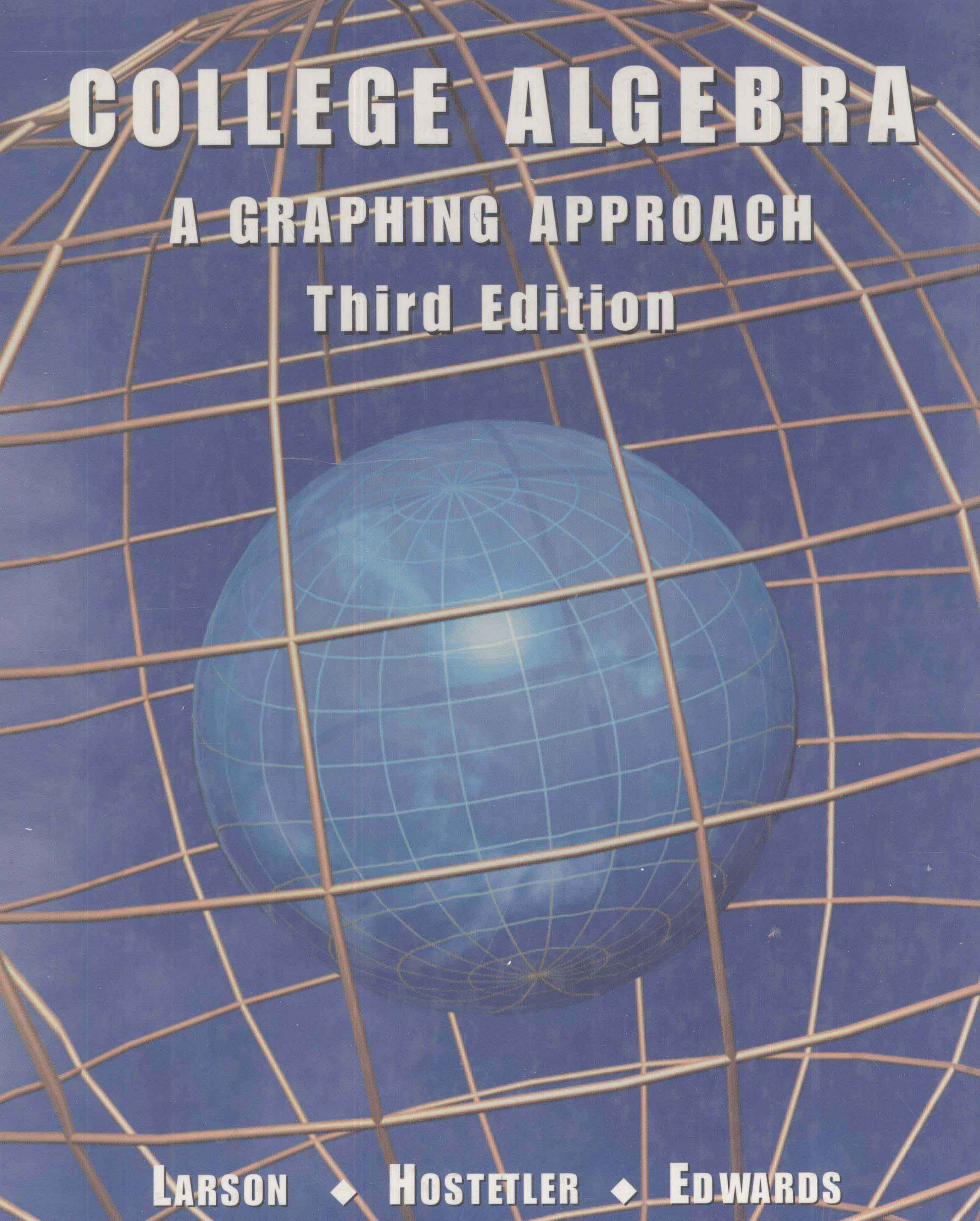
COLLEGE ALGEBRA

A GRAPHING APPROACH

Third Edition



LARSON ◆ HOSTETLER ◆ EDWARDS

College Algebra
A Graphing Approach

Third Edition

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We have included examples and exercises that use real-life data as well as technology output from a variety of software. This would not have been possible without the help of many people and organizations. Our wholehearted thanks go to all for their time and effort.

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College Algebra

A Word from the Authors

Welcome to *College Algebra: A Graphing Approach*, Third Edition. In this revision we have focused on student success, accessibility, and flexibility.

Accessibility: Over the years we have taken care to write a text for the student. We have paid careful attention to the presentation, using precise mathematical language and clear writing to create an effective learning tool. We believe that every student can learn mathematics and we are committed to providing a text that makes the mathematics within it accessible to all students.

In the Third Edition, we have revised and improved upon many text features designed for this purpose. Our pedagogical approach includes presenting solutions to examples from multiple perspectives—algebraic, graphic, and numeric. The side-by-side format allows students to see that a problem can be solved in more than one way, and to compare the accuracy of the solution methods.

Technology has been fully integrated into the text presentation. Also, the *Exploration* and *Study Tip* features have been expanded. *Chapter Tests*, which give students an opportunity for self-assessment, now follow every chapter in the Third Edition. The exercise sets now contain both *Synthesis* exercises, which check students' conceptual understanding, and *Review* exercises, which reinforce skills learned in previous sections and chapters. Students also have access to several media resources—videotapes, *Interactive College Algebra: A Graphing Approach* CD-ROM, and a *College Algebra: A Graphing Approach* website—that provide additional text-specific support.

Student Success: During our past 30 years of teaching and writing, we have learned many things about the teaching and learning of mathematics. We have found that students are most successful when they know what they are expected to learn and why it is important to learn it. With that in mind, we have restructured the Third Edition to include a thematic study thread in every chapter.

Each chapter begins with a study guide called *How to Study This Chapter*, which includes a comprehensive overview of the chapter concepts (*The Big Picture*), a list of *Important Vocabulary* that is integral to learning *The Big Picture* concepts, a list of study resources, and a general study tip. The study guide allows students to get organized and prepare for the chapter.

An old pedagogical recipe goes something like this, "First I'm going to tell you what I'm going to teach you, then I will teach it to you, and finally I will go over what I taught you." Following this recipe, we have included a set of learning objectives in every section that outlines what students are expected to learn, followed by an interesting real-life application that illustrates why it is important to learn the concepts in that section. Finally, the chapter summary (*What did you learn?*), which reinforces the section objectives, and the chapter *Review Exercises*, which are correlated to the chapter summary, provide additional study support at the conclusion of each chapter.

Our new *Student Success Organizer* supplement takes this study thread one step further, providing a content-based study aid.

Flexibility: From the time we first began writing in the early 1970s, we have always viewed part of our authoring role as that of providing instructors with flexible teaching programs. The optional features within the text allow instructors with different pedagogical approaches to design their courses to meet both their instructional needs and the needs of their students. In addition, we provide several print and media resources to support instructors, including a new *Instructor Success Organizer*.

We hope you enjoy the Third Edition.

Ron Larson

Robert P. Hostetler

Bruce H. Edwards

Acknowledgments

We would like to thank the many people who have helped us prepare the text and the supplements package. Their encouragement, criticisms, and suggestions have been invaluable to us.

