Explorations in College Algebra

Linda Almgren Kime
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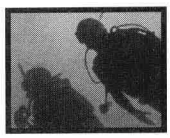

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To all of our students, who have been thoughtful critics
and enthusiastic participants in the development of these materials

and

to our parents and other members of our families
who provided the inspiration for us to pursue mathematics.

To the Students

FROM A STUDENT

Have you ever thought of math as something that you like to do? No, I'm not crazy. You might think of math as merely a bunch of numbers and variables and long hours of monotonous repetition, but this class makes math fun and interesting.

Math will come alive right in front of you. This class, unlike many other math courses, is different in that it shows you practical applications of math to real life situations and gives meaning to math. The numbers and variables are symbolic representations of things in the world. Math will finally make sense.

This class is also different in that there will be lecture, but much of the class time will be spent *doing* math. I found this to be very helpful in that you're actually doing the math right there and then and not trying to decipher and make sense of your lecture notes at home where it's too late to ask questions.

At first things will take a little time to get used to. But watch out. Once you see the big picture of this class it can be very addictive. All you have to do is give it a chance to show you what it's all about. This class opens up your mind and helps you think critically and thoughtfully about math as you never have before. I hope you enjoy it as much as I have.

Sincerely,
Philip Wan
Nursing/Pre-Med Major, UMass/Boston

FROM THE AUTHORS

Mathematics offers a way of looking at the world. Just as musical training increases awareness of sounds, or knowledge of history gives a deeper perspective on the present, mathematics heightens your perception of underlying order and systematic patterns in the world around you.

Our approach to learning algebra starts with questions from the physical and social sciences. You will learn to use algebra in your search for answers. You will also be asked to *read* and analyze mathematical arguments from a variety of different sources and viewpoints. You will be asked to *reason* through your own quantitative arguments, using both traditional algebraic tools and contemporary technological ones. And you will be asked to *write* about your conclusions.

The classes may not look much like the ones you're used to. The professor might lecture a lot less, and you may find yourself working with your classmates in small groups. An exploratory approach results in a lot of questions and discussions. This process of exploring is as important as memorizing the "facts."

We hope that you learn mathematics in a way that will prove useful throughout your life.

With warm regards,
Linda and Judy
Cambridge, Massachusetts, June 1997

Preface

This book grew from a desire to reshape the standard college algebra course. We wanted to make algebra interesting and relevant to our students. Thus we designed the materials to shift the focus from learning a set of discrete mechanical rules to exploring how algebra is used to answer questions about the physical and social world around us.

The materials are in the spirit of the reform movement. The content and approach reflect the standards established by the National Council of Teachers of Mathematics (NCTM) and the American Mathematical Association of Two-Year Colleges (AMATYC), and the recommendations of the Mathematical Association of America (MAA). Our goal is to adequately prepare students for future mathematics or other quantitatively based courses. This text, while designed for college algebra, has been used successfully for other types of courses including pre-calculus for nonmath majors and quantitative reasoning courses.

OUR PROCESS OF DEVELOPMENT

The materials were first used in an experimental course, Explorations in College Algebra, funded in part by a grant from the National Science Foundation, and taught at the University of Massachusetts, Boston. The text and accompanying software have evolved from the ongoing collaboration of faculty from a broad consortium of schools. The Rough Draft was tested at fourteen beta sites. Their feedback shaped the Preliminary Edition, which was in turn critiqued by additional mathematics faculty, teaching assistants, and students. We also consulted with a wide range of discipline specialists, such as economists, physicists, bankers, biologists, political scientists, and astronomers, in order to understand what kinds of questions and answers were relevant to these fields. We wanted to know how they actually *used* mathematics, not so much in classrooms as in their work.

We conducted an extensive survey of faculty members teaching college algebra courses. The results helped us decide which topics to include in the text, but also convinced us that there is no such thing as a generic college algebra course.

Our students graciously allowed us to experiment, revise, and experiment again. They enthusiastically provided honest and insightful advice, and suggestions for improvement. Their input, perhaps more than any others, shaped the content and ideas of the course.

GENERAL PRINCIPLES

The following general principles served as our guide.

- Algebra is a powerful tool that can help answer questions that arise in the social,

physical, and life sciences. Algebraic procedures and concepts can be developed from the investigation of practical problems.

- Mathematical ideas can be represented and understood in multiple ways—through words, numbers, graphs, and symbols.
- Students can make connections among various representations and understand concepts more deeply by becoming actively engaged in doing mathematics, as well as by communicating their ideas to others.
- Materials should be flexible. They should accommodate a broad range of teaching and learning styles in a variety of settings.
- Technology should be used when appropriate. While technology is not required to teach this course, its use can greatly enhance student understanding.
- Conceptual understanding should be accompanied by sufficient practice in building skills.

