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COMBINATORIAL PHYSICS

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Preface

It is nearly fifty years since the authors of this book began a collaboration based on their common interest in the foundations of physics. During that time others have made very major contributions. The fruits of this cooperative enterprise, particularly of the later part of it, are set out here. One problem has led to another. Over that time it became clear that such existing preconceptions as the space and time continua formed an inadequate basis for a physics which has to incorporate a quantum world of discrete character. Here we argue that the impossibility of reconciliation between continuous and discrete starting points means that we must start from the discrete or combinatorial position. Otherwise the quantum theory will remain with confusion and muddle at its centre.

If intuitive clarity is to come from the combinatorial approach it turns out to be hard won because continuum ideas are so deeply embedded in orthodox physics. It has been possible to travel only part of the way but that has been far enough to reveal another positive aspect of the approach. Discreteness is intimately related, according to our theory, to the existence of *scale-constants* —those dimensionless constants commonly thought by physicists to be of some fundamental significance. We should therefore be able to calculate these. Here we are able to detail the calculation of one, the fine-structure constant, which Paul Dirac emphasized for much of his life as an outstanding problem in the completion of quantum electrodynamics. Our value agrees with the experimentally determined one to better than one part in 10^5. The calculation of dimensionless constants will bring to mind the name of Eddington, and although his work was the original cause of the authors' meeting, and although they agree with him in seeking a combinatorial origin for these constants, their mathematical method, and certainly their calculations of the values of those, have nothing in common with his.

This is a book about physics but philosophers will find that some issues — once their province — which they thought dead and decently buried, are resurrected to new life here. Notable amomg these is the place of mind. The great originators of the quantum theory knew that the action of the mind (or the "observer") had to be part of the theoretical structure, but this development has been aborted. In combinatorial theory there is no escape from the issue. In somewhat the same way, since computing is essentially combinatorial, people in and around computer science may find our representation of physics more natural to them than it is to some orthodox physicists. None the less it is primarily the physics community we seek to inspire to carry further a project of which this is the beginning.

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CHAPTER 1

Introduction and Summary of Chapters

This book is an essay in the conceptual foundations of physics. Its purpose is to introduce what we shall call a combinatorial approach. Its method will be to view physical theory under the aspect of a particular point of view which is combinatorial in character. In the course of the book questions will be asked and discussed which may have a long history, but which are not seen as live issues at the present because of the special philosophical stance of present-day physicists. However for combinatorial physics they are very much alive.

The idea which underlies combinatorial physics is that of *process*. The most fundamental knowledge that we can have is of step-by-step unfolding of things; so in a sequence. This is the kind of knowledge we have of quantum processes, and that fact becomes specially evident in the experimental techniques of high-energy physics.

The contrasting view, which has been the main guide of physics in the past, is of a background of physical things whose spatial relationships are on an equal footing in the sense that it can only be an accident in what order we happen across them. To put it another way, when the sequence or process is fundamental then we have to specify the steps by which we get to a particular point, whereas in the conventional picture we imagine we are free to move about in the space in which phenomena occur, without its being necessary to be explicit about it.

Suggestions for fundamental revision of the conceptual framework of physics are unlikely to engage the attention of physicists unless they bring about major improvements in the technical understanding of physics — particularly in new explanations and calculations of experimental results. The strong position of the combinatorial theory is that it has been used to deduce some experimental quantities which have not been deduced from more conventional theory. The closeness to the experimental values of these deductions makes it very unlikely that their success is fortuitous. Moreover it would be agreed on all sides that the experimental quantities in question play a sufficiently important part in physics to warrant attention's being paid to any theory which claims to calculate them.

* * * * *

We have to be more explicit about what we mean by a combinatorial physical theory. Combinatorial physics is physics in which the mathematical relations are combinatorial, and combinatorial mathematics is mathematics in which we study the ways in which symbols are combined. The term 'combinatorial' is often defined ostensively by giving examples. Ryser[1] gives the following examples: interest in magic squares, study of permutations and (indeed) of combinations, finite projective geometry and problems connected with covering of spaces with arrangements of shapes in circumstances in which the space is divided into a finite number of sections and there is a convention which enables us to decide unambiguously whether or not a given section is covered. Such conventions exist, for example, for the use of chess boards and most other gaming boards, and the conventions enable us to decide all questions of the relation of combinatorial structures to physically defined spaces. Ryser sums up his exemplification thus: "Combinatorial mathematics cuts across the many subdivisions of mathematics, and this makes a formal definition difficult. But by and large it is concerned with the study of the arrangements of elements into sets."