Third Edition Reviewers

Jamie Whitehead Ashby, Texarkana College; Teresa Barton, Western New England College; Diane Burleson, Central Piedmont Community College; Alexander Burstein, University of Rhode Island; Victor M. Cornell, Mesa Community College; Marcia Drost, Texas A & M University; Kenny Fister, Murray State University; Susan C. Fleming, Virginia Highlands Community College; Nicholas E. Geller, Collin County Community College; Betty Givan, Eastern Kentucky University; John Kendall, Shelby State Community College; Donna M. Krawczyk, University of Arizona; JoAnn Lewin, Edison Community College; David E. Meel, Bowling Green University; Beverly Michael, University of Pittsburgh; Jon Odell, Richland Community College; Laura Reger, Milwaukee Area Technical College; Craig M. Steenberg, Lewis-Clark State College; Mary Jane Sterling, Bradley University; Ellen Vilas, York Technical College. In addition, we would like to thank all the college algebra instructors who took the time to respond to our survey.

We would like to extend a special thanks to Ellen Vilas for her contributions to this revision.

We would like to thank the staff of Larson Texts, Inc. and the staff of Meridian Creative Group, who assisted in proofreading the manuscript, preparing and proofreading the art package, and typesetting the supplements.

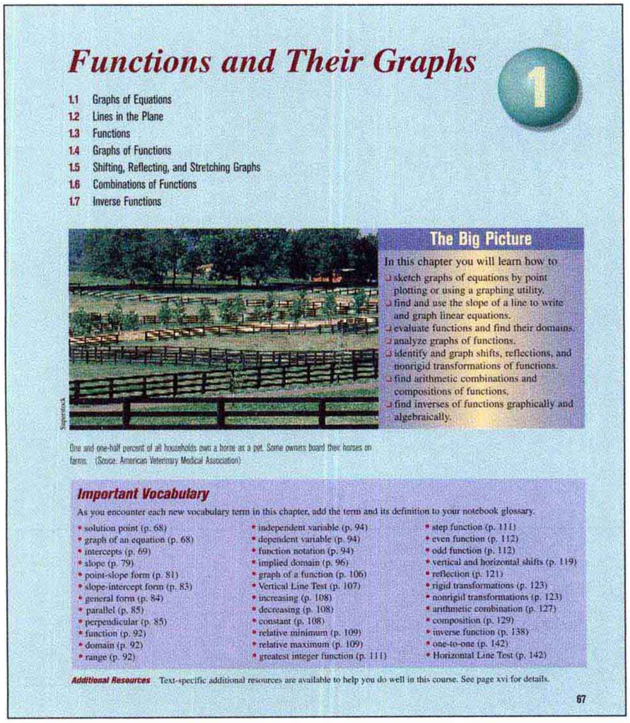
On a personal level, we are grateful to our wives, Deanna Gilbert Larson, Eloise Hostetler, and Consuelo Edwards for their love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write to us. Over the past two decades we have received many useful comments from both instructors and students, and we value these very much.

Ron Larson
Robert P. Hostetler
Bruce H. Edwards

Features Highlights

Student Success Tools



Functions and Their Graphs 1

1.1 Graphs of Equations
1.2 Lines in the Plane
1.3 Functions
1.4 Graphs of Functions
1.5 Shifting, Reflecting, and Stretching Graphs
1.6 Combinations of Functions
1.7 Inverse Functions

The Big Picture

In this chapter you will learn how to

- sketch graphs of equations by point plotting or using a graphing utility.
- find and use the slope of a line to write and graph linear equations.
- evaluate functions and find their domains.
- analyze graphs of functions.
- identify and graph shifts, reflections, and nonrigid transformations of functions.
- find arithmetic combinations and compositions of functions.
- find inverses of functions graphically and algebraically.

Superstock

One and one-half percent of all households own a horse as a pet. Some owners board their horses on farms. (Souce: American Veterinary Medical Association)

Important Vocabulary

As you encounter each new vocabulary term in this chapter, add the term and its definition to your notebook glossary.

- solution point (p. 68)
- graph of an equation (p. 68)
- intercepts (p. 69)
- slope (p. 79)
- point-slope form (p. 81)
- slope-intercept form (p. 83)
- general form (p. 84)
- parallel (p. 85)
- perpendicular (p. 85)
- function (p. 92)
- domain (p. 92)
- range (p. 92)
- independent variable (p. 94)
- dependent variable (p. 94)
- function notation (p. 94)
- implied domain (p. 96)
- graph of a function (p. 106)
- Vertical Line Test (p. 107)
- increasing (p. 108)
- decreasing (p. 108)
- constant (p. 108)
- relative minimum (p. 109)
- relative maximum (p. 109)
- greatest integer function (p. 111)
- step function (p. 111)
- even function (p. 112)
- odd function (p. 112)
- vertical and horizontal shifts (p. 119)
- reflection (p. 121)
- rigid transformations (p. 123)
- nonrigid transformations (p. 123)
- arithmetic combination (p. 127)
- composition (p. 129)
- inverse function (p. 138)
- one-to-one (p. 142)
- Horizontal Line Test (p. 142)

Additional Resources Text-specific additional resources are available to help you do well in this course. See page xvi for details.

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New Chapter Openers include:

▶ **The Big Picture**

An objective-based overview of the main concepts of the chapter.

▶ **Important Vocabulary**

Mathematical terms integral to learning *The Big Picture* concepts.

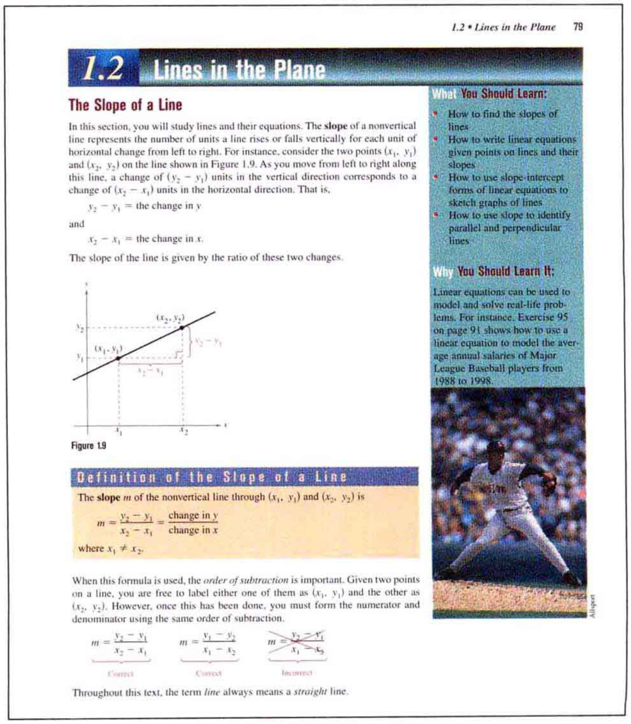
New Section Openers include:

▶ **"What you should learn"**

Objectives outline the main concepts and help keep students focused on *The Big Picture*.

▶ **"Why you should learn it"**

A real-life application or a reference to other branches of mathematics illustrates the relevance of the section's content.



1.2 • Lines in the Plane 79

1.2 Lines in the Plane

The Slope of a Line

In this section, you will study lines and their equations. The **slope** of a nonvertical line represents the number of units a line rises or falls vertically for each unit of horizontal change from left to right. For instance, consider the two points (x_1, y_1) and (x_2, y_2) on the line shown in Figure 1.9. As you move from left to right along this line, a change of $(y_2 - y_1)$ units in the vertical direction corresponds to a change of $(x_2 - x_1)$ units in the horizontal direction. That is,

$$y_2 - y_1 = \text{the change in } y$$

and

$$x_2 - x_1 = \text{the change in } x.$$

The slope of the line is given by the ratio of these two changes.

Figure 1.9

Definition of the Slope of a Line

The **slope** m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

where $x_1 \neq x_2$.

