical engineering thermodynamics and traditional

undergraduate levels of knowledge of ordinary differential equations and elementary partial differential equations.

Specific aspects of a given separation process are studied in the chapter devoted to those aspects for all separation processes. To study a particular separation process in great detail, one therefore has to go to different chapters. The footprints of a given separation process are provided at the beginning of the book (Tables 1–7); there are quite a few tables to cover a variety of separation processes. The list of processes is large; however, it is far from being all inclusive. The introductory chapter, which provides additional details about various chapters, as well as about the book, is preceded by a notation section. All references appear at the end of the book.

The description of the extent of separation achieved in a closed vessel for a mixture of molecules is treated in Chapter 1. Chapter 2 illustrates how to describe the separation of molecules in open separators under steady and unsteady state operation; a description of separation for a size-distributed system of particles is also included. Chapter 3 introduces various forces developing species-specific velocities, fluxes and mass-transfer coefficients, and illustrates how the spatial variation of the potential of the force field can develop multicomponent separation ability. The criteria for chemical equilibrium are then specified for different types of multiphase separation systems, followed by an illustration of integrated flux expressions for two-phase and membrane based systems.

Chapter 4 develops the extent of separation achieved in a closed vessel to a variety of individual processes under each of the three broad categories of separation processes. Chapter 5 demonstrates how separation can be considerably enhanced by chemical reactions in phase equilibrium based and membrane based processes under both equilibrium- and rate-controlled conditions. For open separators having bulk flow in and out, including continuous stirred tank separators (CSTs), Chapter 6 provides first the equations of change for molecular species concentration in single-phase and two-phase systems, the trajectory equation for a particle in a fluid and the general equation of change for a particle population. Chapter 6 then treats individual separation processes under each of the three

broad categories of separation processes when the bulk flow is parallel to the direction of the force and in CSTS mode.

Chapter 7 follows this latter approach of treating individual separation processes under each of the three broad categories of separation processes when the bulk flow of feed-containing phase is perpendicular to the direction of the force. Chapter 8 follows the same approach when the bulk flows of two phases/regions in the separator are perpendicular to the direction(s) of the force(s). Chapter 9 briefly elaborates on cascades, which were already introduced in the countercurrent multistaged flow systems of Chapter 8. Chapter 10 introduces the energy required for a number of separation processes. Chapter 11 illustrates a few common separation sequences in a number of common industries involved in bioseparations, water treatment, chemical and petrochemical separations and hydrometallurgy. Conversion factors between various systems of units are provided in an Appendix.

Virtually all separation processes taught to chemical engineering students in a variety of courses have been covered via the approach illustrated in Chapters 3, 4, 6, 7 and 8; in addition, many particle separation methods have been treated. The structural similarity in the separation method between apparently unrelated separation processes becomes quite clear. A few basic principles equip the students with the capability to understand a wide variety of separation processes and techniques, including emerging ones. To aid the student, there are 118 worked examples, 300 problems, 340 figures, 100 tables and 1011

references. A website will provide guidance for computer simulations for a few selected problems.

The introductory chapter provides references to articles and books which influenced the development of various aspects of this book. I have benefitted considerably from the comments on selected chapters of the book by reviewers, anonymous or otherwise. Comments by Professors C. Stewart Slater, of Rowan University, Steven Cramer, of Rensselaer Polytechnic Institute, and Ranil Wickramasinghe, of Colorado State University (now at University of Arkansas), were particularly useful.

Many doctoral students and postdoctoral fellows were of invaluable help during the long gestation period of this book, either in formulating solutions of the problems or in developing illustrative drawings. I want to mention in particular Amit Sengupta, Theoharris Papadopoulos, Xiao-Ping Dai, Meredith Feins, Dimitrios Zarkadas, Quixi Fan, Praveen Kosaraju, Fei He, Atsawin Thongsukmak, Sagar Roy, Dhananjay Singh, John Tang and John Chau. The first two students helped me when we were at Stevens Institute of Technology. Sarah Matthews of Cambridge University Press patiently provided manuscript preparation guidelines and encouraging comments during an ever-shifting timetable. Irene Pizzie did an extraordinary job as the copy editor. Brenda Arthur of New Jersey Institute of Technology tirelessly typed the draft of the whole manuscript over a considerable length of time, while carrying out many other duties.

I must also mention at the end my wife, Keka, without whose patience, help and understanding this book would never have been finished.

# Notation

Equation numbers identify where the symbols have been introduced or defined.

The following styles have been adopted.	
<b>Bold</b>	vector quantity
<b>Overlines</b>	
—	quantity averaged over time or a specific coordinate direction, multicomponent system, Laplace transformed dependent variable
— =	averaged quantity
^	quantity in a mixture, per unit mass of bulk phase
<b>Underlines</b>	
—	hypothetical binary system quantity (2.4.23), (2.4.24)
~, =	vector quantity, tensor quantity
<b>Brackets</b>	
$\langle v_{ij} \rangle$	average value of $v_{ij}$ over surface area $S_j$
$a$	ellipsoid semiaxis dimension; see also (3.1.10a); constant in relations (3.1.49), (4.3.7) and (4.3.43a); interfacial area per unit volume, defined by (7.2.191), (7.3.25)
$a_1; a_2, a_3, \dots$	constant in (3.3.105), (4.1.42b), (4.3.29) and (5.2.147); constants in (7.2.73), (7.3.50) and (7.3.139)
$a_1$	constant in (7.2.198a)
$a_A$	stoichiometric coefficient for species A
$a_H$	Hamaker constant (3.1.16)
$a_i; a_i(T); a_{ij}; a_{ii};$	activity of species $i$ ; equilibrium
$\hat{a}_i; a_m; a_{ms\ell}$	constant (7.1.63); atom fraction of $i$ th isotope of an element in region $j$ (1.3.6), value of $a_i$ in region $j$ and liquid phase, respectively; amplitude (7.1.72b); constant in (4.3.43c); membrane surface per unit channel length (7.2.70)
$a_p$	surface area of a particle
$a_{s1}, a_{s2}; a_{su}$	activity of solvent $s$ in regions 1 and 2, respectively; activity of salt in water
$a_{sp}$	pore surface area per unit volume of the porous medium of porosity $\varepsilon$
$a_v, a_{vc}$	surface area of a particle per unit particle volume, value of $a_v$ in a cake (6.3.135j)
$a_{\pm}$	mean electrolyte activity (3.3.119d)
$A; A_1, A_2, A_3$	amplitude (7.3.18), pure-water permeability constant in reverse osmosis and diffusive ultrafiltration; constants in (7.1.90b), three surface areas in control volume of Figure 7.2.6(b)
$A(r)$	cross-sectional area of a cone in centrifugal elutriation
$A_c$	cross-sectional area of duct
$A_{hexi}; A_{ij}; A_{mi};$	heat exchanger surface area; constant in equations for activity coefficients (4.1.34d); surface area of membrane;
$A_m^t; A_i(T); A_i^0$	total membrane surface area; modified equilibrium constant (7.1.66); constant (7.1.72b)
$A_{1p}$	constant in crystal growth rate equation (6.4.27)
$A_p$	projected area of a particle (3.1.64); transport coefficient in solution-diffusion-imperfection model (3.4.60a)
$A_p^+(x)$	cumulative crystal surface area distribution fraction (6.4.17)
$A_T$	total particle surface area per unit volume of total mixture (Example 2.4.2); total crystal surface area per unit liquid volume (6.4.16)
$Am^+, Am^-, Am^{\pm}$	three forms of amino acid
$b$	ellipsoid semiaxis dimension; proportionality constant in osmotic pressure relation (3.4.61b); half of channel gap; width of region of gas flow completely cleaned up by a fiber (6.3.42a); constant in crystal growth rate expression (6.4.35); parameter (7.2.18); liquid envelope radius (7.2.208)



