

INTRODUCTION TO

TECH- NICAL

MATHEMATICS

With Problem Solving • Second Edition

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Second Edition

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PREFACE

This Second Edition of *Introduction to Technical Mathematics, with Problem Solving* leaves most of the first edition intact, with a few improvements. We've added a section on slope, Chapter Tests to the Instructor's Manual, and two new chapters (plus three inserted sections and an Appendix) on problem solving.

The text is designed for an introductory course in technical mathematics. It is intended for students with minimal mathematics backgrounds who wish to prepare for further study in technical areas. The material has no prerequisites except a familiarity with elementary arithmetic operations, and even these are reviewed in the first three chapters.

The added material on problem solving can be considered optional (see chart near the end of this Preface) and could be used for a separate course, or unit, on problem solving.

The book's topical coverage is comprehensive but is presented in an easy-to-read manner so that students should be able to use the text without difficulty. The text may be used in technical/vocational schools and colleges, in community colleges, and in secondary schools offering pre-technical mathematics.

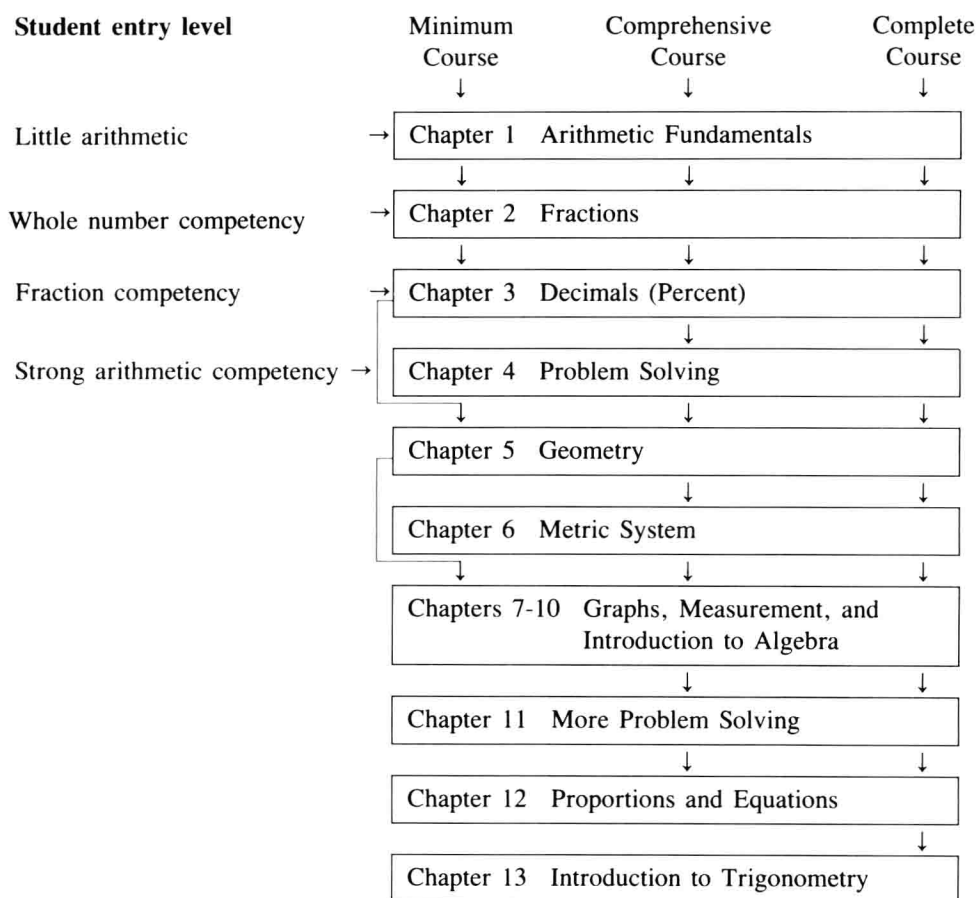
A wealth of applications is contained in the nearly 750 worked out examples and some 5,000 exercises and problems. The applications cover all the major areas of student need, including construction, mechanics, and electronics, as well as appropriate applications from relevant sciences. Formulas, rules of operations, and summaries of important procedures are clearly displayed and boxed when introduced.