The first person to have seen a very profound difference between the combinatorial approach and the rest of formalised thinking was Leibniz. The term 'combinatorial', used in this context, originates with Leibniz's "Dissertate de arte combinatoria", and according to Polya[2] the originator saw that difference in a very striking form. ... "Leibniz planned more and more applications of his combinatorial art or 'combinatorics': to coding and decoding, to games, to mortality tables, to the combination of observations. Also he widened more and more the scope of the subject. He sometimes regards combinatorics as one half of a general Art of Invention; this half should deal with synthesis, whereas the other half should deal with analysis. Combinatorics should deal, he says in another passage, with the same and the different, the similar and the dissimilar, the absolute and the relative, whereas ordinary mathematics deals with the one and the many, the great and the small, the whole and the part."

This insight of Leibniz seems borne out by what is happenning in formal areas of thinking at the present time. The general connection between combinatorial mathematics and the impulse behind computing science has been widely remarked. There is more to it than that computer programmes are constructed in formal steps with information stored in (binary) patterns. It is the connectivity of the computer programme which is its vital characteristic, and which is an aspect of the ars combinatoria. Weyl[3] says, "Modern computing machines translate our insight into the combinatorial structure of mathematics into practice by mechanical and electronic devices." When one is subjecting oneself to the discipline of the computer programming, which in its essentials is arranging the mathematical operations in sequential order so that each brings the next appropriate one in its train without intervention on the part of the mathematician, then Weyl is saying that one has somehow, through exercise of that discipline, displayed that insight.

Arguing this way, one comes to feel that in spite of all the modern sophistication about the impact of computing on the foundations of mathematics, not enough attention has been paid to the vast difference between the classical idea of the mathematician and that required by the computer revolution. This difference shows up clearly when it comes to making mathematical models. The former makes up his mind about how to get from point to point in a mathematical structure on the basis of some purpose that he imagines to be directing his efforts, whereas, by contrast a computer model is meant to have these instructions incorporated in the program.

In contemporary thinking there are a variety of essential principles which are welded together in a mathematical framework which we call quantum mechanics. Uncertainty, exclusion, complementarity and the whole theory of wave-functions and differential operators and eigenstates are some of these. From this point of view anyone who undertakes to reformulate the foundations of quantum physics will be expected to replicate this same mathematical structure because that simply is what quantum theory is. At this early stage we should warn the reader not to have this unquestioning expectation.

Our case is that all the results of current quantum theory are combinatorial in origin and — at any rate in principle — we should expect to obtain them all by continuation of our combinatoric method. Moreover we argue that all the principles which are normally thought to be characteristic of the quantum theory take their familiar form because of the need to reconcile the discrete character of quantum events with the classical theory-language.

Plan of the book by chapters

CHAPTER 1. INTRODUCTION
The book is an essay in the foundations of physics; it presents a combinatorial approach; ideas of *process* fit with a combinatorial approach; quantum physics is naturally combinatorial and high energy physics is evidently concerned with process. Definition of 'combinatorial'; the history of the concept takes us back to the bifurcation in thinking at the time of Newton and Leibniz; combinatorial models and computing methods closely related.

CHAPTER 2. SPACE
Theory-language defined to make explicit the dependence of modern physics on Newtonian concepts, and to make it possible to discuss limits to their validity; Leibniz' relational, as opposed to absolute, space discussed; the combinatorial aspect of the monads.

CHAPTER 3. COMPLEMENTARITY AND ALL THAT
Bohr's attempt to save the quantum theory by deducing the wave-particle duality, and thence the formal structure of the theory, from a more general principle (complementarity) examined: the view of complementarity as a philosophical gloss on a theory which stands up in its own right shown to misrepresent Bohr: Bohr's argument rejected — leaving the quantum theory still incomprehensible.