$b_1, b_2, b_3$	constants in relations (3.1.143f)	$C_{\bar{i}}; \bar{C}_i; C_i^t; C_i^*$	molar concentration of species $i$ ; an
$b_1$	constant in (3.3.105), (5.2.147)	$C_{ig}^*; C_{il}^t, C_{il}^i$	average of the molar concentration of
$b_f; b_{\bar{i}}; b_{if}; b_{ie}; b_1^i$	membrane feed channel height (Figure 7.2.3(a)); constant in equilibrium relation (3.3.81); constant in (7.1.73), constant in Freundlich isotherm (3.3.112c); constant in Langmuir isotherm (3.3.112a); constant in equation (4.1.42a)		species $i$ in the feed and the permeate (5.4.74), (6.3.158b); total molar concentration of species $i$ in the porous medium per unit volume (3.1.118b); nondimensional species $i$ concentration (5.3.35j), hypothetical gas-phase species $i$ concentration (8.1.47); initial bed concentration of solute $i$ in phases 1 and 2
$b'_{i-k}, b'_k$	proportionality constant in (5.2.154), equilibrium constant in (5.2.155)		values of $C_{il}, C_{i2}$ at $z = 0$
$b_m, b'_m$	constants in (4.3.43a,c)	$C_{i10}, C_{i20}$	liquid-phase concentration of pure solute $i$ at spreading pressure $\pi$ and temperature $T$ providing the same surface phase concentration of $i$ as the mixture
$\mathbf{B}$	magnetic induction vector (3.1.19)	$C_i^o(\pi)$	
$B; B^o; B_i^o; B_p$	constant, density function of the birth rate of new particles (6.2.50g), value of $B$ as $r_p \rightarrow 0$ ; value of $B^o$ for crystal growth rate $gr_i$ ; duct perimeter (7.3.41)		
$Bi$	Biot number (3.4.35)	$C_{igel}$	molar species $i$ concentration in gel
$B_{1p}, B_{2p}$	second virial coefficients for interaction between polymers 1 and 2 (4.1.34p)	$C_{ij}$	molar concentration of species $i$ in region $j$ or location $j$ or stream $j$ ; $j = b$ , bulk; $j = E$ , extract; $j = f$ , feed region; $j = g$ , gas phase; $j = k$ , $k$ th phase; $j = \ell$ , liquid; $j = \ell'$ , adhering liquid phase on crystal; $j = m$ , membrane; $j = o$ , organic; $j = p$ , permeate, product; $j = r$ , raffinate; $j = R$ , ion exchange resin phase, raffinate; $j = s$ , solution, solid phase or pore surface; $j = w$ , water, mol/liter
$c; c_C$	velocity of light; gap between plates at entrance (Figure 7.3.10), stoichiometric coefficient for species $C$		total molar concentration of species $i$ in region $j$ including complexed or dissociated forms; value of $C_{ij}$ on $n$ th plate/stage
$C$	clearance of a solute (8.1.390)		species $i$ concentrations in phase $j$ at locations 0 (or initial concentration), $\ell$ and $\delta$
$C_1, C_2; C_2(x)$	integration constants (3.3.10b); molar concentration of species 2 at location $x$	$C_{ij}^t; C_{ijn}$	
$C_{2f}$	molar concentration of species 2 in initial mixture, mol/liter		
$C^{(2)}$	defined in (6.3.75)	$C_{ij}^0, C_{ij}^t, C_{ij}^\delta$	
$C_A^0, C_A^\delta$	molar concentration of A at $z = 0, \delta$ (3.1.124)		
$C_{A\ell b}, C_{A\ell i}$	molar concentration of A in phase bulk and phase interface and in liquid at gas-liquid interface (3.4.1b)	$C_{i\ell b}, C_{i\ell e}, C_{i\ell i}$	molar concentrations of species $i$ in bulk solution, at the end of concentration process and at the beginning of concentration process (6.3.173)
$C_{Awb}, C_{Aw i}, C'_{Awb}$	molar concentration of A in bulk water and in water at phase interface (3.4.45e); critical value of $C_{Awb}$ for maximum enhancement (5.3.53)	$C'_{ig}, C'_{igf}$	molar species $i$ concentration per unit gas phase volume in a pore
$C_{Bob}, C_{Boi}$	molar concentration of species B in organic-phase bulk and interface, respectively	$\bar{C}_{ik}, \bar{\bar{C}}_{ik}, \hat{C}_{ik}; \bar{C}_{it}$	intrinsic phase average, phase average and deviation in $C_{ik}$ for species $i$ concentration in phase $k$ (6.2.24a,b), (6.2.28); defined by (7.1.94)
$C_c; C'_{Cb}; C_{CD}$	slip (Cunningham) correction factor (3.1.215); critical value of bulk concentration of C for maximum enhancement (5.3.29b); particle drag coefficient (3.1.64), (6.3.4)	$C_{im}^d, C_{im}^H; C_{im}^p$	species $i$ concentration in membrane: for Henry's law and Langmuir species, respectively, in dual sorption model (3.3.81); membrane pore liquid
$C_{FC}$	molar concentration of fixed charges in ion exchange resin	$C_{imib}, C_{imoi}, C_{iobr}$	molar concentrations of species $i$ at various locations in Figure 3.4.11
$C'_{H1}, C'_{H2}$	dual mode sorption constants for species 1 and 2 (3.3.82a,b)	$C_{iwb}, C'_{iwb}$	molar protein concentrations in resin phase and aqueous solution
		$C_{pr}, C_{pw}$	