When this formula is used, the *order of subtraction* is important. Given two points on a line, you are free to label either one of them as (x_1, y_1) and the other as (x_2, y_2). However, once this has been done, you must form the numerator and denominator using the same order of subtraction.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \frac{y_1 - y_2}{x_1 - x_2} \quad m = \frac{y_2 - y_1}{x_1 - x_2}$$

Correct Correct Incorrect

Throughout this text, the term *line* always means a *straight* line.

What You Should Learn:

- How to find the slopes of lines
- How to write linear equations given points on lines and their slopes
- How to use slope-intercept forms of linear equations to sketch graphs of lines
- How to use slope to identify parallel and perpendicular lines

Why You Should Learn It:

Linear equations can be used to model and solve real-life problems. For instance, Exercise 95 on page 91 shows how to use a linear equation to model the average annual salaries of Major League Baseball players from 1988 to 1998.

Allsport

148 *Chapter 1 • Functions and Their Graphs*

1 Chapter Summary

What did you learn?

Section 1.1	Review Exercises
☐ How to sketch graphs of equations by point plotting and by using a graphing utility	1–22
☐ How to use graphs of equations in real-life problems	23, 24
Section 1.2	
☐ How to find the slopes of lines	25–34
☐ How to write linear equations given points on lines and their slopes	35–46
☐ How to use slope-intercept forms of linear equations to sketch graphs of lines	47–58
☐ How to use slope to identify parallel and perpendicular lines	59–62
Section 1.3	
☐ How to decide whether relations between two variables are functions	63–68
☐ How to use function notation and evaluate functions	69, 70
☐ How to find the domains of functions	71–76
☐ How to use functions to model and solve real-life problems	77, 78
Section 1.4	
☐ How to find the domains and ranges of functions and use the Vertical Line Test for functions	79–86
☐ How to determine intervals on which functions are increasing or decreasing	87–90
☐ How to determine relative maximum and relative minimum values of functions	91–94
☐ How to graph step functions and other piecewise-defined functions	95, 96
☐ How to identify even and odd functions	97, 98
Section 1.5	
☐ How to recognize graphs of common functions	99–102
☐ How to use vertical and horizontal shifts and reflections to sketch graphs of functions	103–118
☐ How to use nonrigid transformations to sketch graphs of functions	119–126
Section 1.6	
☐ How to add, subtract, multiply, and divide functions	127–132
☐ How to find compositions of one function with another function	133–136
☐ How to use combinations of functions to model and solve real-life problems	137, 138
Section 1.7	
☐ How to find inverse functions informally and verify that two functions are inverses of each other	139–142
☐ How to use graphs of functions to decide whether functions have inverses	143–150
☐ How to find inverse functions algebraically	151–156

▶ "What did you learn?" Chapter Summary

This chapter summary provides a concise, section-by-section review of the section objectives. These objectives are correlated to the chapter Review Exercises.

Flexibility and Accessibility

▶ Algebraic, Graphical, and Numerical Approach

- Many examples present solutions from multiple approaches—algebraic, graphical, and numerical.
- Solutions are displayed side-by-side.
- The multiple-approach format shows students different solution methods that can be used to reach the same answer.
- The solution format helps students expand their problem-solving abilities.

330 *Chapter 4 • Exponential and Logarithmic Functions*

Solving Exponential Equations

EXAMPLE 2 Solving Exponential Equations

Solve each equation. **a.** $e^x = 72$ **b.** $3(2^x) = 42$

Algebraic Solution

a. $e^x = 72$ Write original equation.

$\ln e^x = \ln 72$ Take natural log of each side.

$x = \ln 72$ Inverse Property

$x \approx 4.277$ Use a calculator.

The solution is $\ln 72 \approx 4.277$. Check this solution in the original equation.

b. $3(2^x) = 42$ Write original equation.

$2^x = 14$ Divide each side by 3.

$\log_2 2^x = \log_2 14$ Take log (base 2) of each side.

$x = \log_2 14$ Inverse Property

$x = \dfrac{\ln 14}{\ln 2}$ Change-of-base formula

$x \approx 3.807$ Use a calculator.

The solution is $\log_2 14 \approx 3.807$. Check this solution in the original equation.

Graphical Solution

a. Use a graphing utility to graph the left- and right-hand sides of the equation as $y_1 = e^x$ and $y_2 = 72$ in the same viewing window. Use the *intersect* feature or the *zoom* and *trace* features of the graphing utility to approximate the intersection point, as shown in Figure 4.24. So, the approximate solution is 4.277.

b. Use a graphing utility to graph $y_1 = 3(2^x)$ and $y_2 = 42$ in the same viewing window. Use the *intersect* feature or the *zoom* and *trace* features to approximate the intersection point, as shown in Figure 4.25. So, the approximate solution is 3.807.

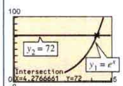


Figure 4.24

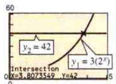


Figure 4.25

EXAMPLE 3 Solving an Exponential Equation

Solve $4e^{2x} = 5$.

Algebraic Solution

$4e^{2x} = 5$ Write original equation.

$e^{2x} = \dfrac{5}{4}$ Divide each side by 4.

$\ln e^{2x} = \ln \dfrac{5}{4}$ Take logarithm of each side.

$2x = \ln \dfrac{5}{4}$ Inverse Property

$x = \dfrac{1}{2} \ln \dfrac{5}{4}$ Solve for x.

$x \approx 0.112$ Use a calculator.

The solution is $\frac{1}{2} \ln \frac{5}{4} \approx 0.112$. Check this solution in the original equation.

Graphical Solution

Rather than graph both sides of the equation you are solving as separate graphs, as you did in Example 2, another way to graphically solve the equation is to first rewrite the equation as $4e^{2x} - 5 = 0$, then use a graphing utility to graph $y = 4e^{2x} - 5$. Use the *zero* or *root* feature or the *zoom* and *trace* features of the graphing utility to approximate the value of x for which $y = 0$. From Figure 4.26, you can see that the zero occurs when $x \approx 0.112$. So, the approximate solution is 0.112.

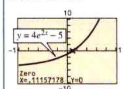
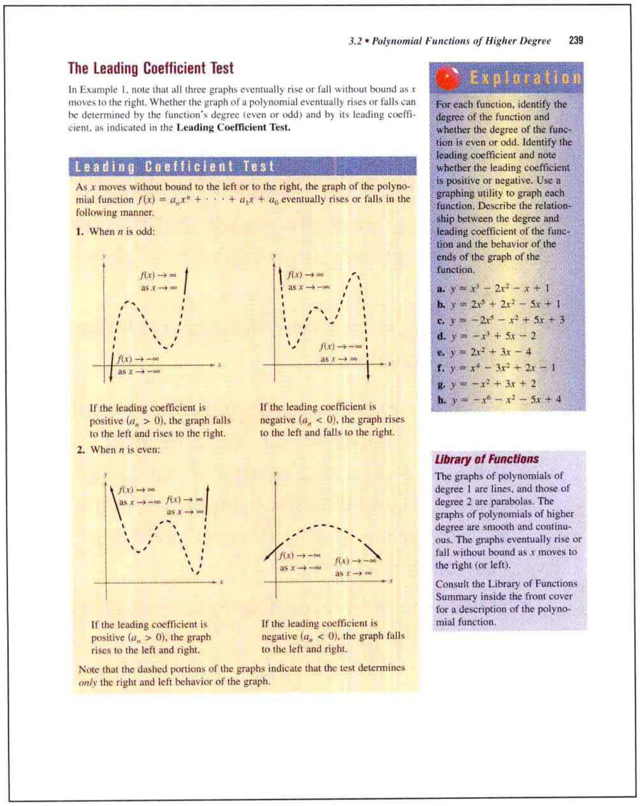


Figure 4.26



3.2 • Polynomial Functions of Higher Degree 239

The Leading Coefficient Test

In Example 1, note that all three graphs eventually rise or fall without bound as x moves to the right. Whether the graph of a polynomial eventually rises or falls can be determined by the function's degree (even or odd) and by its leading coefficient, as indicated in the **Leading Coefficient Test.**

Leading Coefficient Test

As x moves without bound to the left or to the right, the graph of the polynomial function $f(x) = a_n x^n + \cdots + a_1 x + a_0$ eventually rises or falls in the following manner.