CHAPTER 4. THE SIMPLE CASE FOR A COMBINATORIAL PHYSICS
Physics not scale-invariant; it depends on some numbers which come from somewhere outside to provide absolute scales; the classical kind of measurement cannot in the nature of the case provide them; measurement is counting; the coupling constants are the prima-facie candidates; this was Eddington's conjecture; the question is not *whether* we find combinatorial values for these constants, but how we do so; current physics puts the values in *ad hoc*.

CHAPTER 5. A HIERARCHICAL MODEL — SOME INTRODUCTORY ARGUMENTS
The combinatorial model used is hierarchical; the algorithm relating the levels is due to Parker–Rhodes; the construction is presented in several ways each stressing a particular connection with physics: (1) similarity of position, (2) the original combinatorial hierarchy, (3) counter firing, (4) limited recall, (5) self-organization, (6) program universe.

CHAPTER 6. A HIERARCHICAL COMBINATORIAL MODEL — FULL TREATMENT

The elementary process expressed algebraically and interpreted as decision whether to incorporate a presented element as new; new elements labelled; the need for labelling to be consistent gives central importance to discriminately closed subsets; any function which can assign labels equivalent to one which represents process; process defined as always using the smallest possible extension at each step that is allowed by the previous labelling; representation of functions by arrays; representation of arrays by matrices and strings familiar from the simpler treatments of Chapter 5; summary of the argument.

CHAPTER 7. SCATTERING AND COUPLING CONSTANTS

The primary contact with experiment in quantum physics comes through counting in scattering processes; coupling constants are ratios of counts which specify the basic interactions; this outline picture has to be modified to get the experimental values; history of attempts to calculate the fine-structure constant reviewed; explanations of and calculations of the non-integral part due to McGoveran and to Kilmister given. The latter follows better from the principles of construction of the hierarchy algebra of Chapter 6.

CHAPTER 8. QUANTUM NUMBERS AND THE PARTICLE

Comments provided on high energy physics and the particle/quantum number concept from the standpoint regarding the basic interactions of Chapter 7; the particle is the conceptual carrier of a set of quantum numbers; the view of the particle as a Newtonian object with modifications is flawed; an alternative basis for the classification of the quantum numbers due to Noyes is described; it is compared with the Standard Model.

CHAPTER 9. TOWARDS THE CONTINUUM

We have no representation of physical space, let alone the continuum; the conventional understanding of dimensionality replaced by a 3D argument based on the hierarchy algebra; the finite velocity of light necessarily follows from the pure-number fine-structure constant; it leads to a very primitive form of relativity; this is developed; the quadratic forms which appear in the Lorentz transformation as well as in Pythagoras' theorem are discussed; measurement is defined.

CHAPTER 10. OBJECTIVITY AND SUBJECTIVITY — SOME "ISMS"

The philosophical position of the book is assessed to see how it fits with some familiar positions — mostly ending in "ism": subjectivism; realism; the anthropic principle; constructivism; reductionism; the critical philosophy; positivism; operationalism; particles.

CHAPTER 2

Space

The most important consequence of the change to combinatorial mathematics in physics arises over the representation of physical space. What we abandon is the automatic freedom to consider a problem from several points of view and involving several mathematical techniques which have no coherence but which we take as simultaneously relevant because we ascribe them all to the same point of space. The combinatorial approach has to construct a process which is equivalent as far as possible to the classical putting together of results to give a sort of composite picture of what is happenning at a point of space. In this book we shall only be able to take the first steps in this programme — just enough to show what is needed. There is much that comes first.

If one wishes to replace spatial relationships as the primary data by sequential development, then one should first look at the reasons for the profound hold that the former way exerts. It postulates a continuum background of space within which things have positions, and the changes in those positions give rise to a second continuum of time. These continua are imagined as perfectly smooth, perfectly homogeneous, infinitely divisible. They are modelled mathematically by the continuum of all real numbers as that was formalized by Dedekind and Cantor between 1870 and 1880. The irreducibly simplest entities in these continua of space and time are particles — idealized as single points whose position in space changes continuously with time — and fields. Fields are spread through space and have at each spatial point a magnitude which again varies smoothly both with time and with changes of the point at which the field is specified. Upon this basis, a scheme of dynamical concepts was elaborated which seemed to provide a complete description of the motion of things in the space continuum. The work of Newton was the cornerstone of this edifice. Its subsequent elegance (epitomized by Thomson and Tait's "Natural Philosophy") compelled a feeling of universality: one felt one was in possession of the means to describe reality direct. Indeed this scheme became for the physicist the mathematical elaboration of commonsense and the automatic vehicle for his thought.