A set of cumulative exercises, titled "Review Exercises," is provided at the end of most sections. These exercises help the student maintain skills, and provide a quick and continual check on progress in the course.

Calculators may be used at the instructor's option throughout the book. Calculator exercises are given at the end of most sections after chapter 3. These exercises are related to the topics presented in the section but include numbers that make calculations by hand a very tedious process. Calculator exercises are not included in the first three chapters so that manipulative skills can be emphasized in these chapters. Chapter 13 utilizes the calculator in trigonometry. Tables of roots and functions and information on interpolation have been moved to Appendix A.

Answers to all odd-numbered exercises are given at the end of the book. Answers to all problems appear at the end of problem-solving sections. All answers to exercises, odd- and even-numbered, as well as a set of Chapter Tests, are in the Instructor's Manual available from the publisher. **Special Note:** Those instructors who skip the first three chapters can use the Chapter Tests in the manual as diagnostic tests to be sure that students are ready to begin after chapter 3.

The following chart outlines chapter coverage needed for a minimum, a comprehensive, and a complete course. Entry levels into each type of course are indicated for students having either little arithmetic competency, competency with whole numbers, competency with fractions, or strong arithmetic competency.



Note: Section 1.6 should be studied by students prior to Chapter 7

The First Edition benefited from critical reviews and comments. We would like to thank the following reviewers for their help: Glenn R. Boston, George W. Brewer, Rueben C. Drake, Tom O. Eller, Lanny Hendrickson, John Hutchinson, Stanley Kohli, Richard Rowe, Richard Semmler, Morris L. Shoss, Gerald Skidmore, Lawrence Trivieri, and Frank Weeks.

The Second Edition benefited from comments and suggestions made by John Belveal, Rose State College; Gene Comero, Rochester Technical College; Barbara Cox, Pratt Community College; Stephen Lane, Big Bend Community College; Dale McCormick, Women Unlimited; Shelley Sauer, Fox Valley Technical College; and Arthur Schiffer, Greenville Technical College.

The authors each appreciate the efforts of the

others, blending areas of expertise into the whole that comprises this book: George L. Henderson, arithmetic, problem solving, and trigonometry; Walter W. Leffin, geometry and measurement; Mary VanBeck Voelker, algebra, equations, and proportions; Fred C. Janusek, applications.

Many other people—teachers, technicians, publishers, editors, etc.—have made appreciated suggestions. We especially want to thank our editor, Laurie Prossnitz, and publisher, Neil J. Rowe, of the Waveland Press, who motivated and helped with the Second Edition.

Walter W. Leffin
George L. Henderson
Mary VanBeck Voelker
Fred C. Janusek

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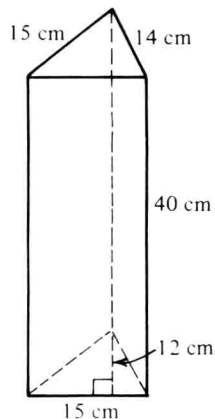
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12. Compute the surface area of the triangular container. Include the area of the top and bottom of the container.