1. When n is odd:

If the leading coefficient is positive ($a_n > 0$), the graph falls to the left and rises to the right.

If the leading coefficient is negative ($a_n < 0$), the graph rises to the left and falls to the right.

2. When n is even:

If the leading coefficient is positive ($a_n > 0$), the graph rises to the left and right.

If the leading coefficient is negative ($a_n < 0$), the graph falls to the left and right.

Note that the dashed portions of the graphs indicate that the test determines *only* the right and left behavior of the graph.

Exploration

For each function, identify the degree of the function and whether the degree of the function is even or odd. Identify the leading coefficient and note whether the leading coefficient is positive or negative. Use a graphing utility to graph each function. Describe the relationship between the degree and leading coefficient of the function and the behavior of the ends of the graph of the function.

a. $y = x^3 - 2x^2 - x + 1$
b. $y = 2x^5 + 2x^2 - 5x + 1$
c. $y = -2x^5 - x^2 + 5x + 3$
d. $y = -x^3 + 5x - 2$
e. $y = 2x^2 + 3x - 4$
f. $y = x^4 - 3x^2 + 2x - 1$
g. $y = -x^2 + 3x + 2$
h. $y = -x^6 - x^2 - 5x + 4$

Library of Functions

The graphs of polynomials of degree 1 are lines, and those of degree 2 are parabolas. The graphs of polynomials of higher degree are smooth and continuous. The graphs eventually rise or fall without bound as x moves to the right (or left).

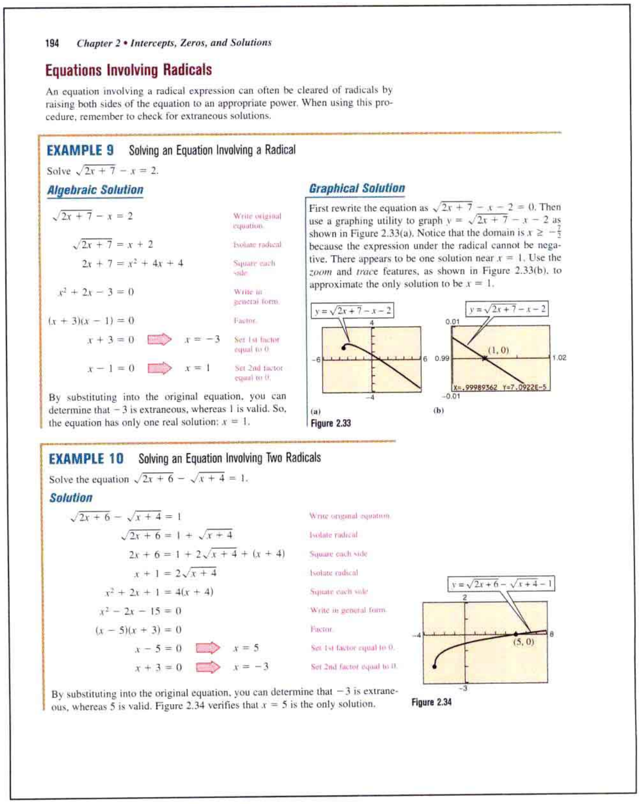
Consult the Library of Functions Summary inside the front cover for a description of the polynomial function.

Exploration

- Before introducing selected topics, *Exploration* engages students in active discovery of mathematical concepts and relationships, often through the power of technology.
- *Exploration* strengthens students' critical thinking skills and helps them develop an intuitive understanding of theoretical concepts.
- *Exploration* is an optional feature and can be omitted without loss of continuity in coverage.

▶ Examples

- Each example was carefully chosen to illustrate a particular mathematical concept or problem-solving skill.
- Every example contains step-by-step solutions, most with side-by-side explanations that lead students through the solution process.
- Many examples provide side-by-side solutions utilizing two separate approaches.



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Equations Involving Radicals

An equation involving a radical expression can often be cleared of radicals by raising both sides of the equation to an appropriate power. When using this procedure, remember to check for extraneous solutions.

EXAMPLE 9 Solving an Equation Involving a Radical

Solve $\sqrt{2x+7} - x = 2$.

Algebraic Solution

$\sqrt{2x+7} - x = 2$ — Write original equation.

$\sqrt{2x+7} = x + 2$ — Isolate radical.

$2x + 7 = x^2 + 4x + 4$ — Square each side.

$x^2 + 2x - 3 = 0$ — Write in general form.

$(x+3)(x-1) = 0$ — Factor.

$x + 3 = 0 \Rightarrow x = -3$ — Set 1st factor equal to 0.

$x - 1 = 0 \Rightarrow x = 1$ — Set 2nd factor equal to 0.

By substituting into the original equation, you can determine that -3 is extraneous, whereas 1 is valid. So, the equation has only one real solution: $x = 1$.

Graphical Solution

First rewrite the equation as $\sqrt{2x+7} - x - 2 = 0$. Then use a graphing utility to graph $y = \sqrt{2x+7} - x - 2$ as shown in Figure 2.33(a). Notice that the domain is $x \geq -\frac{7}{2}$ because the expression under the radical cannot be negative. There appears to be one solution near $x = 1$. Use the *zoom* and *trace* features, as shown in Figure 2.33(b), to approximate the only solution to be $x = 1$.

(a) (b)
Figure 2.33

EXAMPLE 10 Solving an Equation Involving Two Radicals

Solve the equation $\sqrt{2x+6} - \sqrt{x+4} = 1$.

Solution

$\sqrt{2x+6} - \sqrt{x+4} = 1$ — Write original equation.

$\sqrt{2x+6} = 1 + \sqrt{x+4}$ — Isolate radical.

$2x + 6 = 1 + 2\sqrt{x+4} + (x+4)$ — Square each side.

$x + 1 = 2\sqrt{x+4}$ — Isolate radical.

$x^2 + 2x + 1 = 4(x+4)$ — Square each side.

$x^2 - 2x - 15 = 0$ — Write in general form.

$(x-5)(x+3) = 0$ — Factor.

$x - 5 = 0 \Rightarrow x = 5$ — Set 1st factor equal to 0.

$x + 3 = 0 \Rightarrow x = -3$ — Set 2nd factor equal to 0.

By substituting into the original equation, you can determine that -3 is extraneous, whereas 5 is valid. Figure 2.34 verifies that $x = 5$ is the only solution.

Figure 2.34

Revised Exercises and Applications

Exercises

- Exercise sets consist of a variety of computational, conceptual, and applied problems.
- Exercise sets carefully graded in difficulty allow students to gain confidence as they progress.
- Each exercise set now concludes with two new types of exercises:
  - *Synthesis* exercises promote further exploration of mathematical concepts, critical thinking skills, and writing about mathematics. These exercises require students to synthesize the main concepts presented in the section and chapter.
  - *Review* exercises reinforce previously learned skills and concepts.

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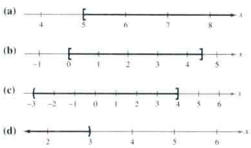
2.5 Exercises

In Exercises 1–4, match the inequality with its graph. [The graphs are labeled (a), (b), (c), and (d).]