The set of concepts that makes up the scheme has an interlocking and closed character which makes it unique in the history of thought. Other disciplines have aspired to this cohesion, but have only very partially been able to achieve it. Through its long elaboration classical mechanics has become like a language which we learn to use rather than like an exploration which may turn out right or may turn out wrong, though of course it was like that once. When we learn classical mechanics we find that the definitions of the concepts chase themselves round in circles. The mass of a body is given a numerical value through the behaviour of that body under a given applied force. The force is specified numerically by the acceleration of the body, but only if we already know the mass, and so on. At the time of Newton, there was no unanimity on the use of the concepts, and if one were asked to give a definition of one of them, one would refer to some experimental situation which made the use one was proposing plausible: one would have to argue. Now things are interestingly different, for the set of interlocking concepts defines its own appropriate application, and therefore cannot conflict with experiment.

It is easy to give an example. It is now known that the spiral nebulae rotate like a solid body, so that parts of the nebula at different distances out on a radius stay on that radius. Not merely that, but there may be no simple relation between the rotation and the swept back look: in certain cases the arms may advance point first. After those shocks one is almost disappointed to find that the thing does rotate in the plane in which its arms lie, like a catherine wheel. Yet, one would have sworn, if ever there were an object whose rough dynamics was plain for all to see, it would have been the spiral nebula — a loose aggregate of matter having some angular momentum, and with some radial motion shown by distribution of material along the arms, but with the arms appearing swept back because of greater angular velocities near the centre.

The fascinating subject of galactic dynamics is, however, not our topic now: we are using it only to make a methodological point. This is that no amount of evidence will induce us to doubt the asumption that the universe out there would seem as it does to us here, and exhibit the same mechanics, if we could be transported there. We know we have to give up that presumption as we approach the atomic scale: that we retain it in the face of possible contrary evidence at the increasingly large scale is something between a guiding principle and a prejudice. It should be re-evaluated in the light of the information we actually have each time we use it. The strongest case for the principle or prejudice is that galaxies have similar form over large variations in their red-shift, and so can be regarded to a first approximation as invariant units. The spiral galaxies are the largest structures that we encounter on our way to the 'limit of the observable universe'. There, the extrapolation principle compels us to entertain the idea of a universe which goes on for ever and ever, which is unsatisfactory when we need to operate theoretically with the "observable universe".

It is a strange fact that this immunity of the deductive language of classical physics to experimental check has received little or no attention in contemporary writing on the philosophy of physics. The authors1 coined the term *theory-language* to refer to a theory which had reached this stage of development. They tried to use the 'logic of facts' of Wittgenstein's Tractatus Logico-philosophicus, to describe it: "... a physical theory consists of propositions which may be thoughts, sentences written or spoken, or manipulations with bits of the physical world ... the experimental thing has meaning only as part of a theory. The theory may have different degrees of complexity, and there will be experimental procedures corresponding to each degree. Thus the theory is a kind of language, but experiments in the theory are the same language. One cannot use experiments in a complex language to criticise a simpler theoretical language."

What has to be replaced is the 'platonic receptacle' view of space: space is what holds whatever we care to put into it. Of course the receptacle view is deep in our thinking: it seems to be a way of operating physical theory by projecting our most immediate sensuous knowledge of the world onto each theoretical statement — it is almost as though we need to picture what bodily actions we should take to correspond to it, before we can understand it. If we go back to our example of the spiral nebula, it is as though we transport ourselves in the imagination to that place and think what would happen to us in order to formulate the theoretical description. Our attempt to formalize the classical theory-language was essentially to replace this sensuous correspondence. We have to remember that when we speak of replacing the continuum we are replacing this too.

If we associate the 'classical theory-language' predominantly with Newton, it is not surprising to find the alternative associated predominantly with that other giant — his contemporary — Leibniz. The differences in conception of the two men to be seen in their respective innovations which became two versions of the calculus, were pointers to a greater difference. We have already associated Leibniz' differing view with the combinatorial aspect of his thought. This seems to have been intrinsic to his view of the world in rather the way we have postulated with our view of the centrality of process. If, for ease of expression, we let a computing model stand for combinatorial process,