ORGANIZATION

The text includes more material than college algebra courses usually include in one semester. Instructors will need to make choices and may wish to cover the chapters in a different order. It is recommended, however, that students become familiar with the material covered in Chapters 1, 2, 3, and 4 before attempting to do the other chapters. The rest of the chapters may be covered in any order, with two exceptions: the material on exponents in Chapter 7 should be covered prior to material in the other chapters in Part II; and Chapter 12, "Logarithmic Links: Logarithmic, Exponential, and Power Functions" assumes Sec. 7.7, "Logarithms Base 10," and Chapter 8, "Growth and Decay." Entering students should have had elementary high school algebra and be familiar with the use of symbols to represent unknown numerical quantities.

Part I focuses on algebraic applications in the social sciences. Chapter 1 introduces the basic issues in working with and writing about data. This chapter is the least like a traditional algebra chapter, but perhaps the most relevant to students' everyday lives. Students can be introduced to the basics of using a graphing calculator or computer. The amount of time spent on this chapter (from 2 days to 3 weeks) will vary a great deal from instructor to instructor. The time spent is probably dictated by what mathematics courses most of the students would take next and how long it takes for students to become familiar with a particular technology.

Chapter 2 introduces the fundamental concept of functions and their representations in words, tables, graphs, and equations. In Chapter 3, students encounter the notion of the *average rate of change,* which becomes a central thread throughout the course.

Chapter 4 is the first of three chapters dedicated to *linear functions* and their applications. It is motivated by looking at phenomena in which the rate of change is constant and focuses on constructing, graphing, and interpreting linear functions. Chapter 5 uses a case study approach to introduce a major tool of the social sciences—fitting lines to data. Students use real U.S. Census data on 1000 families to describe and analyze the relationship between education and income. Chapter 6 deals with *systems of linear equations* and their uses in economics, including a comparison of the costs of different heating systems and the benefits of different income tax plans.

Part II opens with Chapter 7, "Deep Time and Deep Space." Studying the age and

size of objects in the known universe provides a context for learning *scientific notation* and strategies for comparing objects of widely differing sizes. Students learn to estimate answers, a critical skill in this age of calculators and computers. The basic laws of manipulating *exponents* are applied to different bases and powers. *Logarithms* base 10 are used to scale graphs and to construct measurement systems for numbers with different orders of magnitude.

In Chapter 8, the growth of *E. coli* bacteria motivates the study of *exponential functions* of the form $y = Ca^x$. Models of exponential growth or decay are applied to human populations, radioactive decay, and Medicare costs. Semi-log plots are introduced to identify exponential patterns in data. *Power functions* are introduced in Chapter 9 through studying the relationship between size and shape in biological organisms. Students learn about general *polynomial functions* as the sum of power functions with positive integer exponents.

Chapters 10 and 11 cover the family of *quadratic functions*. A case study of a classic free-fall experiment in Chapter 10 gives students some insights into how Galileo uncovered the basic laws of motion. In Chapter 11, "Parabolic Reflections," the properties of quadratic functions and their graphs are examined.

Logarithmic functions are motivated in Chapter 12 by the need to solve exponential equations studied in Chapter 8. Chapter 12 extends the discussion in Section 7.7 of logarithms as numbers to logarithms as functions. It introduces e, the natural logarithm, and exponential functions of the form $y = Ce^{rx}$. Data are graphed on linear, semi-log, and log-log plots to find an appropriate function model.

SPECIAL FEATURES AND THEIR USES

In addition to the usual collection of chapters, chapter overviews and summaries, exercises, and answer sections, these materials contain many special features. These features represent an array of resources from which the instructor and student can pick and choose. Suggestions for using each resource component appear at the appropriate point in the text and in the Instructor's Manual. The following list contains an overview of the features and some suggested ways to use them.

Explorations

Explorations, located at the end of each chapter, allow students to experiment with ideas. The Explorations can be used with students working in small groups or individually, in or out of the classroom, or assigned as special projects. They are generally more open ended than exercises, and are designed to be used in parallel with reading the text. An Exploration icon indicates when a particular Exploration could be started, rather than waiting until the end of the chapter.

CD-ROM

The text comes with a CD-ROM that contains multiple additional resources for the course. The course software, data files, and Graphing Calculator Workbook are all on the CD-ROM and are discussed in the following section on technology use. The CD also contains the Student's Solution Manual, additional exercises, readings, and Explorations. A separate Instructor's Resource CD can be ordered from Wiley. It contains the materials on the text CD as well as the instructor's Manual and Instructor's Solution Manual. The CD icon appears in the text when a resource is available on CD-ROM.

Anthology of Readings

The *Anthology of Readings* is in the Appendix with additional readings on the CD-ROM. The Anthology contains a variety of articles from newspapers, government publications, scientific and popular magazines, etc. Some readings highlight a controversy about a topic discussed in a chapter, others offer expanded coverage of a mathematical topic only mentioned in the text. Each reading is referenced with an icon at an appropriate point in the text, exercises, or Explorations. Instructors, however, may wish to find new and creative ways to use these or other readings.