1. $x < 3$ 2. $x \geq 5$
3. $-3 < x \leq 4$ 4. $0 \leq x \leq \frac{9}{2}$

In Exercises 5–8, determine whether each given value of x is a solution of the inequality.

Inequality	*Values*	
5. $5x - 12 > 0$	(a) $x = 3$	(b) $x = -3$
	(c) $x = \frac{5}{2}$	(d) $x = \frac{3}{2}$
6. $-5 < 2x - 1 \leq 1$	(a) $x = -\frac{1}{2}$	(b) $x = -\frac{5}{2}$
	(c) $x = \frac{4}{3}$	(d) $x = 0$
7. $-1 < \frac{3 - x}{2} \leq 1$	(a) $x = 0$	(b) $x = \sqrt{5}$
	(c) $x = 1$	(d) $x = 5$
8. $\lvert x - 10 \rvert \geq 3$	(a) $x = 13$	(b) $x = -1$
	(c) $x = 14$	(d) $x = 9$

In Exercises 9–18, solve the inequality and sketch the solution on the real number line. Use a graphing utility to verify your solution graphically.

9. $-10x < 40$ 10. $2x > 3$
11. $4(x + 1) < 2x + 3$ 12. $2x + 7 < 3$
13. $1 < 2x + 3 < 9$ 14. $-2 < 3x + 1 < 10$
15. $-8 \leq 1 - 3(x - 2) < 13$
16. $0 \leq 2(x + 4) < 20$
17. $-4 < \frac{2x - 3}{3} < 4$ 18. $0 \leq \frac{x + 3}{2} < 5$

Graphical Analysis **In Exercises 19–24, use a graphing utility to approximate the solution.**

19. $6x > 12$ 20. $3x - 1 \leq 5$
21. $5 - 2x \geq 1$ 22. $3(x + 1) < x + 7$
23. $-9 < 6x - 1 < 1$ 24. $-10 < 4(x - 3) \leq 8$

In Exercises 25–28, use a graphing utility to graph the equation and graphically approximate the values of x that satisfy the specified inequalities. Then solve each inequality algebraically.

Equation	*Inequalities*	
25. $y = 2x - 3$	(a) $y \geq 1$	(b) $y \leq 0$
26. $y = \frac{2}{3}x + 1$	(a) $y \leq 5$	(b) $y \geq 0$
27. $y = -\frac{1}{2}x + 2$	(a) $0 \leq y \leq 3$	(b) $y \geq 0$
28. $y = -3x + 8$	(a) $-1 \leq y \leq 3$	(b) $y \leq 0$

In Exercises 29–36, solve the inequality and sketch the solution on the real number line.

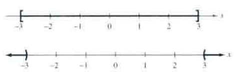
29. $\lvert 5x \rvert > 10$ 30. $\lvert x - 20 \rvert \leq 4$
31. $\lvert x - 7 \rvert < 6$ 32. $\lvert x - 20 \rvert \geq 4$
33. $\lvert x + 14 \rvert + 3 > 17$ 34. $\left\lvert \frac{x - 3}{2} \right\rvert \geq 5$
35. $\lvert 1 - 2x \rvert < 5$ 36. $3\lvert 4 - 5x \rvert \leq 9$

In Exercises 37 and 38, use a graphing utility to graph the equation and graphically approximate the values of x that satisfy the specified inequalities. Then solve each inequality algebraically.

Equation	*Inequalities*	
37. $y = \lvert x - 3 \rvert$	(a) $y \leq 2$	(b) $y \geq 4$
38. $y = \left\lvert \frac{1}{2}x + 1 \right\rvert$	(a) $y \leq 4$	(b) $y \geq 1$

In Exercises 39–44, use absolute value notation to define each interval (or pair of intervals) on the real number line.

39.
40.



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75. ***Forestry*** The number of board feet V in a 16-foot log is approximated by the model

$$V = 0.77x^2 - 1.32x - 9.31, \quad 5 \leq x \leq 40$$

where x is the diameter (in inches) of the log at the small end. (One board foot is a measure of volume equivalent to a board that is 12 inches wide, 12 inches long, and 1 inch thick.)
(a) Use a graphing utility to graph the function.
(b) Estimate the number of board feet in a 16-foot log with a diameter of 16 inches. Use a graphing utility to verify your answer.
(c) Estimate the diameter of a 16-foot log that scaled 500 board feet when the lumber was sold. Use a graphing utility to verify your answer.

76. ***Automobile Aerodynamics*** The number of horsepower y required to overcome wind drag on a certain automobile is approximated by

$$y = 0.002s^2 + 0.005s - 0.029, \quad 0 \leq s \leq 100$$

where s is the speed of the car in miles per hour.
(a) Use a graphing utility to graph the function.
(b) Graphically estimate the maximum speed of the car if the power required to overcome wind drag is not to exceed 10 horsepower. Verify your result algebraically.

77. ***Graphical Analysis*** For certain years from 1950 to 1990, the average annual per capita consumption C of cigarettes by Americans (18 and older) can be modeled by $C = 3248.89 + 108.64t - 2.97t^2$ for $0 \leq t \leq 40$, where t is the year, with $t = 0$ corresponding to 1950. (Source: U.S. Department of Agriculture)
(a) Use a graphing utility to graph the model.
(b) Use the graph of the model to approximate the maximum average annual consumption. Beginning in 1966, all cigarette packages were required by law to carry a health warning. Do you think the warning had any effect? Explain.
(c) In 1960, the U.S. population (18 and over) was 116,530,000. Of those, about 48,500,000 were smokers. What was the average annual cigarette consumption *per smoker* in 1960? What was the average daily cigarette consumption *per smoker*?

78. ***Data Analysis*** The number y (in millions) of VCRs in use in the United States for the years 1987 through 1996 are shown in the table. The variable t represents time (in years), with $t = 7$ corresponding to 1987.

t	7	8	9	10	11	12	13	14	15	16
y	43	51	58	63	67	69	72	74	77	79

(Source: Television Bureau of Advertising, Inc.)
(a) Use a graphing utility to sketch a scatter plot of the data.
(b) Use the regression capabilities of a graphing utility to fit a quadratic model to the data.
(c) Use a graphing utility to graph the model in the same viewing window as the scatter plot.
(d) Do you think the model can be used to estimate VCR utilization in the year 2005? Explain.

Synthesis

True or False? **In Exercises 79 and 80, determine whether the statement is true or false. Justify your answer.**

79. The function $f(x) = -12x^2 - 1$ has no x-intercepts.
80. The graphs of $f(x) = -4x^2 - 10x + 7$ and $g(x) = 12x^2 + 30x + 1$ have the same axis of symmetry.

81. ***Think About It*** The profits P (in millions of dollars) for a company are modeled by a quadratic function of the form $P = at^2 + bt + c$, where t represents the year. If you were president of the company, which of the models would you prefer? Explain your reasoning.
(a) a is positive and $t \geq -b/(2a)$.
(b) a is positive and $t \leq -b/(2a)$.
(c) a is negative and $t \geq -b/(2a)$.
(d) a is negative and $t \leq -b/(2a)$.

Review

In Exercises 82–85, determine algebraically any points of intersection of the graphs of the equations. Verify your results using the *intersect* feature of a graphing utility.

