



ALGEBRA AND TRIGONOMETRY

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Algebra and Trigonometry

Second Edition

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To Our Wives

**Mary Alice
Julia**

Preface

This book is designed to provide the essentials of college algebra and trigonometry for students who have had two to three years of high school mathematics. As in the first edition, the goal has been to produce a text that can be understood by an average student with a minimal amount of outside assistance, and to do so without any sacrifice in rigor.

We have done our best to present sound mathematics in an informal manner that stresses meaningful motivation, detailed explanations, extensive use of pictures and diagrams, numerous examples, and emphasis on the real-world origins of basic concepts (especially functions). There are a wide variety of exercises, ranging from routine to challenging, including a large number of thought-provoking problems that are well within the reach of the average student.

The flexible design of the text (which is fully explained in the To the Instructor section on page xv) makes it suitable for a wide variety of courses ranging from one-term courses in college algebra, applied college algebra, or trigonometry for noncontinuing students to longer sequences intended to prepare students for calculus.

PEDAGOGICAL CHANGES

Approximately three fourths of the book has been completely rewritten to improve clarity and convenience. Major changes include the following:

Complex numbers are introduced immediately after quadratic equations (Section 2.5).

The presentation of inverse functions has been revised and appears in Section 4.6.

An easier method of graphing polynomial functions is now used (Section 5.2).

The coverage of logarithms begins with a section on natural logarithms (Section 6.2) because of their central role in calculus. However, Section 6.3 on logarithms to an arbitrary base is written so that it may be covered first by instructors who prefer the conventional approach.

Trigonometry (Chapter 7) begins with right triangles, at the request of many instructors.

The solution algorithm for trigonometric equations (Section 8.4) has been rewritten. It allows students easily to solve equations other than those whose solutions are special angles.

Conic sections are treated in greater depth (including applications and rotation of axes) in a separate chapter (Chapter 10). However, the material is arranged to allow a fast treatment of conics centered at the origin if time is short.

NEW MATERIAL

Many optional sections have been added, covering the following topics:

Variation

Descartes' Rule of Signs

Additional trigonometric graphs

Polar coordinates

Vectors in the plane

Partial fractions

Matrix algebra

Higher order determinants

Systems of inequalities

Linear programming

Sequences

Infinite series

Probability theory

OTHER FEATURES

Useful features from the first edition have been retained and others have been added, including:

Geometry Review Frequently used facts from plane geometry are summarized, with examples, in an appendix at the end of the book.

Warnings Students are alerted to common errors and misconceptions by clearly marked “warning boxes.”

Chapter Reviews Every chapter now concludes with a list of important concepts (referenced by section and page number), a summary of important facts and formulas, and a set of review questions. Whenever possible, these questions tie together material from several sections in the chapter or from other chapters.

Artwork All the graphs and figures have been redrawn by computer for this edition. Whenever possible, figures are placed in the body of the text at the point they are needed by the reader, rather than being stuck in the margin for the printer's convenience.

Figures are placed in the exercises only when essential. We believe that coming up with an appropriate sketch is a significant part of many problems and a skill that must be developed by students.

Exercises Each exercise set begins with routine calculation and drill problems, then proceeds to exercises that are less mechanical and require some thought. Answers for all odd-numbered problems are given at the back of the book.

Some sets include a section labeled “Unusual Problems,” a few of which are quite challenging. Many of them, however, are not difficult, but simply different from what students may have seen before and are well within the reach of most students.

Calculators The use of calculators is encouraged whenever appropriate, and nonroutine calculations that may cause trouble are explained. The need to understand the underlying concepts in order to use a calculator intelligently is continually stressed and common misuses of calculators are pointed out. (Tables are supplied in appendices for those who insist upon them.)

Graphing calculators are not needed to read this text, but their use is discussed for the benefit of students who have them. For instructors who want to emphasize the use of graphing calculators, there is a Graphing Calculator Supplement available from the publisher.

SUPPLEMENTS

A Student Solutions Manual with Graphing Calculator Supplement is available for purchase. Part I, written by Dorothy Smith of the University of New Orleans, contains detailed solutions to the odd-numbered exercises, chapter summaries, and additional practice problems. Part II, written by James Angelos of Central Michigan University, explains how to use the Casio and TI graphing calculators and uses examples from the text. It also provides additional practice problems for students to try.