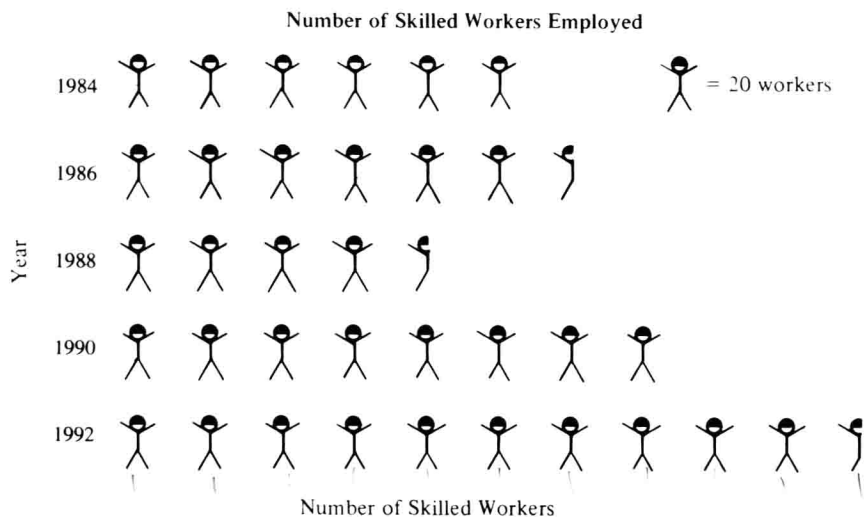


EXERCISES

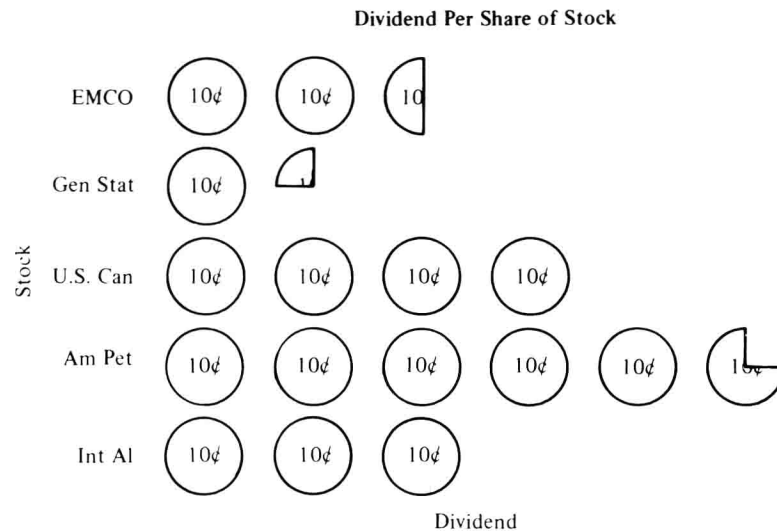
8.1

Use the figure shown here for Exercises 1–6.

- How many skilled workers were employed in 1986?
- How many skilled workers were employed in 1990?
- How many more skilled workers were employed in 1992 than in 1984?
- How many more skilled workers were employed in 1990 than in 1988?
- In 1989 there were 75 skilled workers employed. How many figures would be needed to show this on the graph?
- In 1993 there were 290 skilled workers employed. How many figures would be needed to show this on the graph?

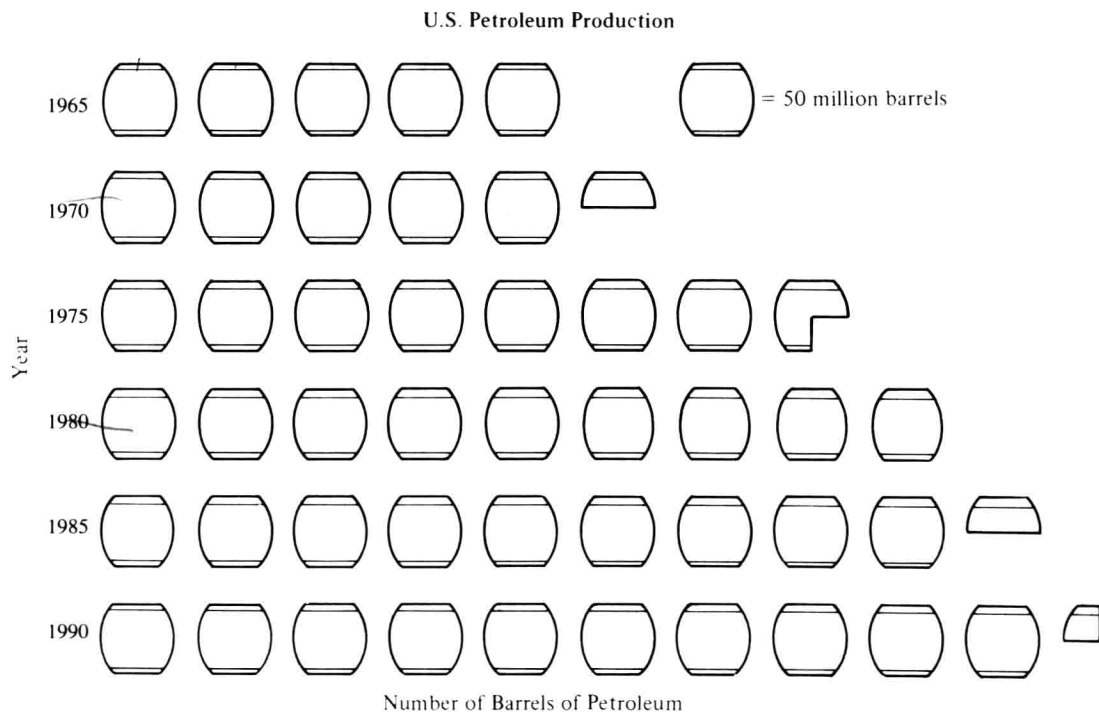


Use the following figure for Exercises 7–10.