$C_{sf}, C_{sm}, C_{sp}$	molar solvent concentration in feed, membrane and permeate, respectively	$D_{im}, D_{imo}, D_{ip}$	diffusion coefficient of $i$ in membrane (6.3.149); value of $D_{im}$ for $C_{im} = 0$ (3.4.67b), $D_i$ in a pore (3.4.89c), (6.3.145a); effective $D_i$ in the pores of a particle
$C_i, C_{tj}, C_{vp}$	total molar concentration; $C_t$ in a mixture in region $j$ ; volume of particles per unit fluid volume (7.2.176)	$\bar{D}_{ik}, \bar{D}_{im}$	multicomponent diffusion coefficient of species pair $(i, k)$ and $(i, m)$ in Maxwell–Stefan approach
$d; d_h; d_i; d_{tm}$	diameter of tube/pipe/vessel; hydraulic diameter (Table 3.1.8); effective diameter of a molecule of gas species $i$ (3.3.90a); logarithmic mean diameter (8.1.417)	$D_{iK}$	Knudsen diffusion coefficient for species $i$ (3.1.115c)
$d_i; d_{gr}; d_{imp}$	force-type term (3.1.178), (3.1.181); grain diameter; diameter of an impeller	$D_{iM}$	effective binary diffusivity of species $i$ in a mixture (3.1.184), (3.1.185)
$d_{ion}, d_p, d_w; d_{p1}, d_{p2}$	mean diameter of a molecular ion; mean diameter of a particle (6.1.4b); wire diameter (3.1.23); diameters of particles 1 and 2	$D_{is}, D_{is}^0, D_{is}^N$	binary diffusion coefficient for solute $i$ /solvent $s$ ; value of $D_{is}$ at infinite dilution; Nernst–Planck binary diffusion coefficient for species $i$ /solvent $s$
$d_{32}$	Sauter mean diameter of a drop or particle (6.4.88), (6.4.89)	$D_{12}$	binary diffusion coefficient for species 1 and 2
$D$	diffusion coefficient of species in countertransport through liquid membrane	$De, (De)_{mv}$	density function of particles which disappear (die) (6.2.50h); see (7.2.170a) and (7.2.172)
$D_A, D_C, D_B$	diffusion coefficient of species A and C, respectively; dialysate in hemodialysis (8.1.389)	$Df, DF$	decontamination factor (2.2.1c); dilution factor (6.4.106), (7.2.91c)
$D_{eff}$	effective diffusion coefficient (5.4.64a)	$e$	charge of an electron, $1.60210 \times 10^{-19}$ coulomb
$D_{gr}$	crystal growth diffusivity (6.4.45)	$e_i$	constant in adsorption isotherm for solute $i$ (3.3.113d)
$D_p; D_p(\phi)$	diffusion coefficient for particle (3.1.68), (6.2.52); shear-induced particle diffusivity (3.1.74), (7.2.126), (7.2.131a)	$en_b, en_m, en_{MN}, en_{MP}, en_p$	molecular energy in the bulk, due to intramolecular interactions, due to intermolecular interactions, due to interaction between molecules and pores and total energy for molecules in the pore (3.3.89d)
$D_r$	desalination ratio (1.4.25), (2.2.1a)	$E; E; E_c; E_{Di}; E_e$	electrical force field; its magnitude (3.1.8), (6.1.22), (6.3.8f), extraction factor (8.1.281), stage efficiency (6.4.72); electrical field strength $E_c$ (7.3.32a); activation energy for diffusion of species $i$ in polymer (4.3.46b); extraction factor for extraction section (8.1.303)
$D_{i,eff}, D_{i,eff,r}$	effective diffusion coefficient of $i$ in liquid (6.2.18), (6.3.16b); value of $D_{i,eff}$ in $r$ - and $z$ -directions; value of $D_{i,eff}$ in phase/region 1	$E_{BRS}, E_{GrS}$	enrichment of species $i$ by pervaporation (6.3.193b), particle collection efficiency (7.2.200b); overall column efficiency (8.1.195); point efficiency (8.3.13); extraction factor for the scrubbing section; electrical field strength in $y$ -direction (7.3.48); particle collection efficiency (7.2.219), (7.2.214)
$D_{i,eff,k}; \bar{D}_{i,eff,k}$	effective diffusion coefficient of $i$ in phase $k$ (6.2.33); dispersion tensor (6.2.31)	$E_i; E_o; E_{oG}; E_s; E_y;$	inertial impaction based single fiber capture efficiency (6.3.42a); particle capture efficiency by interception (7.2.224)
$D_{ij}, D_{Bj}, D_{is}$	diffusion coefficient of species $i$ in region $j$ ; $j = l$ , liquid; $j = s$ , solvent		
$D_A^T, D_B^T, D_{is}^T$	thermal diffusion coefficient for species A and B (3.1.44); for species $i$ in solution		
$D_{AB}, D_{BR}$	binary diffusion coefficient for mixture of gases A and B; diffusion coefficient of particles due to Brownian motion (7.2.216)		
$D_{iD}, D_{iH}$	diffusion coefficients in dual sorption–dual transport model (3.4.78)		
$D_{ie}$	effective diffusion coefficient of $i$ in a porous medium (3.1.112d)		

$E_{ME}, E_{MR}, E_{MV}, E_N$	Murphree extract stage efficiency, Murphree raffinate stage efficiency (6.4.70), (6.4.71); Murphree vapor efficiency (8.1.198); Newton particle separation efficiency (2.4.14a)	$F; \hat{F}, \hat{F}_p; F$	force on a particle; value of $F$ per unit particle mass; degrees of freedom (4.1.22)
$E_T, E_T^1; E_{T_i}$	total efficiency in solid–fluid separation (2.4.4a), overall filter efficiency (6.3.45), (7.2.201); reduced efficiency of Kelsall (2.4.16a); $E_T$ for $i$ th solid–fluid separator (2.4.17c,d)	$F(r_p)$	probability distribution function corresponding to $f(r_p)$ (2.4.1c), crystal size distribution function (6.4.11)
$f$	friction factor (6.1.3a), fractional consumption of chemical adsorbent (5.2.19d)	$F_b, F_i^{\text{ext}}$	electrostatic force on 1 gmol of a charged species in solution (6.3.8a); magnitude of external force on 1 gmol of species $i$
$f_2$	fraction of the solute in ionized form ( $i = 2$ ) in RO (5.4.4)	$F_{\text{acrx}}$	acoustic radiation force in $x$ -direction (3.1.48)
$f(r); f(r_p); f(\varepsilon)$	molar density function in a continuous/semi-continuous mixture with characteristic property $r$ ; particle size probability density function (2.4.1a), pore size distribution function in a membrane; defined by (7.2.222a,c)	$F_{\text{rad}}$	radiation pressure force (3.1.47), (7.3.267)
$f_f(r_p), f_1(r_p), f_2(r_p)$	value of $f(r_p)$ for feed stream, overflow and underflow based on particle weight fraction in a given size range (2.4.1b)	$F^{\text{BR}}$	force on very small particle due to random Brownian motion (3.1.43)
$f_A, f_i^0, f_i$	fugacity of species A, species $i$ ; standard state fugacity of species $i$	$F_i^{\text{ELK}}$	electrokinetic force on particle in double layer (3.1.17)
$f_g(\text{gr})$	probability density function of crystal growth rate (6.4.41a)	$F_i^{\text{ELS}}$	electrostatic force on particle $i$ , Coulomb's law (3.1.15)
$f_{il}$	fugacity of pure species $i$ in liquid phase	$F_i^{\text{Lret}}$	London attraction force (3.1.16)
$f_m, f'_m$	quantities characteristic of a membrane polymer (4.3.46a,d)	$F_k^{\text{m}}$	force on species $k$ in mass flux $j_k$ , force relation (3.1.202)
$f_{ij}^0, f_{ig}^0, f_{il}^0$	standard state fugacity of species $i$ in region $j$ ; $j = g$ , gas phase; $j = \ell$ , liquid phase	$F_{\text{net}}^{\text{ext}}$	net external force; for gravity see (3.1.5)
$f_{i,i}^d, f_p^d$	frictional coefficient for species $i$ and spherical particle	$F_{\text{drag}}, F_p^{\text{inert}}, F_{pz}^{\text{ext}}$	frictional force on a particle; inertial force on a particle (6.2.45); external force on a particle in $z$ -direction
$\hat{f}_{ig}, \hat{f}_{ij}, \hat{f}_{il}, \hat{f}_{ijpl}$	value of fugacity of $i$ in a mixture in gas phase, phase $j$ and liquid phase, respectively; value of $\hat{f}_{ij}$ for a planar surface	$F_{ii}, F_{ii}^{\text{ext}}$	total force and total external force on 1 gmol of species $i$ (3.1.50)
$f_{io}^d$	value of $f_i^d$ for a sphere of equivalent volume (3.1.91e)	$F_{tp}^{\text{ext}}, F_{tpx}^{\text{ext}}, F_{tpy}^{\text{ext}}, F_{tpz}^{\text{ext}}$	total external force on a particle (3.1.59), (6.2.45); components of $F_{tp}^{\text{ext}}$ in $x$ -, $y$ - and $z$ -directions
$f_{im}^d, f_{sm}^d$	frictional coefficient for solute $i$ and solvent $s$ in a membrane	$F_{\text{TA}}$	force on 1 gmol of species A due to a temperature gradient (3.1.44)
$f_f(M), f_l(M), f_v(M); f_\infty, f_\lambda$	value of $f(r)$ , where $r = M$ , molecular weight, for the feed mixture, liquid fraction and the vapor fraction, respectively; defined by (7.1.59a), (7.2.187), respectively	$\mathcal{F}$	Faraday's constant, 96 485 coulomb/gm-equivalent
$f_{Mi}, f_{Qm}, f_{yo}$	fraction of the total metal ion concentration in the aqueous phase present as $M^{n+}$ (5.2.97); probability density functions (7.3.79), (7.3.80)	$g; g_c; g_m$	acceleration due to gravity; conversion factor; a quantity characteristic of a membrane polymer (4.3.46a)
		$g^{\text{ext}}, g_x^{\text{ext}}, g_y^{\text{ext}}, g_z^{\text{ext}}$	external body force per unit mass; its components in $x$ -, $y$ - and $z$ -directions
		$gr_i$	intrinsic growth rate of $i$ th crystal (6.4.41a)
		$G; G_g$	superficial mass average velocity based on empty flow cross section, $G$ for gas phase
		$G; G_a, G_b; G_i; G_o$	growth rate of crystal (6.4.25), (6.4.3b); value of $G$ under condition a, condition b; factor representing contribution of species $i$ properties to $Q_{im}$ (4.3.56a); constant

