HANDBOOK *of* NUMERICAL ANALYSIS

P. G. CIARLET *and* J. L. LIONS • Editors

Volume
VI

Numerical Methods for Solids (Part 3)

Numerical Methods for Fluids (Part 1)

NORTH-HOLLAND

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Numerical Methods for Solids (Part 3)

Numerical Methods for Fluids (Part 1)

Handbook of Numerical Analysis

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General Preface

During the past decades, giant needs for ever more sophisticated mathematical models and increasingly complex and extensive computer simulations have arisen. In this fashion, two indissociable activities, *mathematical modeling* and *computer simulation*, have gained a major status in all aspects of science, technology, and industry.

In order that these two sciences be established on the safest possible grounds, mathematical rigor is indispensable. For this reason, two companion sciences, *Numerical Analysis* and *Scientific Software*, have emerged as essential steps for validating the mathematical models and the computer simulations that are based on them.

Numerical Analysis is here understood as the part of *Mathematics* that describes and analyzes all the numerical schemes that are used on computers; its objective consists in obtaining a clear, precise, and faithful, representation of all the "information" contained in a mathematical model; as such, it is the natural extension of more classical tools, such as analytic solutions, special transforms, functional analysis, as well as stability and asymptotic analysis.

The various volumes comprising the *Handbook of Numerical Analysis* will thoroughly cover all the major aspects of Numerical Analysis, by presenting accessible and in-depth surveys, which include the most recent trends.

More precisely, the Handbook will cover the *basic methods of Numerical Analysis*, gathered under the following general headings:

- Solution of Equations in $\mathbb{R}^n$,
- Finite Difference Methods,
- Finite Element Methods,
- Techniques of Scientific Computing,
- Optimization Theory and Systems Science.

It will also cover the *numerical solution of actual problems of contemporary interest in Applied Mathematics*, gathered under the following general headings:

- Numerical Methods for Fluids,
- Numerical Methods for Solids,
- Specific Applications.

"Specific Applications" include: Meteorology, Seismology, Petroleum Mechanics, Celestial Mechanics, etc.

Each heading is covered by several *articles*, each of which being devoted to a specialized, but to some extent "independent", topic. Each article contains a thorough description and a mathematical analysis of the various methods in actual use, whose practical performances may be illustrated by significant numerical examples.

Since the Handbook is basically expository in nature, only the most basic results are usually proved in detail, while less important, or technical, results may be only stated or commented upon (in which case specific references for their proofs are systematically provided). In the same spirit, only a "selective" bibliography is appended whenever the roughest counts indicate that the reference list of an article should comprise several thousand items if it were to be exhaustive.

Volumes are numbered by capital Roman numerals (as Vol. I, Vol. II, etc.), according to their *chronological appearance*.

Since all the articles pertaining to a given *heading* may not be simultaneously available at a given time, a given heading usually appears in more than one volume; for instance, if articles devoted to the heading "Solution of Equations in $\mathbb{R}^n$" appear in Volumes I and III, these volumes will include "Solution of Equations in $\mathbb{R}^n$ (Part 1)" and "Solution of Equations in $\mathbb{R}^n$ (Part 2)" in their respective titles. Naturally, all the headings dealt with within a given volume appear in its title; for instance, the complete title of Volume I is "Finite Difference Methods (Part 1) — Solution of Equations in $\mathbb{R}^n$ (Part 1)".

Each article is subdivided into *sections*, which are numbered consecutively throughout the article by *Arabic numerals*, as Section 1, Section 2, ..., Section 14, etc. Within a given section, *formulas, theorems, remarks, and figures*, have their own independent numberings; for instance, with Section 14, formulas are numbered consecutively as (14.1), (14.2), etc., theorems are numbered consecutively as Theorem 14.1, Theorem 14.2, etc. For the sake of clarity, the article is also subdivided into *chapters*, numbered consecutively throughout the article by *capital Roman numerals*; for instance, Chapter I comprises Sections 1 to 9, Chapter II comprises Sections 10 to 16, etc.

P.G. CIARLET
J.L. LIONS
May 1989

Contents of Volume VI

GENERAL PREFACE v

NUMERICAL METHODS FOR SOLIDS (PART 3)

Iterative Finite Element Solutions in Nonlinear Solid Mechanics, *R.M. Ferencz and T.J.R. Hughes* 3
Obituary – Juan Carlos Simo 179
Numerical Analysis and Simulation of Plasticity, *J.C. Simo* 183

NUMERICAL METHODS FOR FLUIDS (PART 1)

Navier–Stokes Equations: Theory and Approximation, *M. Marion and R. Temam* 503

Contents of the Handbook

VOLUME I

FINITE DIFFERENCE METHODS (PART 1)

Introduction, *G.I. Marchuk* 3
Finite Difference Methods for Linear Parabolic Equations, *V. Thomée* 5
Splitting and Alternating Direction Methods, *G.I. Marchuk* 197

SOLUTION OF EQUATIONS IN $\mathbb{R}^n$ (PART 1)

Least Squares Methods, *Å. Björck* 465

VOLUME II

FINITE ELEMENT METHODS (PART 1)

Finite Elements: An Introduction, *J.T. Oden* 3
Basic Error Estimates for Elliptic Problems, *P.G. Ciarlet* 17
Local Behavior in Finite Element Methods, *L.B. Wahlbin* 353
Mixed and Hybrid Methods, *J.E. Roberts and J.-M. Thomas* 523
Eigenvalue Problems, *I. Babuška and J. Osborn* 641
Evolution Problems, *H. Fujita and T. Suzuki* 789

VOLUME III

TECHNIQUES OF SCIENTIFIC COMPUTING (PART 1)