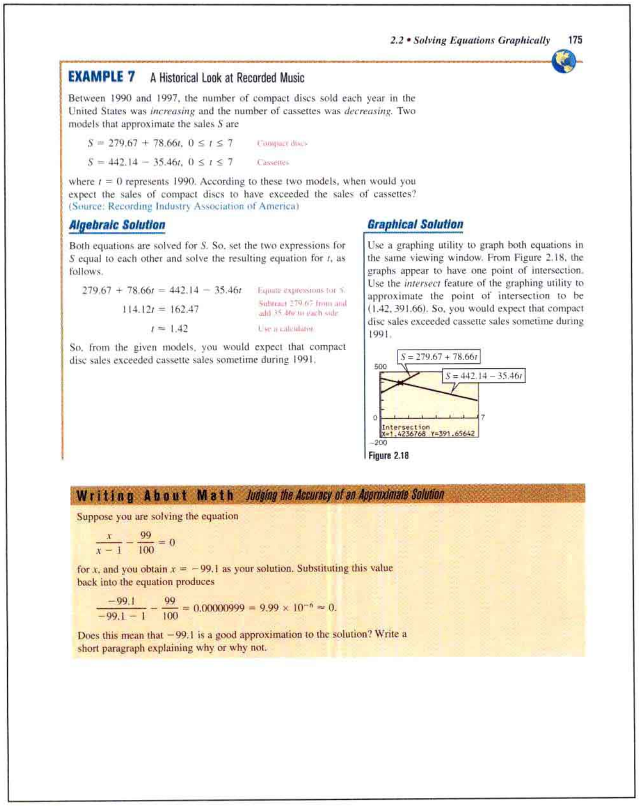
82. $x + y = 8$, $-\frac{2}{3}x + y = 6$ 83. $y = 3x - 10$, $y = \frac{1}{4}x + 1$
84. $y = 9 - x^2$, $y = x + 3$ 85. $y = x^3 + 2x - 1$, $y = -2x + 15$

In Exercises 86–89, perform the operation and write the result in standard form.

86. $(6 - i) - (2i + 11)$ 87. $(2i + 5)^2 - 21$
88. $(3i + 7)(-4i + 1)$ 89. $(4 - i)^3$

Real-Life Applications

- A wide variety of real-life applications, many using current, real data, are integrated throughout the examples and exercises.
- The icon indicates an example that involves a real-life application.



EXAMPLE 7 A Historical Look at Recorded Music

Between 1990 and 1997, the number of compact discs sold each year in the United States was *increasing* and the number of cassettes was *decreasing*. Two models that approximate the sales S are

$S = 279.67 + 78.66t,\ 0 \le t \le 7$ Compact discs

$S = 442.14 - 35.46t,\ 0 \le t \le 7$ Cassettes

where $t = 0$ represents 1990. According to these two models, when would you expect the sales of compact discs to have exceeded the sales of cassettes? (Source: Recording Industry Association of America)

Algebraic Solution

Both equations are solved for S. So, set the two expressions for S equal to each other and solve the resulting equation for t, as follows.

$279.67 + 78.66t = 442.14 - 35.46t$ Equate expressions for S.

$114.12t = 162.47$ Subtract 279.67 from and add 35.46t to each side.

$t \approx 1.42$ Use a calculator.

So, from the given models, you would expect that compact disc sales exceeded cassette sales sometime during 1991.

Graphical Solution

Use a graphing utility to graph both equations in the same viewing window. From Figure 2.18, the graphs appear to have one point of intersection. Use the *intersect* feature of the graphing utility to approximate the point of intersection to be (1.42, 391.66). So, you would expect that compact disc sales exceeded cassette sales sometime during 1991.

Figure 2.18

Writing About Math *Judging the Accuracy of an Approximate Solution*

Suppose you are solving the equation

$$\frac{x}{x-1} - \frac{99}{100} = 0$$

for x, and you obtain $x = -99.1$ as your solution. Substituting this value back into the equation produces

$$\frac{-99.1}{-99.1-1} - \frac{99}{100} = 0.00000999 = 9.99 \times 10^{-6} \approx 0.$$

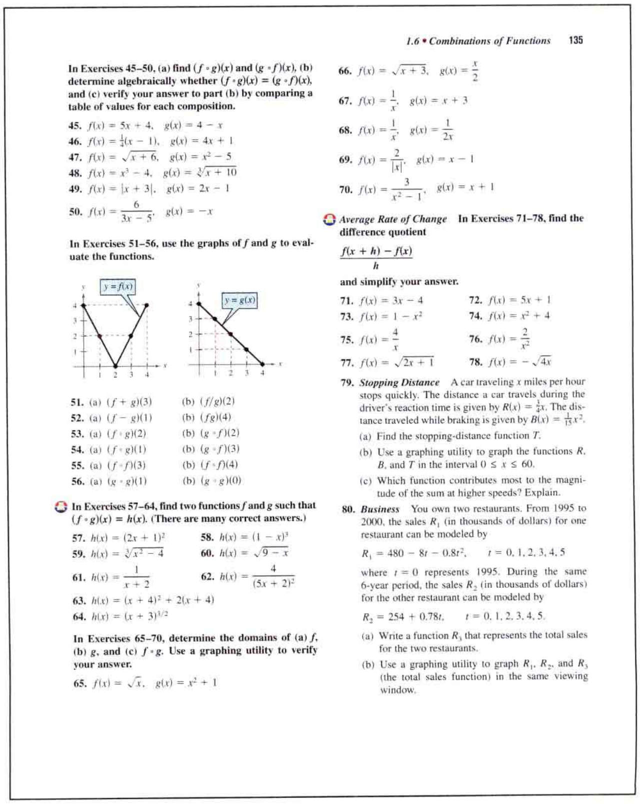
Does this mean that -99.1 is a good approximation to the solution? Write a short paragraph explaining why or why not.

Algebra of Calculus

- Special emphasis is given to the algebraic techniques used in calculus.
- Algebra of Calculus examples and exercises are integrated throughout the text.
- The symbol indicates an example or exercise in which the Algebra of Calculus is featured.

Additional Features

Carefully crafted learning tools designed to create a rich learning environment can be found throughout the text. These learning tools include a Library of Functions, Study Tips, Historical Notes, Writing About Math, Chapter Projects, Chapter Review Exercises, Chapter Tests, Cumulative Tests, and an extensive art program.



In Exercises 45–50, (a) find $(f \circ g)(x)$ and $(g \circ f)(x)$, (b) determine algebraically whether $(f \circ g)(x) = (g \circ f)(x)$, and (c) verify your answer to part (b) by comparing a table of values for each composition.

45. $f(x) = 5x + 4,\ g(x) = 4 - x$
46. $f(x) = \frac{1}{4}(x - 1),\ g(x) = 4x + 1$
47. $f(x) = \sqrt{x + 6},\ g(x) = x^2 - 5$
48. $f(x) = x^3 - 4,\ g(x) = \sqrt[3]{x + 10}$
49. $f(x) = |x + 3|,\ g(x) = 2x - 1$
50. $f(x) = \frac{6}{3x - 5},\ g(x) = -x$

In Exercises 51–56, use the graphs of f and g to evaluate the functions.

51. (a) $(f + g)(3)$ (b) $(f/g)(2)$
52. (a) $(f - g)(1)$ (b) $(fg)(4)$
53. (a) $(f \circ g)(2)$ (b) $(g \circ f)(2)$
54. (a) $(f \circ g)(1)$ (b) $(g \circ f)(3)$
55. (a) $(f \circ f)(3)$ (b) $(f \circ f)(4)$
56. (a) $(g \circ g)(1)$ (b) $(g \circ g)(0)$

In Exercises 57–64, find two functions f and g such that $(f \circ g)(x) = h(x)$. (There are many correct answers.)