$\overline{G_c}; \overline{G_{Dr}}; \overline{G_{D\mu}}$	convective hindrance factor (3.1.113); drag factor reducing solute diffusion by hindrance (3.1.112e); function of particle volume fraction in hindered settling (4.2.61)	<b>j</b> $\mathbf{j}_i; j_{ix}, j_{iy}, j_{iz}$	unit vector in positive y-direction; mass flux vector of species $i$ , $M_i \mathbf{j}_i$ (3.1.98), Tables 3.1.3A, 3.1.3B, (6.2.5n); components of $\mathbf{j}_i$ in x-, y- and z-directions
$\overline{G_i}; \overline{G_{ij}}$	partial molar Gibbs free energy of species $i$ , ratio of solute $i$ velocity to the averaged pore solvent velocity, convective hindrance factor ( $=\overline{G_c}$ ) (3.1.113), (3.4.89b); value of $\overline{G_i}$ in region $j$	$J_D$ $\mathbf{J}_i; J_i^*; (J_i)_k; J_1, J_1^T$	factor defined by (3.1.143g); molar flux vector of species $i$ (3.1.98), (3.1.99), Tables 3.1.3A, 3.1.3B; value of $\mathbf{J}_i$ in region $k$ ; diffusive molar flux vector of species 1 (4.2.63); temperature gradient driven molar flux vector of species 1 (4.2.62)
$G_r(r_p)$ $G_{ij}; G_{crit}$	grade efficiency function (2.4.4b) total Gibbs free energy of all molecules in region $j$ (3.3.1), (4.2.23); defined by (7.1.58e)	$J_{iz}, J_{sz}; J_{iy}^*$ $J_{iz}^*, J_{jz}^*, J_{sz}^*, J_{Az}^*$	z-components of flux vectors $\mathbf{J}_i$ and $\mathbf{J}_s$ ; y-component of flux vector $\mathbf{J}_i^*$
$Gr; Gr(\beta_r, \sigma_v); Gz$	Grashof number (3.1.143e); function defined by (7.2.174); Graetz number (8.1.276)	$J_{Ay}^T$ $J_{\nu z}$	z-components of flux vectors $\mathbf{J}_i^*, \mathbf{J}_j^*, \mathbf{J}_s^*$ and $\mathbf{J}_A^*$ total molar flux of species A in y-direction (5.4.51) volume flux through membrane in z-direction (3.4.60c), (6.3.155a)
$h; h_o$	membrane flow channel height, distance between particle and collector (3.1.17), constant in (4.1.9a) for Henry's law constant, $(1/h)$ is a characteristic thickness of double layer (6.3.31a), height of liquid in a capillary at any time $t$ ; value of $h$ as $t \rightarrow \infty$ (6.1.11)	<b>k; k</b> $k^B$ $k_a, k_d$	unit vector in positive z-direction; region or phase, constant in (2.2.8a-c), $(2\pi/\lambda)$ (3.1.48) Boltzmann's constant (3.1.72), (3.3.90c)
$h_+, h_-, h_G$	contributions of different species to $h$ (4.1.9c)	$k_b, k_f$	rate constants for adsorption and dissociation, respectively (4.1.77a) backward and forward reaction rate constants (5.4.42)
$h_{min}$ $H; H_j; H_i$ $H_i^C, [H_i^C]'; H_i^P$ $H_{lf}, H_{vf}$	minimum value of $h$ plate height, stack height; molar enthalpy of feed; value of $H$ for species $i$ (6.3.22), Henry's law constants for species $i$ in gas-liquid equilibrium; (3.3.59) (4.1.7); (3.4.1b), (5.2.6); (5.2.7); (8.1.49), (3.4.1a); molar enthalpy of liquid fraction and vapor fraction, respectively, of the feed	$k_{Ao}, k_{Aw}$ $k_c, k_g, k_{xj}, k_y$ $k'_c, k'_g, k'_x, k'_y$ $k_d, k_s, k_\ell$	mass-transfer coefficient of species A in organic or water phase mass-transfer coefficients for species $i$ (3.1.139), (3.4.3) values of $k_c, k_g, k_x$ and $k_y$ for equimolar counterdiffusion (3.1.124)
$H_{o\ell}, H_{o\ell P}$	height of a transfer unit defined by (8.1.96)	$k_{igc}, k_{igx}, k_{igy}$	mass-transfer coefficients in crystallization (3.4.23a,b); liquid film mass transfer coefficient (5.3.3)
<b>H<sup>m</sup></b> $H_A, H_A^o, H_A^c$ $H_1, H_2$ $H_D, H_M, H_S, H_{SM}$ $\overline{H}_i$ HTU	magnetic field strength vector Henry's law constants for species A (3.4.1a,b), (3.4.8); defined by (7.1.20b) components of plate height (7.1.107e-i) partial molar enthalpy of species $i$ height of a transfer unit (6.4.85), (8.1.54b), (8.1.57b), (8.1.65e), (8.1.245b), (8.1.247a), (8.1.357b)	$k_{T}; k'_T$ $k_{gf}, k_{gs}$	mass-transfer coefficients for species $i$ in gas phase when the concentration gradient is expressed in terms of $C$ , molar concentration of species $i$ in gas phase, $x$ , mole fraction of species $i$ in gas phase and similarly $y$ , mole fraction in gas phase, respectively thermal diffusion ratio (3.1.45); thermal diffusion constant (4.2.64) gas film mass-transfer coefficient on feed side and strip side of a liquid membrane (5.4.97a), (5.4.99a)
<b>i</b>	unit vector in positive x-direction, current density (3.1.108c)	$k_{ef}, k_{es}; k_{pp}$	liquid film mass-transfer coefficient on feed side and strip side of a liquid membrane (5.4.97b), (5.4.99b); particle mass-transfer coefficient (7.2.217b)
$I; \mathbf{I}; I_j; I(C_{isbl}^+)$	ionic strength of the solution (3.1.10c), (4.1.9b); purity index (1.4.3b), current; value of purity index for region $j$ (1.4.3b); integral (7.2.86)		