7. What was the dividend on one share of Gen Stat stock?
8. What was the dividend on 100 shares of U.S. Can stock?
9. What was the total dividend on 5 shares of
- EMCO, 12 shares of Int Al, and 10 shares of Am Pet?
10. What was the total dividend on 20 shares of Gen Stat, 3 shares of Am Pet, and 16 shares of U.S. Can?

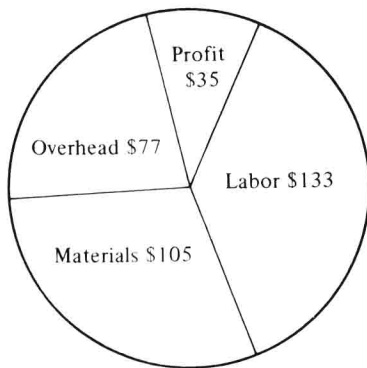
Refer to the following figure for Exercises 11–14.



11. How many barrels of petroleum were produced in 1965?
12. How many barrels of petroleum were produced in 1975?
13. How many more barrels of petroleum were produced in 1980 than were produced in 1970?
14. How many more barrels of petroleum were produced in 1990 than were produced in 1985?

Refer to the following figure for Exercises 15–18.

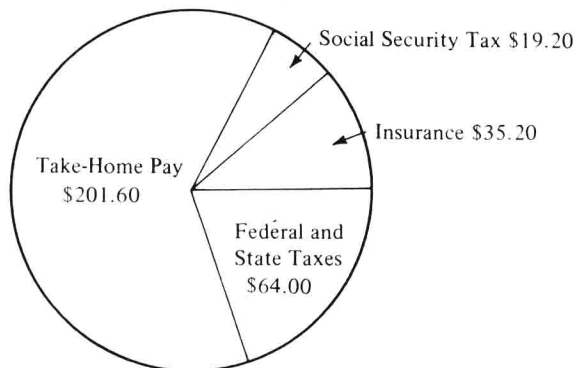
Distribution of the Wholesale Price of \$350 for a Major Appliance



15. What percent of the price of the appliance is to cover the cost of materials?
16. What percent of the price of the appliance is profit?
17. What percent of the price of the appliance is to cover the cost of labor?
18. What percent of the price of the appliance is to cover the overhead costs?

Use the following figure for Exercises 19–22.

Distribution of a Weekly Wage of \$320



19. What percent of the week's wages is take-home pay?
20. What percent of the week's wages is paid toward insurance?
21. What percent of the week's wages is paid as social security tax?
22. What percent of the week's wages is for federal and state taxes?

Construct a picture graph for each set of data in Exercises 23–26.

23. **Production of Cement in the U.S.**

1980	45,000,000 tons
1982	52,500,000 tons
1984	67,500,000 tons
1986	60,000,000 tons
1988	80,000,000 tons
1990	94,000,000 tons

24. **Profits for the First Six Months of the Year**

January	\$4500
February	\$8000
March	\$6250
April	\$5000
May	\$9500
June	\$7750

25. **Number of People Employed in a Manufacturing Company**

Administration	50
Skilled Workers	150
Unskilled Workers	325
Maintenance and Service	75
Sales	100

26. **State Gasoline Taxes in Selected States**

Texas	5¢
Oregon	7¢
Maine	9¢
Idaho	8.5¢
Oklahoma	6.5¢
Montana	7.75¢
Connecticut	10¢

27. Complete the following table and then construct a circle graph for the data.

Distribution of Costs for a New Home

Item	Cost	Percent of total cost	Part of 360° (Number of degrees)
Land	\$ 7,000	_____	_____
House	\$40,000	_____	_____
Landscaping	\$ 3,000	_____	_____
Furnishings	\$10,000	_____	_____
Total	\$60,000	_____	_____

28. Complete the following table and then construct a circle graph for the data.

Average Work Day

Item	Time	Percent of total time	Part of 360°
Work	8 hours	_____	_____
Sleep	7 hours	_____	_____
Meals	2 hours	_____	_____
Recreation	3 hours	_____	_____
Miscellaneous	4 hours	_____	_____

Construct a circle graph for each set of data in Exercises 29–32.