57. $h(x) = (2x + 1)^2$ **58.** $h(x) = (1 - x)^3$
59. $h(x) = \sqrt[3]{x^2 - 4}$ **60.** $h(x) = \sqrt{9 - x}$
61. $h(x) = \frac{1}{x + 2}$ **62.** $h(x) = \frac{4}{(5x + 2)^2}$
63. $h(x) = (x + 4)^2 + 2(x + 4)$
64. $h(x) = (x + 3)^{3/2}$

In Exercises 65–70, determine the domains of (a) f, (b) g, and (c) $f \circ g$. Use a graphing utility to verify your answer.

65. $f(x) = \sqrt{x},\ g(x) = x^2 + 1$
66. $f(x) = \sqrt{x + 3},\ g(x) = \frac{x}{2}$
67. $f(x) = \frac{1}{x},\ g(x) = x + 3$
68. $f(x) = \frac{1}{x},\ g(x) = \frac{1}{2x}$
69. $f(x) = \frac{2}{|x|},\ g(x) = x - 1$
70. $f(x) = \frac{3}{x^2 - 1},\ g(x) = x + 1$

Average Rate of Change **In Exercises 71–78, find the difference quotient**

$$\frac{f(x + h) - f(x)}{h}$$

and simplify your answer.

71. $f(x) = 3x - 4$ **72.** $f(x) = 5x + 1$
73. $f(x) = 1 - x^2$ **74.** $f(x) = x^2 + 4$
75. $f(x) = \frac{4}{x}$ **76.** $f(x) = \frac{2}{x^2}$
77. $f(x) = \sqrt{2x + 1}$ **78.** $f(x) = -\sqrt{4x}$

79. ***Stopping Distance*** A car traveling x miles per hour stops quickly. The distance a car travels during the driver's reaction time is given by $R(x) = \frac{3}{4}x$. The distance traveled while braking is given by $B(x) = \frac{1}{15}x^2$.
(a) Find the stopping-distance function T.
(b) Use a graphing utility to graph the functions R, B, and T in the interval $0 \le x \le 60$.
(c) Which function contributes most to the magnitude of the sum at higher speeds? Explain.

80. ***Business*** You own two restaurants. From 1995 to 2000, the sales R_1 (in thousands of dollars) for one restaurant can be modeled by

$R_1 = 480 - 8t - 0.8t^2,\quad t = 0, 1, 2, 3, 4, 5$

where $t = 0$ represents 1995. During the same 6-year period, the sales R_2 (in thousands of dollars) for the other restaurant can be modeled by

$R_2 = 254 + 0.78t,\quad t = 0, 1, 2, 3, 4, 5.$

(a) Write a function R_3 that represents the total sales for the two restaurants.
(b) Use a graphing utility to graph R_1, R_2, and R_3 (the total sales function) in the same viewing window.

Supplements

Resources

Website (*college.hmco.com*)
Many additional text-specific study and interactive features for students and instructors can be found at the Houghton Mifflin website.

For the Student

Student Success Organizer

Study and Solutions Guide by Bruce H. Edwards (University of Florida)

Graphing Technology Guide by Benjamin N. Levy and Laurel Technical Services

Instructional Videotapes by Dana Mosely

Instructional Videotapes for Graphing Calculators by Dana Mosely

For the Instructor

Instructor's Annotated Edition

Instructor Success Organizer

Complete Solutions Guide by Bruce H. Edwards (University of Florida)

Test Item File

Problem Solving, Modeling, and Data Analysis Labs by Wendy Metzger (Palomar College)

Computerized Testing (Windows, Macintosh)

HMClassPrep Instructor's CD-ROM

An Introduction to Graphing Utilities

Graphing utilities such as graphing calculators and computers with graphing software are very valuable tools for visualizing mathematical principles, verifying solutions to equations, exploring mathematical ideas, and developing mathematical models. Although graphing utilities are extremely helpful in learning mathematics, their use does not mean that learning algebra is any less important. In fact, the combination of knowledge of mathematics and the use of graphing utilities allows you to explore mathematics more easily and to a greater depth. If you are using a graphing utility in this course, it is up to you to learn its capabilities and to practice using this tool to enhance your mathematical learning.

In this text there are many opportunities to use a graphing utility, some of which are described below.

Some Uses of a Graphing Utility

A graphing utility can be used to

- check or validate answers to problems obtained using algebraic methods.
- discover and explore algebraic properties, rules, and concepts.
- graph functions, and approximate solutions to equations involving functions.
- efficiently perform complicated mathematical procedures such as those found in many real-life applications.
- find mathematical models for sets of data.

In this introduction, the features of graphing utilities are discussed from a generic perspective. To learn how to use the features of a specific graphing utility, consult your user's manual or the website for this text found at *college.hmco.com*. Additionally, keystroke guides are available for most graphing utilities, and your college library may have a videotape on how to use your graphing utility.

The Equation Editor

Many graphing utilities are designed to act as "function graphers." In this course, you will study functions and their graphs in detail. You may recall from previous courses that a function can be thought of as a rule that describes the relationship between two variables. These rules are frequently written in terms of x and y. For example, the equation $y = 3x + 5$ represents y as a function of x.

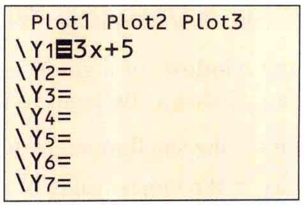


Figure 1

Many graphing utilities have an equation editor that requires an equation to be written in "$y =$" form in order to be entered, as shown in Figure 1. (You should note that your equation editor screen may not look like the screen shown in Figure 1.) To determine exactly how to enter an equation into your graphing utility, consult your user's manual.

The Table Feature

Most graphing utilities are capable of displaying a table of values with x-values and one or more corresponding y-values. These tables can be used to check solutions of an equation and to generate ordered pairs to assist in graphing an equation.