$k_{s+}, k_{s-}$	rate constants for forward and backward interfacial reactions (5.3.40)	$K_{ijc}; k_{ijx}, k'_{ijx}$	molar concentration based overall mass-transfer coefficient for phase $j$ (8.1.1c); $j$ phase mass-transfer coefficient (8.1.60); value of $k_{ijx}$ for equimolar counterdiffusion (8.1.62a)
$k'_{il}; \bar{k}'_{il}$	distribution ratio of species $i$ between regions 1 and 2, also called capacity factor (1.4.1); distribution ratio defined by (2.2.19) for species $i$ between streams 1 and 2	$K_o, K_w$	overall mass-transfer coefficient based on organic or aqueous phase, ionization product for water (5.4.41c)
$k_{cR}, k_{cE}$	mass-transfer coefficient in the continuous phase, raffinate based, extract based (6.4.97a,b)	$K_{is}, K_{ps}$	values of $K$ for ion $i$ /protein (p)-salt (s) exchange on an ion exchange resin (7.1.109d), (3.3.122b)
$k_{gr}, k_{nu}$	rate constant for crystal growth and nucleation, respectively (6.4.51)	$K_{xE}, K_{xR}$	overall mass-transfer coefficient $K_x$ (3.4.5) based on extract phase and raffinate phase, respectively (6.4.77), (6.4.81)
$k_{il}; k_{ilB}, k_{ilA}, k_{ilO}; k_{imo}$	mass-transfer coefficients for species $i$ in liquid phase; value of $k_{il}$ for condition 3, condition 4, channel inlet; species $i$ mass-transfer coefficient through organic filled membrane pore	$l; \ell$	length of a device, length of molten zone in zone melting (6.3.109b), characteristic dimension of the separator; constant in (2.2.8a-c), length
$k_1$	first-order reaction rate constant (5.3.7)	$\ell_{ik}, \ell_{ki}$	phenomenological coefficients (3.1.203)
$k_{1m}$	membrane mass-transfer coefficient for species 1 (4.3.1)	$\ell_{loc}$	characteristic length of a local volume corresponding to a point in volume averaging Section 6.2.1.1
$K$	equilibrium constant for a chemical reaction (3.3.68), or an ion exchange process (3.3.121i), a constant (6.3.49)	$\ell_x, \ell_y, \ell_z$	dimensions of a rectangular separator, Figure 3.2.1
$K^x, K^s$	mole fraction based $K$ for a chemical reaction (5.2.35); defined by (5.4.100)	$L; L_f, L^+$	length of a separator, dimension of length, characteristic crystal size; molar feed flow rate; nondimensional $L$ (7.2.38)
$K_1, K_2, K_3$	constants in membrane transport (6.3.155a,b)	$L_{ib}, L_{is}, L_{ss}$	phenomenological coefficients for binary system ( $i, s$ ) (3.1.208), (3.1.209)
$K_{Ao}, K_{Aw}, K_{AB}$	overall mass-transfer coefficient of species A based on organic or water phase; equilibrium constant (7.1.42c)	$L_{ik}, L_{ki}, L_{iT}$	phenomenological coefficients (3.1.205)
$K_c, K_g, K_x, K_y$	overall mass-transfer coefficients (3.4.5), (3.4.6)	$L_p; L_p^a; L_p^b$	hydraulic transport parameter in Kedem-Katchalsky model (6.3.158a); value of $L_p$ in perfect region; value of $L_p$ in leaky region
$K_{cE}$	$K_c$ based on extract phase (6.4.80)	$L_T; L_{min}; L_{MTZ}; LUB$	separator length ( $= L$ ); (7.1.60); Figure 7.1.5(b); (7.1.21g)
$K_C^A$	equilibrium constant for ion exchange reaction (5.2.122)	$m; m_B; m_i; m_1; m_i^0$	velocity profile constant (7.3.134); moles of B; moles of species $i$ in separator; moles of species 1 in separator; total number of moles of species $i$ in separator
$K_d$	equilibrium constant for protein-ion exchange resin binding (4.1.77c), ionization equilibrium constant (5.2.4)	$m_{ij}; m_{ij}^0; m_{ij}(n); m_{ij}^a$	moles of species $i$ in region $j$ , total number of moles of $i$ in region $j$ at $t = 0$ ; moles of $i$ in region $j$ after $n$ th contact; number of atoms of $i$ th isotope of the element in region $j$
$K_{d1}, K_{d2}$	dissociation constant for solutes 1 and 2, respectively (5.2.61a), (6.3.29)	$m_{i\sigma}$	moles of species $i$ in interfacial region $\sigma$
$K_i; K_1, K_2, K_3; K_i^a; K_i^\infty$	equilibrium ratio of species $i$ between regions 1 and 2 (1.4.1) or (3.3.61); value of $K_i$ for species 1, 2 and 3; values of $K_i$ in terms of activities (4.1.3); $f_{il}^o \gamma_{il}^\infty / P$ in dilute solution stripping (4.1.19b)		
$K_{il}; K_\ell$	overall liquid-phase mass-transfer coefficient for species $i$ (7.1.5a); reaction equilibrium constant in the liquid phase based on molar concentrations (5.2.52a)		