Historical Perspective on Interpolation, Approximation and Quadrature, *C. Brezinski* 3
Padé Approximations, *C. Brezinski and J. van Iseghem* 47
Approximation and Interpolation Theory, *Bl. Sendov and A. Andreev* 223

NUMERICAL METHODS FOR SOLIDS (PART 1)

Numerical Methods for Nonlinear Three-Dimensional Elasticity, *P. Le Tallec* 465

SOLUTION OF EQUATIONS IN $\mathbb{R}^n$ (PART 2)

Numerical Solution of Polynomial Equations, *Bl. Sendov, A. Andreev and N. Kjurkchiev* 625

VOLUME IV

FINITE ELEMENT METHODS (PART 2)

Origins, Milestones and Directions of the Finite Element Method – A Personal View, *O.C. Zienkiewicz* 3
Automatic Mesh Generation and Finite Element Computation, *P.L. George* 69

NUMERICAL METHODS FOR SOLIDS (PART 2)

Limit Analysis of Collapse States, *E. Christiansen* 193
Numerical Methods for Unilateral Problems in Solid Mechanics, *J. Haslinger, I. Hlaváček and J. Nečas* 313
Mathematical Modelling of Rods, *L. Trabucho and J.M. Viaño* 487

VOLUME V

TECHNIQUES OF SCIENTIFIC COMPUTING (PART 2)

Numerical Path Following, *E.L. Allgower and K. Georg* 3
Spectral Methods, *C. Bernardi and Y. Maday* 209
Numerical Analysis for Nonlinear and Bifurcation Problems, *G. Caloz and J. Rappaz* 487
Wavelets and Fast Numerical Algorithms, *Y. Meyer* 639
Computer Aided Geometric Design, *J.-J. Risler* 715

VOLUME VI

NUMERICAL METHODS FOR SOLIDS (PART 3)

Iterative Finite Element Solutions in Nonlinear Solid Mechanics, *R.M. Ferencz and T.J.R. Hughes* 3
Numerical Analysis and Simulation of Plasticity, *J.C. Simo* 183

NUMERICAL METHODS FOR FLUIDS (PART 1)

Navier–Stokes Equations: Theory and Approximation, *M. Marion and R. Temam* 503

Numerical Methods for Solids (Part 3)

Iterative Finite Element Solutions in Nonlinear Solid Mechanics

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Contents

CHAPTER I. Introduction 7

1. Forward 7
2. The initial boundary-value problem of nonlinear solid mechanics 10
3. Finite element solution of the initial boundary value problem 14
4. Linear equation solving 17

CHAPTER II. Iterative Linear Equation Solving 23

5. The conjugate gradient method 23
6. The preconditioned conjugate gradient method 27
7. Element-by-element preconditioning 32

CHAPTER III. Implementation of Element Operations 39

8. An operational view of element-by-element preconditioning 39
9. General concepts of vectorization 43
10. Vectorized element-by-element preconditioning 48

CHAPTER IV. Applications to Large-Scale Continuum Problems 53

11. Fractal dimension of a finite element mesh 53
12. The computational environment 58
13. Example linear analyses 60
14. Example nonlinear analyses 77

CHAPTER V. Extensions to Contact/Impact Problems 85

15. Penalized symmetry planes 85
16. Penalized slidesurfaces 92

CHAPTER VI. Shell Analysis with Element-by-Element Methods 105

17. NIKE3D's shell element 105
18. Linear analysis of a plate 108
19. Analysis of a stiffened cylindrical shell 115
20. Example nonlinear analyses 122

CHAPTER VII. Experience with a Lanczos-Based Iterative Solution Algorithm 129

21. Derivation of the method 129
22. The Lanczos algorithm in finite precision arithmetic 141
23. Numerical examples 146

CHAPTER VIII. Conclusions 167

REFERENCES 171

SUBJECT INDEX 177

CHAPTER I

Introduction

1. Forward

Until recent times, a general lack of computational capacity has effectively limited implicit finite element calculations in solid and structural mechanics to moderately sized two-dimensional simulations. At the core of any implicit time-integration scheme, whether for linear or nonlinear simulation, is the necessity of solving coupled linear systems of equations. Direct methods of solving linear systems, i.e., algorithms based upon Gaussian elimination, have proved quite adequate for this task in two-dimensional applications, and their simplicity and robustness have led to near universal adoption of these techniques in general-purpose finite element codes. However, the increasing demands for three-dimensional simulation, as well as ever larger two-dimensional applications, require us to confront the unfavorable rates of growth for both storage and floating-point operations associated with traditional direct methods. These difficulties have long inspired the study and development of iterative linear equation solvers by numerical analysts. The computational mechanics community is returning to this topic in search of cost-effective methods for application to large-scale analysis. Furthermore, iterative methods are proving to be a natural avenue for the parallel solution of systems of linear equations. The ongoing evolution of seemingly all hardware towards parallel processing adds further impetus to this topic.

Element-by-element methods were originally motivated by the global operator splitting techniques associated with finite difference methods, specifically the alternating-direction implicit schemes of DOUGLAS [1955, 1962], DOUGLAS and RACHFORD [1956], and PEACEMAN and RACHFORD [1955]. Subsequent generalizations are surveyed by MARCHUK [1975, 1990], TEMAN [1968, 1969] and YANENKO [1971]. The initial goal of element-by-element algorithms was to generalize these global techniques to the local (element) data structure which is intrinsic to finite element implementations. HUGHES, LEVIT and WINGET [1983a] first introduced element-by-element methods as a means of implicit time-integration for the heat conduction problem. Based upon this idea, ORTIZ, PINSKY and TAYLOR [1983] developed an element-by-element time-stepping scheme for structural dynamics. Both these algorithms are unconditionally stable and attain formal second-order accuracy, but display unacceptable spatial truncation errors in certain test problems. Due to that experience, subsequent strate-