29. **Where the Federal Government Obtains Each Budget Dollar**

Individual Income Taxes	39¢
Social Security Taxes	29¢
Corporation Income Taxes	13¢
Borrowing	11¢
Excise Taxes	4¢
Others	4¢

30. **How the Federal Government Spends Each Budget Dollar**

Benefit Payments to Individuals	38¢
National Defense	26¢
Grants to States and Localities	16¢
Interest on Debts	7¢
Federal Operations	13¢

31. **Distribution of Receipts of a Machine Company**

Wages	\$135,000
Raw Materials	\$ 81,000
Interest and Taxes	\$ 24,000
Maintenance	\$ 45,000
Miscellaneous	\$ 15,000

32. **Production of Liquid Petroleum Products (number of barrels)**

Motor Fuel	2,000,000,000
Fuel Oil	1,500,000,000
Jet Fuel	800,000,000
Liquid Gases	300,000,000
Others	400,000,000

Calculator Exercises

33. What is 758.3% of 749.4 to the nearest tenth?
34. What is 105.03% of 49.76 to the nearest hundredth?
35. What is 0.0035% of 178.3 to five decimal places?
36. What is $4\frac{7}{8}\%$ of $3\frac{4}{5}$ to four decimal places?
37. What percent is 25.8 of 117.32 to the nearest thousandth of a percent?
38. What percent is 5.783 of 846.004 to the nearest thousandth of a percent?

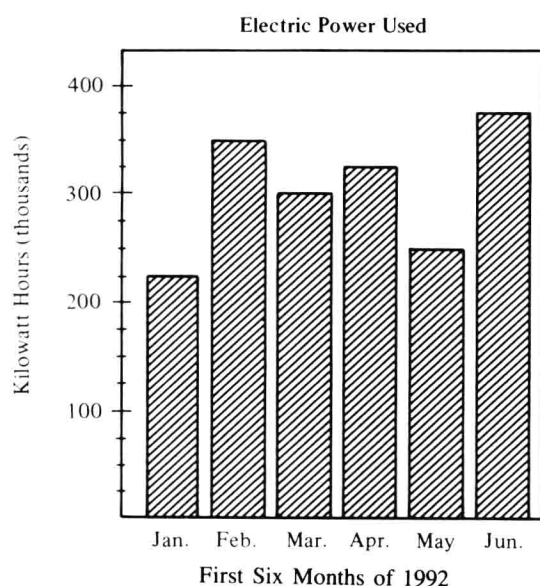
8.2

BAR AND LINE GRAPHS

A **bar graph** uses bars (shaded rectangular regions) to show the relative magnitude of a set of data.

EXAMPLE 1 Answer the following questions from the information illustrated by the bar graph.

- What information is shown along the horizontal axis?
- What information is shown along the vertical axis?
- In what month was power use the greatest?
- Approximately how much power was used during May?
- What is the total amount of power used during the first three months of the year?

**Solution**

- The scale along the horizontal axis lists the months of the year.

- The scale along the vertical axis shows the amount of power used in thousands of kilowatt hours.
- The greatest amount of power was used during the month of June—approximately 375,000 kilowatt hours.
- Approximately 250,000 kilowatt hours of power were used during May.
- During the first three months of 1992 approximately $(225 + 350 + 300)$, or 875,000 kilowatt hours of power were used.

EXAMPLE 2 Construct a bar graph to show the auto sales from January through May.

Month	Amount of Sales
Jan	\$6,500,000
Feb	7,200,000
March	7,000,000
April	7,800,000
May	8,500,000