$m_{ij}$	molality, moles of $i$ per kilogram of solvent in region $j$ : $j = R$ , resin; $j = w$ , aqueous phase	$n_{\max}$ , $n_{\text{med}}$ , $n_{\text{par}}$	peak capacity (3.2.32) (6.3.26a); refractive index of medium and particle
$m_{F,R}$	molality of fixed charges in the resin phase	$n_p$ , $n_p$ ; $n_{py}$ , $n_t$	particle number flux, (3.1.65), (3.1.66), (3.1.68); particle flux in $y$ -direction; number of turns by gas in a cyclone (7.3.146b)
$m_p$ ; $m_{sl}$	mass of particle; solvent moles in stationary liquid phase (7.1.104b)	$N$ ; $N(r_p)$ ; $N(r_{\min}, r_p)$ , $N(r_{p_{\max}})$	total number of stages in a multistage device or in the enriching section of a cascade, anionic species in Donnan dialysis, a metal species, number
$m_{ij}$	total moles of all species in region $j$ ( $j = f$ , feed; $j = 1$ , vapor phase; $j = 2$ , $\ell$ , liquid phase)		concentration of molecules; numbers/cm <sup>3</sup> , number of particles per unit fluid volume in the size range of $r_{\min}$ to $r_p$ ; value of $N(r_{\min}, r_p)$ for $r_{p_{\max}}$
$m_{11}(t)$ , $m_{21}(t)$	moles of species 1 and 2 in region 1 at time $t$	$\tilde{N}$ ; $N_i$	Avogadro's number ( $6.02 \times 10^{23}$ molecules/gmol); plate number for $i$ (6.3.27a)
$m_{11}^r(t)$ , $m_{21}^r(t)$	values of $m_{11}(t)$ and $m_{21}(t)$ in the case of a chemical reaction	$N_A$ ; $N_A^r$ , $N_{Ay}^t$ ; $N_{\text{dil}}$	molar flux of species A in a fixed reference frame without and with reaction; total molar flux of species A in facilitated transport or counter-transport or co-transport in $y$ -direction; normality of diluate solution (8.1.404)
$mo_p^{\text{mag}}$	magnetophoretic mobility (7.3.251)		species $i$ flux; $N_i$ through membrane, $N_j$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$M$ ; $M_B$ , $M_S$ , $M_{Sl}$	molecular weight, a metal species, number of stages in stripping section of a cascade; value of $M$ for species $i$ , for solvent $s$ ; $M$ for coating liquid in stationary phase	$N_i$ ; $N_{im}$ , $N_{jm}$ , $N_{io}$ ; $N_{iy}$ , $N_{iz}$ , $N_{ir}$ , $N_{ij}$ ; $ N_{iy} $	$N_j$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$M_t$	average molecular weight of solution (3.1.56)	$N_{iz}^p$	$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$M_w$	magnetization of wire (3.1.23)		$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$M_{se}$	seed mass density per unit liquid volume (6.4.40a)	$N_{oj}$ ; $N_{o\ell p}$ ; $N_p$	$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$M_T$ ; $M_{Tw}$ , $M_{Tb}$	suspension density of a crystal-containing solution (2.4.2f), (6.4.18); value of $M_T$ for cases $a$ and $b$		$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$Mo^{(n)}$ , $Mo_{\ell}^{(i)}$ , $Mo_{\ell}^{(i)}$ , $Mo_v^{(i)}$	$n$ th moment of the density function (2.4.1g); $i$ th moment of molecular weight density function of feed, liquid and vapor, respectively (6.3.70)	$N_{Ri}$ , $N_S$ ; $N_i(r_p)$ ; $N_{it}$	$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$Mo_{fr}^j$ , $Mo_{gr}^j$	$j$ th moment of crystal size density functions $f(r_p)$ and $f_g(gr)$		$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
MWCO	molecular weight cut off of a membrane		$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$n$ ; $n_c$	number of species/components in a system, number of contacts, stage/plate number, number of positive charges in a metal ion, number of unit bed elements; number of collectors, number of channels (7.3.109)		$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$n(r_p)$ ; $n^o$ ; $\tilde{n}(r_p)$	population density function, particle number density function (2.4.2a); nucleation population density parameter (6.4.7); defined by (6.4.46b)	$N_t$ ; $N_{IoE}$ , $N_{IoR}$	$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$n_i$ , $n_{ix}$ , $n_{iy}$ , $n_{iz}$ ; $n_{ij}$	mass flux vector of species $i$ , $M_i N_i$ ; its components in $x$ -, $y$ - and $z$ -directions; $n_i$ through surface area $S_j$	$No_i$ ; NTU	$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$n_k$	outwardly directed unit normal to the $k$ -phase surface (6.2.26b-d)	$p$ ; $p$	$N_{iz}$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$

$p_A; p_B; p_i; p_j$	partial pressures of species A, species B; species $i$ and species $j$	$q_{fr}$	fraction of light reflected (3.1.47)
$p_{\Delta b}; p_{\Delta b}; p_{B,lm}$	value of $p_A$ in the gas bulk; value of $p_A$ at gas-liquid interface; logarithmic mean of $p_B$ (3.1.131b)	$q_i(C_{i2}), q_{i1}(C_{i2}), \bar{q}_{i1}; q_{i1}^0, q_{i1}^s$	moles of species $i$ in solid phase 1 per unit mass of solid phase, cross-sectional average of $q_{i1}$ , initial value of $q_{i1}$ ; saturation value of $q_{i1}$
$p_{if}; p_{ij}; p_{ip}; p_{iv}$	value of $p_i$ in feed gas, region/stream $j$ , permeate gas, vapor phase	$q_{iR}; q_{is}$	moles of species $i$ per unit mass of ion exchange resin ( $R$ ) or solid adsorbent ( $s$ )
$\bar{p}_{ib}; \bar{p}_{ii}$	species $i$ partial pressure in bulk and particle interface, averaged over bed cross section, Figures 3.4.4(a), (b)	$q_{\max R}$	maximum molar fixed charge density per unit resin mass
$pH; pI$	indicator of hydrogen ion concentration (5.2.65a); isoelectric point for a protein/amino acid; at $pI = pH$ , net charge is zero	$q(\mathbf{r}, \psi, \lambda)$	probability that a molecule having configuration $(\mathbf{r}, \psi, \lambda)$ does not intersect pore wall (3.3.89f)
$pK_i$	$-\log_{10} K_{div}$ (5.2.65b) for $i = 1$ , (5.2.74b) for $i = 2$ , (6.3.29)	$Q$	volumetric fluid flow rate, product species in reaction (5.3.5), hydraulic permeability in Darcy's law (6.1.4g,h), heat transfer rate (6.4.47a,d)
$P; P^0; P_c$	total pressure, system pressure; standard state pressure; critical pressure	$Q_c; Q_d$	electrical charge of a collector; volume flow rate of dialysate solution
$\bar{P}; P_f; P_j$	local solute permeability coefficient (6.3.157b); feed pressure; total pressure of $j$ th region and permeate, respectively; gas pressure (Figure 7.2.1(b)); gas pressure at the end of a capillary of length $l$ (6.1.5d)	$Q_f; Q_g; Q_h; Q_i; Q_o; Q_p; Q_r; Q_{1, 2}$	volumetric feed flow rate to separator; Darcy permeability for gas through packed bed; amount of heat supplied at a high temperature; electrical charge on 1 gmol of charged species $i$ ; amount of heat rejected at a low temperature; volumetric flow rate at membrane channel inlet; electrical charge of a particle; excess particle flux (7.2.123), volumetric flow rate of product stream from separator; heat supplied at the reboiler per mole of feed; volume flow rate of overflow; volumetric rate of underflow/concentrate
$P_i^0(\pi); P_i^*$	equilibrium gas-phase pressure for pure $i$ adsorption at spreading pressure $\pi$ , which is the same for a mixture (3.3.111a); pressure at crossover point for solute $i$ in supercritical extraction		permeabilities of species A, B, $i$ and $j$ , respectively, through membrane in gas permeation and pervaporation, respectively
$P_{atm}; P_{liq}; P_1, P_2$	atmospheric pressure; pressure in the liquid (6.1.12); purification factors (7.2.97)	$Q_{Am}, Q_{Bm}, Q_{im}$	permeabilities of species A, B, $i$ and $j$ , respectively, through membrane in gas permeation and pervaporation, respectively
$Pe; Pe_i; Pe_i^m; Pe_{z,eff}; Pe_{zj}$	Péclet number (3.1.143g), (7.3.34d); $Pe$ number for dispersion of solute $i$ (6.3.23a); pore Péclet number (6.3.145a); $(z v_z/D_{i,eff,z})$ (7.1.18h); $j$ phase $Pe_z$ (8.1.92)	$Q_{ij}$	permeability of species $i$ through region $j$ ( $= A, B, C, D, 1, 2$ ) of the membrane
$P_i^{sat}, P_j^{sat}; P_{iPl}^{sat}, P_{i,curved}^{sat}$	vapor pressure of pure $i$ and pure $j$ , respectively, at system temperature; value of $P_i^{sat}$ on a plane surface; value of $P_i^{sat}$ on a curved surface	$Q_{im}^{ov}$	overall permeability of species in membrane pervaporation for a membrane of thickness $\delta_m$
$P_M^{sat}$	value of $P_i^{sat}$ for pure species $i$ of molecular weight $M$		heat transfer rate of a solution during crystallization and subcooling, respectively
$P_0$	amplitude of pressure wave (3.1.48)	$Q_{crys}, Q_{sub}$	heat transfer rate of a solution during crystallization and subcooling, respectively
$P_R; P_w$	pressure of resin phase and external aqueous solution, respectively	$Q_{im0}$	value of $Q_{im}$ for $C_{im} = 0$
$Po, Pr$	power number (6.4.976), power (3.1.47)	$Q_{sm}, Q_{sc}$	solvent permeability through membrane and cake in cake filtration
$q$	number of variables in a problem, stoichiometric coefficient for species Q, heat flux in a heat exchanger attached to a cooling crystallizer (6.4.47a), the power of $M_T$ in expression (6.4.39a) for $B^0$ , factor (8.1.150)	$\mathbf{r}$	vector of molecular position, radius vector, unit vector in radially outward direction