Something to think about

Short inserts entitled *Something to think about* pose provocative questions that could be used as the basis of a class discussion or an assignment or for students to ponder on their own.

Writing assignments and class presentations

Many of the exercises or Explorations contain a writing component. Chapter 1 includes suggestions for writing strategies. The notion of a "60 Second Summary" is introduced as a way for students to present key ideas clearly and concisely. Some of the Explorations also ask students to present their results to the class.

TECHNOLOGY COMPONENTS

There are many Explorations and exercises built around graphing calculators and computers, but there are no specific hardware requirements. Some schools use only graphing calculators, others use only computers with the course software, and some use a combination of both. The course can also be taught entirely in a computer lab using a spreadsheet program, a simple function graphing program, and the course software. The accompanying CD-ROM includes several resources to make using technology easier. A CD icon indicates when one of the electronic resources might prove useful. The CD can be used with either a PC (with Windows) or Mac and includes:

Custom Software

These programs offer an easy-to-use interactive environment that helps students visualize specific mathematical concepts, and provides practice in basic skills. The software can be used for classroom demonstrations or in a lab as the basis for several of the Explorations, or at home by the student.

Graph Link Files

The CD-ROM contains numerous graph link files for the TI-82 or TI-83 graphing calculators and instructions for file downloading. These short programs contain all the major data sets used in the course. They can all be transferred at once to every student's calculator at the beginning of the course.

Graphing Calculator Workbook

Basic instructions for using TI-82 and TI-83 calculators are included in the Graphing Calculator Workbook. The instructions are linked directly with the chapters, and introduce calculator skills as needed. The CD-ROM contains additional worksheets with problems that provide additional practice in basic skills. If students are using graphing calculators for the first time, we strongly

suggest they read through the workbook instructions before doing the Explorations or exercises for each chapter. The workbook may be ordered in hard copy, as well.

Spreadsheet Files

All the major data sets (including the entire FAM 1000 U.S. Census data set used in the case study in Chapter 5) exist as Excel files on the CD-ROM.

SKILL-BUILDING COMPONENTS

In addition to doing selected end of chapter exercises, students can build skills in manipulating algebraic expressions through:

Algebra Aerobics

Algebra Aerobics are short collections of problems integrated throughout the text, usually at the end of sections, that provide basic practice in skills either recently introduced or assumed as prerequisites. Answers to these drill problems are provided in the Appendices. Students are encouraged to read the text with pencil and paper in hand, and to stop and work through these skill problems and check their answers before moving on.

Custom Software

Certain programs in the custom software provide skill-building practice. Instructors can assign these programs for homework, or students on their own may wish to use them to practice.

Worksheets on the CD-ROM

The CD-ROM contains additional worksheets and answers to reinforce basic manipulative skills. Instructors are free to print out and xerox the worksheets as handouts for an entire class, or just for students who need extra help in a particular topic. Students may also wish to print them out and use them for practice on their own.

OTHER TEXT SUPPLEMENTS

Instructors can order printed versions of the Graphing Calculator Workbook or the Student's Solution Manual that appear on the CD-ROM packaged with the text. The Instructor's Manual and Instructor's Solution Manual exist on a separate Instructor's Resource CD and in print versions.

MAJOR CHANGES FROM THE PRELIMINARY TO THE FIRST EDITION

- A new Chapter 12 introduces logarithmic functions and exponential functions base e.
- The text has been refined based on feedback from numerous test sites for increased clarity, and additional problems and examples have been added where needed.

- The Algebra Aerobics sections, for skill building, have been increased to provide additional practice with drill exercises.
- The CD-ROM, packaged with every text, provides students with course software and data sets, along with a getting-started Graphing Calculator Manual for the TI-82 and 83. Other resources on the disk: additional skill building material, readings, worksheets, and exercises. The Student's Solution Manual, with answers to the odd exercises found in the text, is also included on the CD.
- The Instructor's Resource CD-ROM contains the Instructor's Manual, Instructor's Solution Manual, and all features contained on the student CD.
- The separate Graphing Calculator Workbook replaces the graphing calculator appendix.

KEEPING IN TOUCH

You may want to visit our Web site at *http://www.wiley.com/college/math/mathem/kimeclark* to look for (or suggest) new ideas for Explorations, readings, and exercises, to leave mail for the authors, or to join the ongoing discussion of the College Algebra Consortium on reforming college algebra. The authors would be delighted to receive reactions to the materials. Please contact us at the University of Massachusetts, Boston, 100 Morrissey Blvd, Boston, MA 02125.

Readers interested in further information about the materials should contact John Wiley & Sons, Inc., by mail at 605 Third Avenue, New York, NY 10158 or by e-mail at *math@wiley.com.*

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