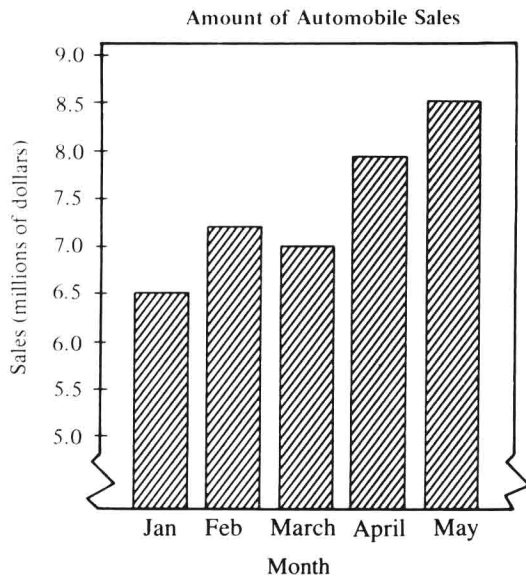
Solution

- Determine what information will be shown along the horizontal axis and what information will be shown along the vertical axis. In this example we place the *years* along the horizontal axis and the *amount of sales*, in millions of dollars, along the vertical axis. With this choice, the bars of the graph will be vertical. (It is also possible to interchange the data on the two axes so that the bars of the graph run horizontally.)
- Determine the scale along each axis so that the full range of data can be shown on the

graph, yet keep the graph to an appropriate size. The units along a given scale must all be the same size. The units on one scale do not have to be the same size as the units on the other scale. Note that it is not necessary to start at zero on a scale if only high values of the scale are to be used.

- (c) Draw bars to represent the information given in the table. Usually the bars are shaded and a space is left between bars.
- (d) Give the graph a title and label each axis.

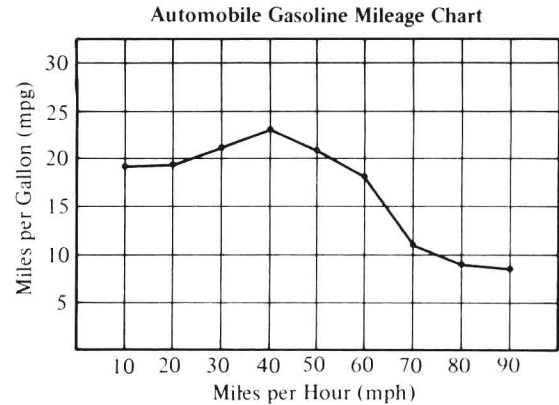
The completed graph for Example 2 is shown here. (*Note:* The jagged lines at the bottom of the graph are used to show that a portion of the graph from 0.0 to 5.0 has not been drawn.)



A line graph can be used to show the trend of a set of data that is continuous. Two or more sets of data can be compared by plotting both sets of data on the same line graph.

EXAMPLE 3 Answer the following questions from the information shown by the line graph.

- (a) At what speed does the automobile get the highest mileage per gallon (mpg)?
- (b) What is the approximate mpg at 30 miles per hour (mph)?
- (c) What is the approximate mpg at 55 miles per hour?
- (d) At what speed(s) does the automobile get approximately 20 mpg?
- (e) What is the decrease in the number of miles per gallon between 60 mph and 70 mph?

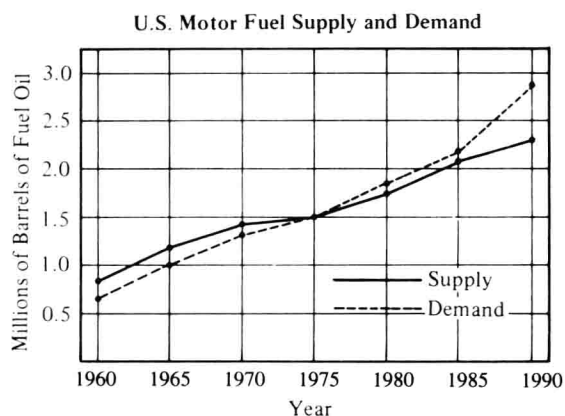


Solution

- (a) The automobile gets the highest mileage per gallon (approximately 23 mpg) at 40 miles per hour.
- (b) At 30 mph the mileage is approximately 21 miles per gallon.
- (c) At 55 mph the mileage is approximately 19 miles per gallon.
- (d) The automobile gets 20 mpg at approximately 23 mph and 52 mph.
- (e) The decrease in the number of miles per gallon between 60 mph and 70 mph is approximately (18 mph - 11 mpg) or 7 mpg.

EXAMPLE 4 This example shows two sets of data on the same line graph. The solid line represents the supply of motor fuel. The dashed line represents the demand for motor fuel. Answer the following questions from the information shown in the graph.