Instructors who adopt this text may also receive, free of charge, the following items:

Instructor's Manual with Transparency Masters Also written by Dorothy Smith, this manual contains detailed solutions to all the exercises to assist the instructor in the classroom and in grading assignments. In addition, 200 transparency masters of important figures from the text are provided for use in preparing classroom lectures.

Test Bank This manual, written by Larry Small of Los Angeles Pierce College, contains over 1200 multiple-choice and open-ended questions arranged in five forms per chapter. There are also three final examinations. Master answer sheets and a complete answer section are included.

Computerized Test Bank (IBM version) The computerized test bank contains all the questions from the Test Bank and allows instructors to prepare quizzes and examinations easily and quickly.

A&T Software (IBM version) This software is ideal for tutorial and review purposes. It includes both algorithmic and theoretical questions, referenced by chapter and section. Hints are provided upon request, and the student's score is recorded. A demonstration disk is available.

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Our continued thanks go to the many people who offered assistance and advice with the first edition, without which the present edition would not have been possible. The new edition has also benefited from the comments of the following reviewers:

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Their sharp eyes and helpful suggestions significantly improved the book.

Finally, we want to express our appreciation to the staff of Saunders College Publishing for their assistance, with special thanks to Bob Stern, Maureen Iannuzzi, and Alexa Barnes.

Thomas W. Hungerford

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October 1990

To the Instructor

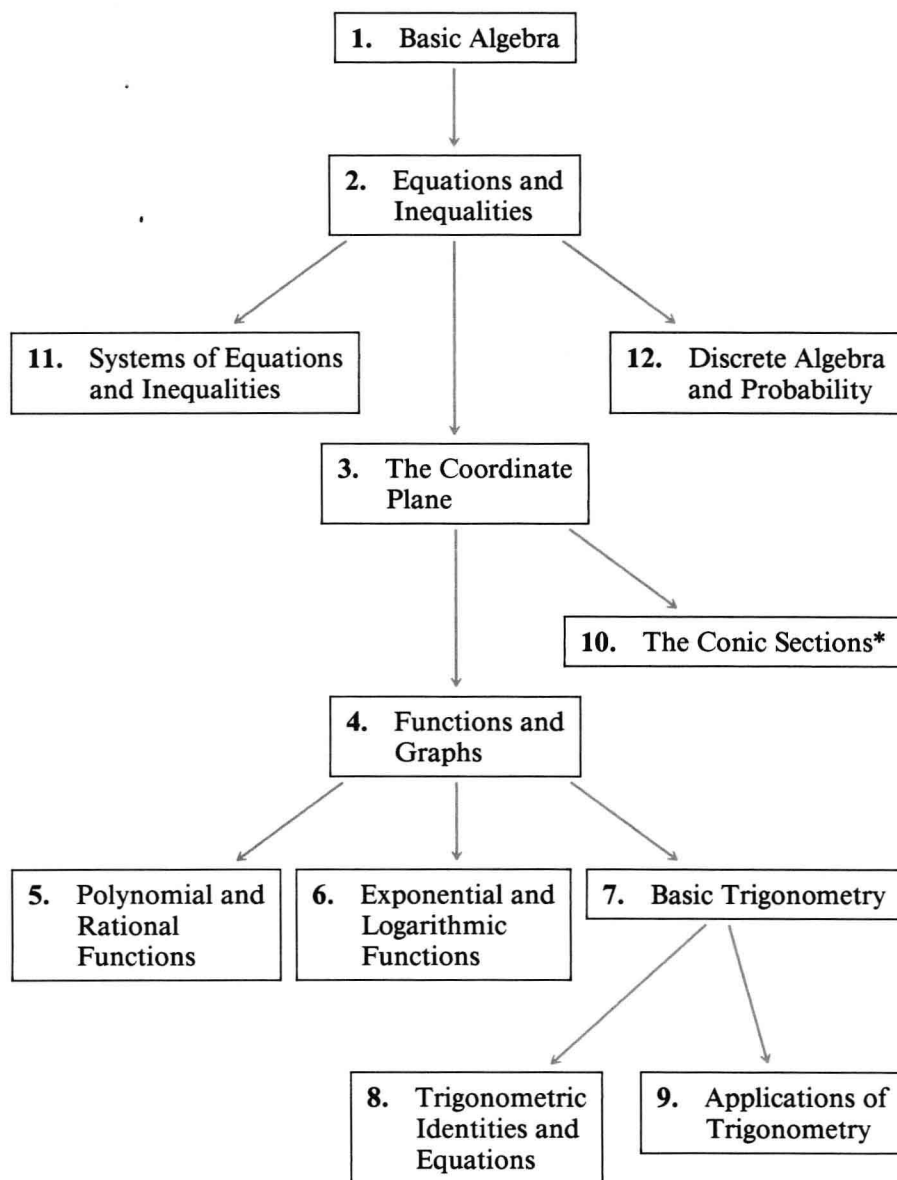
Every effort has been made to make this text as flexible as possible, so that you can easily adapt it to the needs of your own class. With minor exceptions (usually an occasional example or exercise), the interdependence of chapters is given by the chart on the next page.