To use the *table* feature, enter an equation into the equation editor in "$y =$" form. The table may have a setup screen, which allows you to select the starting x-value and the table step or x-increment. You may then have the option of automatically generating values for x and y or building your own table using the *ask* mode. In the *ask* mode, you enter a value for x and the graphing utility displays the y-value.

```
Plot1 Plot2 Plot3
\Y1=3x/(X+2)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Figure 2

For example, enter the equation

$$y = \frac{3x}{x + 2}$$

into the equation editor, as shown in Figure 2. In the table setup screen, set the table to start at $x = -4$ and set the table step to 1. When you view the table, notice that the first x-value is -4 and each value after it increases by 1. Also notice that the Y_1 column gives the resulting y-value for each x-value, as shown in Figure 3. The table shows that the y-value when $x = -2$ is ERROR. This means that the variable x may not take on the value -2 in this equation.

X	Y1	
-4	6	
-3	9	
-2	ERROR	
-1	-3	
0	0	
1	1	
2	1.5	

X=-4

Figure 3

With the same equation in the equation editor, set the table to *ask* mode. In this mode you do not need to set the starting x-value or the table step, because you are entering any value you choose for x. You may enter any real value for x—integers, fractions, decimals, irrational numbers, and so forth. If you enter $x = 1 + \sqrt{3}$, the graphing utility may rewrite the number as a decimal approximation, as shown in Figure 4. You can continue to build your own table by entering additional x-values in order to generate y-values.

X	Y1	
2.7321	1.7321	

X=

Figure 4

If you have several equations in the equation editor, the table may generate y-values for each equation.

Creating a Viewing Window

A **viewing window** for a graph is a rectangular portion of the coordinate plane. A viewing window is determined by the following six values.

Xmin = the smallest value of x

Xmax = the largest value of x

Xscl = the number of units per tick mark on the x-axis

Ymin = the smallest value of y

Ymax = the largest value of y

Yscl = the number of units per tick mark on the y-axis

When you enter these six values into a graphing utility, you are setting the viewing window. Some graphing utilities have a standard viewing window, as shown in Figure 5.

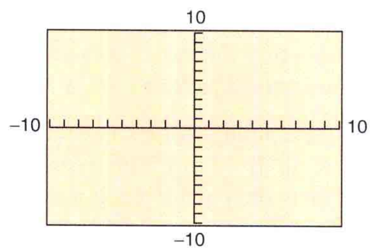


Figure 5

By choosing different viewing windows for a graph, it is possible to obtain very different impressions of the graph's shape. For instance, Figure 6 shows four different viewing windows for the graph of

$$y = 0.1x^4 - x^3 + 2x^2.$$

Of these, the view shown in part (a) is the most complete.

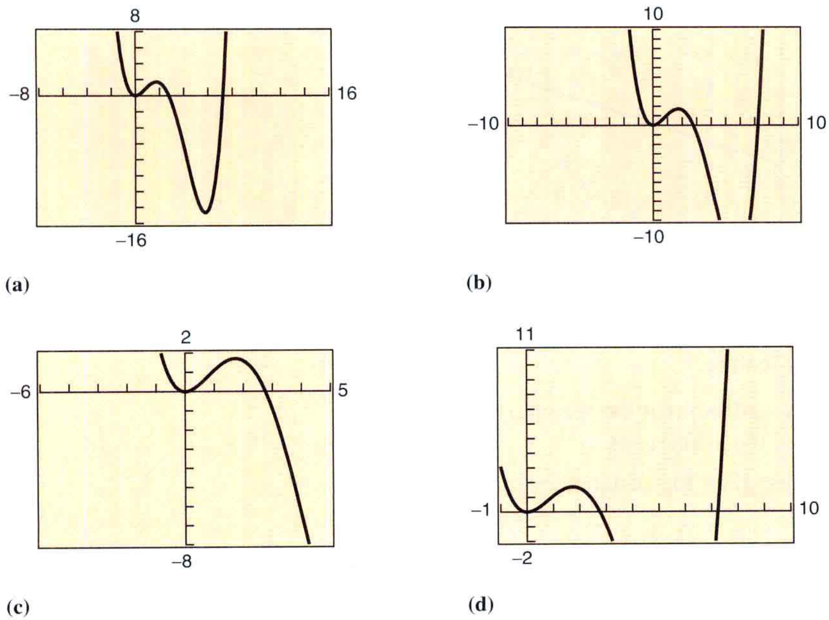


Figure 6

On most graphing utilities, the display screen is two-thirds as high as it is wide. On such screens, you can obtain a graph with a true geometric perspective by using **a square setting**—one in which

$$\frac{\text{Ymax} - \text{Ymin}}{\text{Xmax} - \text{Xmin}} = \frac{2}{3}.$$

One such setting is shown in Figure 7. Notice that the x and y tick marks are equally spaced on a square setting, but not on a standard setting.

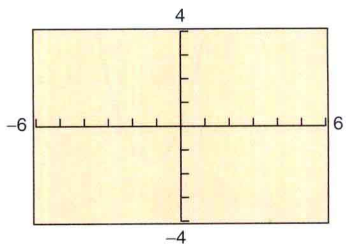


Figure 7

To see how the viewing window affects the geometric perspective, graph the semicircles $y_1 = \sqrt{9 - x^2}$ and $y_2 = -\sqrt{9 - x^2}$ in a standard viewing window. Then graph y_1 and y_2 in a square window. Note the difference in the shapes of the circles.

Zoom and Trace Features

When you graph an equation, you can move from point to point along its graph using the *trace* feature. As you trace the graph, the coordinates of each point are displayed, as shown in Figure 8. The *trace* feature combined with the *zoom* feature allows you to obtain better and better approximations of desired points on a graph. For instance, you can use the *zoom* feature of a graphing utility to approximate the x-intercept(s) of a graph [the point(s) where the graph crosses the x-axis]. Suppose you want to approximate the x-intercept(s) of the graph of $y = 2x^3 - 3x + 2$.

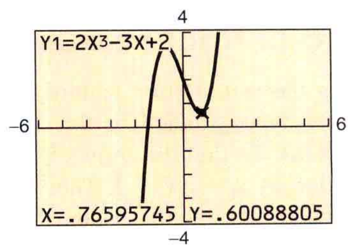
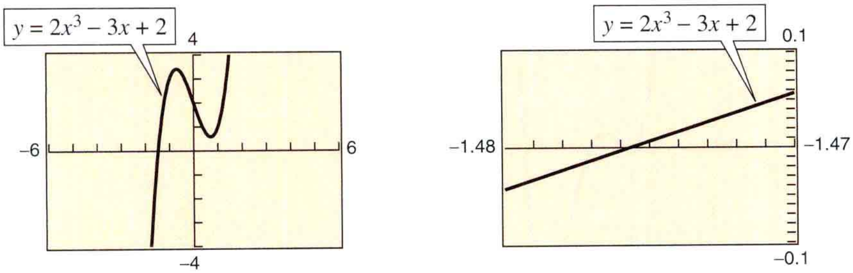


Figure 8

Begin by graphing the equation, as shown in Figure 9(a). From the viewing window shown, the graph appears to have only one x-intercept. This intercept lies between -2 and -1. By zooming in on the intercept, you can improve the approximation, as shown in Figure 9(b). To three decimal places, the solution is $x \approx -1.476$.



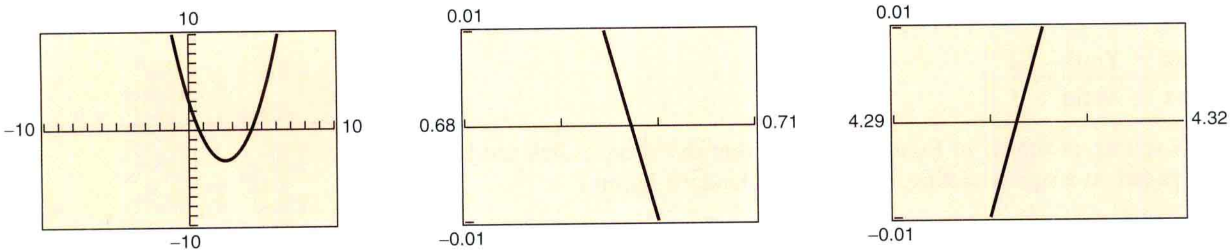
(a) **(b)**

Figure 9

Here are some suggestions for using the *zoom* feature.

1. With each successive zoom-in, adjust the x-scale so that the viewing window shows at least one tick mark on each side of the x-intercept.
2. The error in your approximation will be less than the distance between two scale marks.
3. The *trace* feature can usually be used to add one more decimal place of accuracy without changing the viewing window.

Figure 10(a) shows the graph of $y = x^2 - 5x + 3$. Figures 10(b) and 10(c) show "zoom-in views" of the two x-intercepts. From these views, you can approximate the x-intercepts to be $x \approx 0.697$ and $x \approx 4.303$.



(a) **(b)** **(c)**

Figure 10

Zero or Root Feature

Using the *zero* or *root* feature, you can find the real zeros of functions of the various types studied in this text—polynomial, exponential, and logarithmic functions. To find the zeros of a function such as $f(x) = \frac{3}{4}x - 2$, first enter the function as $y_1 = \frac{3}{4}x - 2$. Then use the *zero* or *root* feature, which may require entering lower and upper bound estimates of the root, as shown in Figure 11.