$r$	radial coordinate, any characterizing property of a continuous mixture	$R_1, R_2$	intrinsic RO rejection of the unionized species 1 and ionized species 2
$r_1, r_2; r_{1i}, r_{2i}; r_c; r_f$	radii of curvature of interface (3.3.47); liquid outlet radii in tubular centrifuge; critical size of a nucleus (3.3.100b), cyclone radius; free surface radius in tubular centrifuge	$R^+ (R^-)$	ion exchange resin with fixed positive (negative) charge; cation (anion) of a surface active solute
$r_g; r_h; r_i; r_{in}$	radius of gyration of a macromolecule; hydrodynamic viscosity based radius (3.3.90f); radius of spherical solute molecule (3.3.90a); radius of liquid-liquid interface in tubular centrifuge	$R_{ij}; R_{im}; R_{i, reqd}$	permeation resistance of region $j$ ( $j = A, B, C, D$ ) in the membrane to species $i$ ; membrane permeation resistance of species $i$ ; solute $i$ rejection required in a RO membrane
$r_o$	radius of a sphere whose volume is equal to that of an ellipsoid (3.1.91g), radial location of the center of solute peak profile, radius of a cylindrical centrifuge	$R_{ik}, R_{ki}, R_{min}$	phenomenological coefficients (3.1.207); minimum reflux ratio (8.1.170)
$r_{p1}; r_{p, ai}; r_{p1}, r_{p2}; r_{p1}; r_{s1}; r_{i1}; r_w$	radius of particle, pore radius; analytical cut size (2.4.18); particles of two different sizes; dimension of particle of a certain size; particle size for classification (2.4.8); cyclone exit pipe radius; radius of wire	$R_{obs}, R_{true}; Re; Re_L; Re_{imp}$	observed and true solute rejection in ultrafiltration and RO; Reynolds number; $Re$ for a plate of length $L$ (3.1.143a), Table 3.1.5; $Re$ for an impeller (Table 3.1.7), (6.4.96)
$r_{max}, r_{min}; r_{p, 50}$	maximum and minimum radius of membrane pores or particle sizes; equiprobable size	$s$	fractional supersaturation (3.3.98b), power of the $\Delta P$ dependence of $\hat{R}_{co}$ (6.3.138j), Laplace transform complex variable (5.4.35), solution volume fed to bed per unit empty bed cross section (7.1.17a), eluent/salt (7.1.109b), salinity (10.1.13)
$\sqrt{r_p^2}$	hydraulic mean pore radius (3.4.87)	$S_m; S_{mp}; S_{ms}; S_p$	pore surface area/membrane volume (6.3.135e); value of $S_m$ for membrane pore volume or membrane solids volume (6.3.135f,g); sedimentation coefficient (4.2.16b)
$\bar{r}_p, \bar{r}_{p, 1.0}, \bar{r}_{p, i+1, i}$	mean of particle size distribution based on $f(r_p)$ and $n(r_p)$ , respectively (2.4.1e), (2.4.2g), (2.4.2h)	$S_1, S_2$	two different solvents
$\bar{r}_{p, 3.2}$	Sauter mean radius of a drop, bubble or particle (6.4.89)	$S; S_c$	solute transmission/sieving coefficient (6.3.141e), supersaturation ratio (3.3.98c), stripping factor (8.1.135), (8.1.189b); bed/column cross-sectional area
$ra; re; re _{flow}$	rate of arrival of cells; fractional water recovery; value of $re$ in a cell (6.3.172b)	$S_f, S_k, S_M; S_j; S_M, S_{dp}$	cross-sectional area vector of feed entrance, $k$ th feed exit and membrane surface area vector in a separator; cross-sectional area of flow for $j$ th stream; selectivities (7.3.219a,b)
$R; R$	universal gas constant; radius of a tube or capillary, reflux ratio (2.3.5), (2.3.7), (8.1.137), solute rejection in reverse osmosis, solute retention in ultrafiltration	$S_{ij}, \bar{S}_{ij}$	molar entropy of species $i$ ; partial molar entropy of species $i$ (3.3.17b)
$R_c; \hat{R}_{co}, \hat{R}_{cw}$	cake resistance; specific cake resistance per unit cake thickness and unit cake mass, respectively (6.3.135l)	$S_{im}; S_{im0}$	solubility coefficient of species $i$ in membrane; value of $S_{im}$ for $C_{im} = 0$
$R_h; R_i; R'_i; R_L$	hydraulic radius (6.1.4c); solute rejection/retention by membrane for species $i$ , retention ratio for species $i$ (7.3.211); fraction of solute $i$ in mobile phase (7.1.16c); largest radius of a conical tube	$S_{\infty}, S_{\sigma}; \bar{S}_{\sigma}; S_1, S_2$	value of $S$ when $Pe_i^m \gg 1$ (6.3.145b) surface area of interfacial region (Figure 3.3.2A); molar surface area in gas-solid adsorption (3.3.107); sieving coefficients for species 1 and 2
$R_m; R_s; R_{\infty}$	membrane resistance; resolution between neighboring peaks (2.5.7) in chromatography; value of $R_i$ at large $Pe_i^m$	$S_{ij}; S_{mag}$	total entropy for region $j$ (3.3.3); magnetic field force strength (7.3.251)
$R_A, R_i$	molar rate of production of A per unit volume (5.3.7), (6.2.2d), for species $i$		