Interdependence of sections within a particular chapter is indicated in three ways.

1. Sections that are not needed in the sequel (and hence can be omitted or postponed) are marked by a footnote at the beginning of the section.
2. Boldface boxes labeled **Roadmap** are scattered throughout the text at the beginnings of chapters or sections. They provide information about section interdependence and alternate orders of topics.
3. Certain sections are labeled as **Excursions**. Each Excursion is closely related to the section that precedes it and usually has that section as a prerequisite. No Excursion is a prerequisite for any other section of the text.

With rare exceptions, each Excursion is a complete discussion and includes a full complement of examples and exercises. The “Excursion” label is designed solely to make syllabus planning easier and is *not* intended as any kind of value judgment on the topic in question.

For certain audiences, a particular Excursion may be far more important than some of the “regular” sections. In other courses, however, this may not be the case. You decide what’s best for your course.

INTERDEPENDENCE OF CHAPTERS

*Section 10.4 (Rotation of Axes) depends on Chapters 7 and 8.

To the Student

Read this — or you will turn into a toad!

If you want to succeed in this course, remember that *mathematics is not a spectator sport*. You can't expect to learn mathematics without *doing* mathematics, any more than you could learn to swim without getting in the water. You have to take an active role, making use of your chief resources: your instructor, your fellow students, and this book.

When it comes to math textbooks, many students use their books only for finding out what the homework problems are. If they get stuck on a problem, they page back through the text until they find a similar example. If the example doesn't clarify things, they may try reading part of the text (as little as possible). Rarely, if ever, do such students read through an entire section (or subsection) from beginning to end.

If this description fits you, don't feel guilty. Some mathematics texts are unreadable. But don't use your bad past experiences as an excuse for not reading this book. It has been classroom-tested for years by students like yourself. It is understandable by an average student, with a minimal amount of outside assistance. So if you want to get the most out of this course, we strongly suggest that you follow these guidelines:

1. Read the pages assigned by your instructor from beginning to end before starting the homework problems. If you find calculations you don't understand, take pencil and paper and try to work them out. If you don't understand a particular statement, reread the preceding material to see if you missed something.
2. If you have spent a reasonable amount of time trying to figure something out, mark the place with a question mark and continue reading. Plan to ask your instructor about the material you have marked.
3. Now do the homework problems. You should be able to do all, or almost all, of the assigned problems. After you've worked at the homework for a reasonable amount of time and answered as many problems as you can, mark the exercises that are still causing trouble. Plan to ask your instructor about them.

If you follow these guidelines, you will get the most out of this book. But it won't be enough unless you actually *ask* your instructor about the things you don't understand. Unfortunately, many students are afraid to ask questions in class for fear that the questions will seem "dumb." Such students should remember this:

If you have honestly followed the guidelines above and still have unanswered questions, then there are at least six other students in your class who have the same questions.

So it's not a dumb question. Furthermore, your instructor will welcome questions that arise from a serious effort on your part. In any case, your instructor is being paid (with your tuition money) to answer questions. So do yourself a favor and get your money's worth—*ask questions*.

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