- What was the *supply* of motor fuel in 1970?
- What was the *demand* for motor fuel in 1985?
- During what year(s) was the supply of motor fuel greater than the demand?
- In 1980 what was the difference between the supply and demand for motor fuel?
- During what year(s) did the supply of motor fuel equal the demand for motor fuel?



Solution

- The supply of motor fuel in 1970 was approximately 1,400,000 barrels.
- The demand for motor fuel in 1985 was approximately 2,200,000 barrels.
- The supply of motor fuel was greater than the demand from 1960 to 1975.

- In 1980 the demand for motor fuel was approximately 1,000,000 barrels more than the supply.
- The supply and demand for motor fuel were equal in 1975.

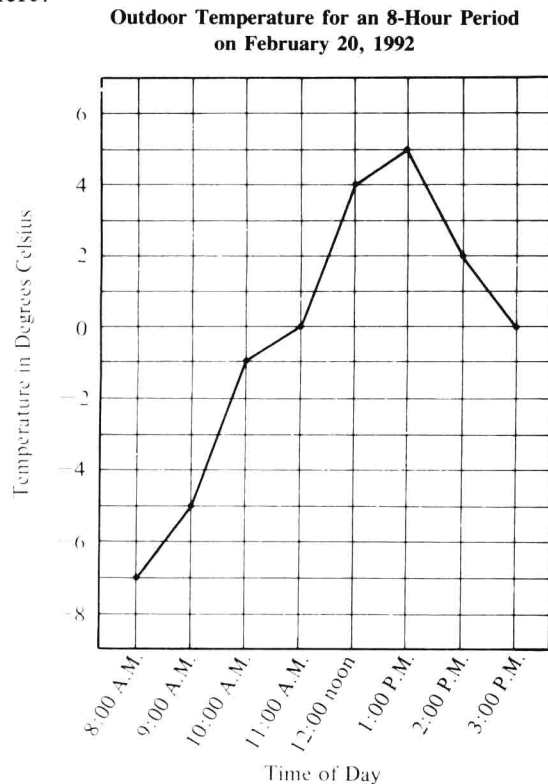
EXAMPLE 5 Using the following data, construct a line graph to show the outdoor temperature during an eight-hour period on February 20, 1992.

Time	Temperature
8:00 A.M.	-7 °C
9:00 A.M.	-5 °C
10:00 A.M.	-1 °C
11:00 A.M.	0 °C
12:00 noon	4 °C
1:00 P.M.	5 °C
2:00 P.M.	2 °C
3:00 P.M.	0 °C

Solution

- Determine what information will be shown along the horizontal axis and what information will be shown along the vertical axis. In this example we place *time of day* along the horizontal axis and *temperature* along the vertical axis.
- Determine the scale along each axis so the full range of data can be shown. The units on a given scale must be equal to each other. However, a unit on the horizontal scale does not have to be equal to a unit on the vertical scale. The scales do not have to start at zero.
- Locate the pairs of numbers on the horizontal and vertical scales and place a dot on the graph to represent each pair of values.
- Connect the dots with line segments.
- Give the graph a title and label each axis.

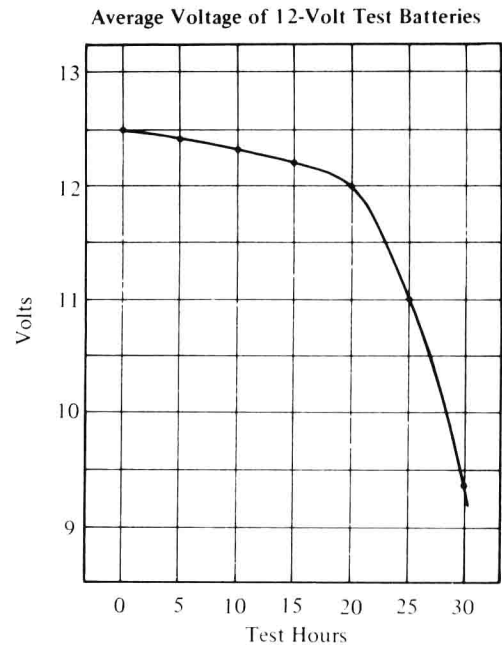
The completed graph for Example 5 is shown here.



A line graph may be drawn as a series of connected line segments (broken line), as shown in the previous two examples, or it may be drawn as a smooth curve. Example 6 shows a curved-line graph.