$S_{\text{obsi}}, S_{\text{true}}$	observed solute transmission/sieving coefficient; true value of $S$	$u_i^0, u_k^0, u_{ij}^0, u_{il}^0, u_{io}^0$ $u_{is}^0, u_{ilb}$	mass fraction of impurity species $i$ ; mass fraction of species $k$ ; mass fraction of species $i$ in region $j$ (1.3.5) or $j$ th stream (2.1.20); value of $u_i$ in melt; value of $u_i$ initially in the solid; value of $u_i$ in the recrystallized solid; value of $u_{il}$ in the bulk melt
$Sc, Sc_c$	Schmidt number (3.1.143a); $Sc$ for continuous phase	$u_o, u_{rci}, u_{ro}$	mass fraction of solute in the solution charged for crystallization; $u_c/(1 - u_c)$ ; $u_o/(1 - u_o)$ (6.4.47b)
$Sh, Sh_c, Sh_D, Sh_p, Sh_z$	Sherwood number (3.1.143a); $Sh$ for continuous phase and dispersed phase, respectively; $Sh$ in a packed bed (7.2.218a); $Sh$ at location $z$ (7.2.64)	$u_i^m$ $u_{iEn}, u_{iRn}$	ionic mobility (3.1.108j) mass fraction of species $i$ in extract and raffinate streams, respectively, from stage $n$
$St$	Stanton number (3.1.143g), Stokes' number (6.3.41)	$ur_{ig}, ur_{is}$ $ur_{ij}$	defined by (8.1.349) and (8.1.350) weight of solute $i$ per unit weight of phase $j$ (9.1.32)
$t, t_{br}, t_c, t_{\text{resi}}, t_{\sigma}$	time; breakthrough time; time for cut point in the chromatographic separator output (2.5.1); residence time; thickness of interfacial region (Figure 3.3.2A)	$\mathbf{U}_i, \mathbf{U}_p$ $\bar{U}_i$	migration velocity vector for species $i$ (3.1.84b); particle velocity vector averaged velocity vector of $i$ th molecules due to all forces
$t_i^+, t_1^+, t_2^+; \bar{t}, \bar{t}_1, \bar{t}_2, \bar{t}_3$	nondimensional time variable for species $i$ (6.3.12); value for $t_i^+$ for species 1, 2 (3.2.9), (3.2.20); breakthrough time (7.1.15c); value of $\bar{t}$ for species 1, 2 and 3	$\bar{U}_i^{\text{new}}$ $U_{ix}, U_{iy}, U_{iz}$	defined by (3.1.103) components of migration velocity of species $i$ in $x$ -, $y$ - and $z$ -directions, (3.1.82)
$t_i^{\text{in}}, t_i^0$	times when species $i$ appears and disappears, respectively, from a chromatographic separator output	$U_{ik}; \langle U_{ik} \rangle^k; \hat{U}_{ik}$	value of $\mathbf{U}_i$ in the $k$ th phase/region; average of $\mathbf{U}_{ik}$ in the $k$ th phase/region; defined as a fluctuation by (6.2.28)
$t_{im}, t_{is}$	transport number of $i$ in membrane or solution (3.1.108d)	$U_p^{\text{int}}, U_{px}^{\text{int}}, U_{py}^{\text{int}}$	internal particle velocity vector (6.2.50d); its component in the direction of the internal coordinate $x^i$ ; internal particle velocity vector for all particles of size $r_p$
$t_{s_1}, t_{s_2}$ $t_{R_1}, t_{R_M}, t_{R_o}$	time required for solvents, $s_1, s_2$ (3.2.24) retention time for species $i$ in capillary electrophoresis (6.3.18a), (7.1.99d); retention time for the mobile phase; retention time based on $\nu_{z,\text{avg}}$ (7.3.207)	$\mathbf{U}_{pi}, \mathbf{U}_{pri}; \mathbf{U}_{pzt}, \mathbf{U}_{pyt}$	terminal velocity vector of particle (3.1.62); radial particle velocity; terminal value of $\mathbf{U}_{pi}$ ; value of $\mathbf{U}_{pi}$ in $z$ -direction (6.3.1), (7.2.211), $y$ -direction (7.3.154)
$T, T_1, T_2, T_c$	absolute temperature; temperature of cooled plate; temperature of the heated plate; critical temperature	$\mathbf{v}, \mathbf{v}_i; \mathbf{v}^*, \mathbf{v}_i^*, \mathbf{v}^+, \mathbf{v}_i^{\text{ref}}; \mathbf{v}_{ij}, \mathbf{v}_{ij}^*$	mass averaged velocity vector of a fluid (also $\mathbf{v}$ ); molar average velocity vector of a fluid (also $\mathbf{v}_i^*$ ); nondimensional $\mathbf{v}$ (6.3.39); reference $\mathbf{v}_i$ ; values of $\mathbf{v}_i$ and $\mathbf{v}_i^*$ on surface area $\mathbf{S}_j$ (2.1.1), (2.1.2)
$T_f, T_g, T_i, T_p$	feed temperature; glass transition temperature of a glassy polymer; value of absolute temperature $T$ of region $i$ ; product temperature	$\mathbf{v}_i, \mathbf{v}_{ij}$	averaged velocity vector of $i$ th species; value of $\mathbf{v}_i$ on surface area $\mathbf{S}_j$ (2.1.1)
$T_G, T_H, T_L, T_R$	temperature of condenser; two temperatures in supercritical extraction; reboiler temperature	$\nu_k; \nu_{kz}; \nu_k^{\text{int}}$	velocity of region $k$ ; $z$ -component of velocity $\nu_k$ of region $k$ ; mass average velocity of the interface of two phases
$T_{cf}, T_{ci}, T_{mi}$	temperature of cooling fluid, critical temperature of species $i$ ; melting temperature for species $i$	$\mathbf{v}_p^r$	particle diffusion velocity relative to that of the fluid phase (3.1.43)
$T_{\text{sat}}$	temperature at which the solution is saturated		
$T_{\text{sol}}$	temperature of the solution due to undergo crystallization		
$Th$	dimensionless group (3.1.46b)		
$u$	number of fundamental dimensions (Section 3.1.4.1)		
$u_c$	mass fraction of solute in the crystallized solution on a solid-free basis		