EXAMPLE 6 Use the following graph to answer questions (a) – (d).

- What is the approximate voltage after 10 test hours?
- After how many hours does the voltage drop below 12 volts?
- What is the approximate voltage after 30 test hours?
- What is the decrease in voltage from 20 test hours to 25 test hours?

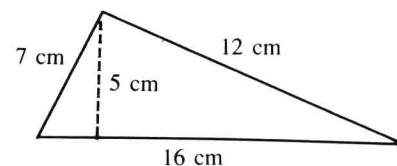


Solution

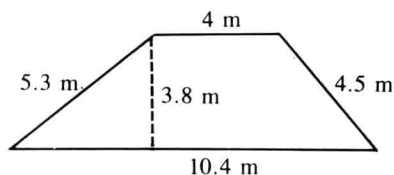
- After 10 test hours the voltage is approximately 12.3 volts.
- The voltage drops below 12 volts after approximately 20 test hours.
- After 30 test hours the voltage is approximately 9.4 volts.
- The decrease in voltage from 20 to 25 test hours is approximately 1 volt.

Review Exercises

- What is the area of the triangle?



2. What is the area of the trapezoid?



3. $\frac{3}{5} \div \frac{8}{7} = \underline{\hspace{2cm}}$

4. $\frac{4}{3} \div \frac{12}{16} = \underline{\hspace{2cm}}$

5. $17.94 \times 0.082 = \underline{\hspace{2cm}}$

6. $0.035 \times 7.04 = \underline{\hspace{2cm}}$

7. Eight is $\underline{\hspace{2cm}}$ % of 160.

8. Nine is $\underline{\hspace{2cm}}$ % of 45.

9. $0.054 \div 4.5 = \underline{\hspace{2cm}}$

10. $8.4 \div 0.012 = \underline{\hspace{2cm}}$

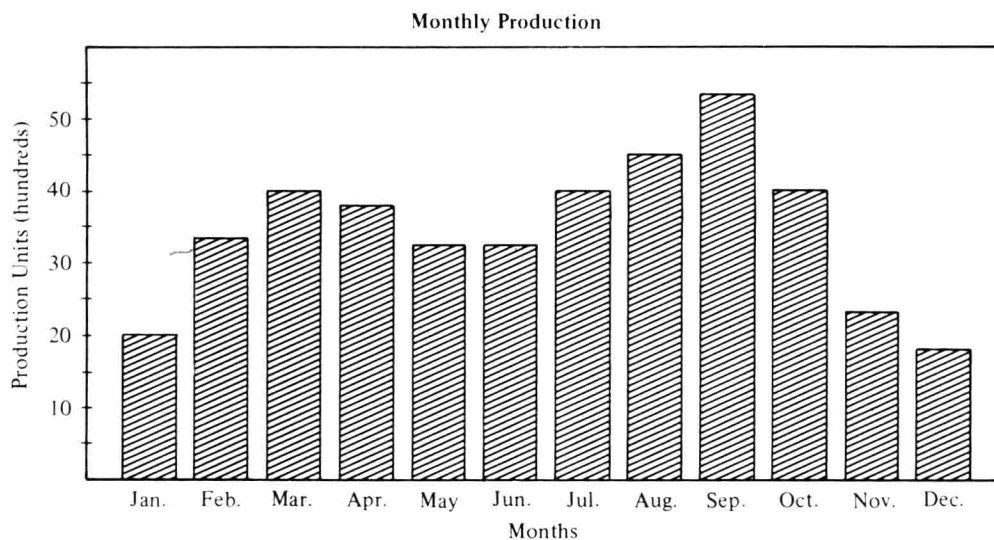
11. Convert $8^{\circ}43'$ to degrees, to the nearest thousandth of a degree.

12. Convert $13^{\circ}17'$ to degrees, to the nearest thousandth of a degree.

EXERCISES

8.2

Use the following graph for Exercises 1–6.



- Which month had the greatest production? Approximately how many units were produced that month?
- Which month had the least production? Approximately how many units were produced that month?
- During which month(s) was production about one-third of the production during September?
- During which month(s) was production about twice the production during November?
- What was the approximate average production for the first three months of the year?
- What was the approximate average production for the months of July, August, and September?