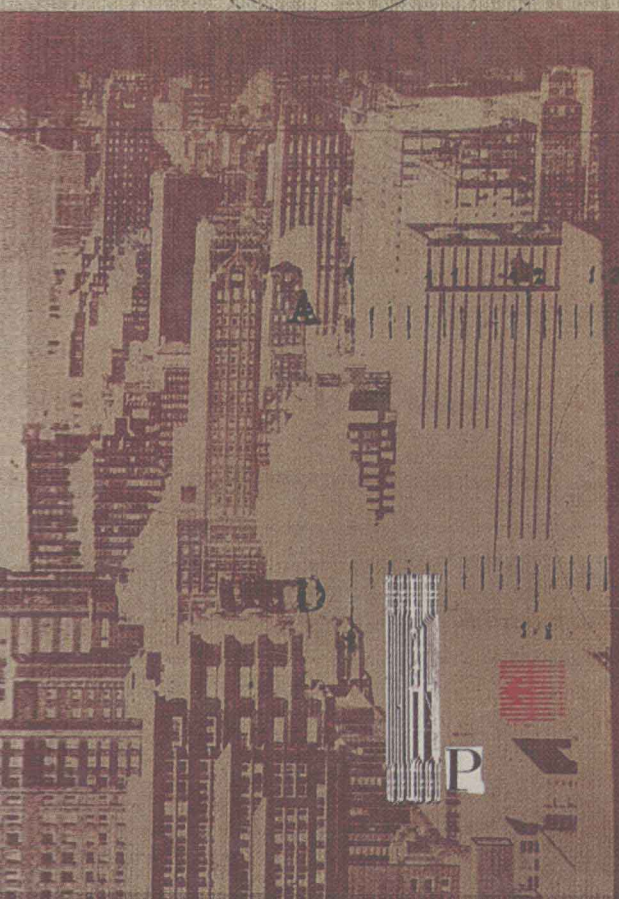
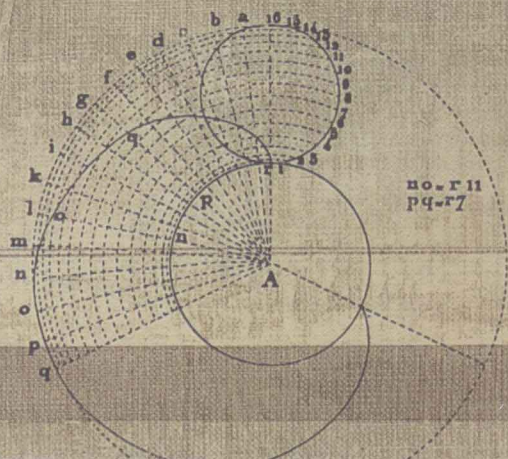


Basic Technical Mathematics with Calculus

Second Edition

Peter Kuhfittig



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Peter Kuhfittig

Milwaukee School of Engineering



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Purpose of This Book

The main purpose of the second edition of *Basic Technical Mathematics with Calculus* is to enhance the student-oriented features that have contributed to the success of the first edition. Based on classroom experience and on suggestions from users of the first edition, a number of changes have been made:

New Features

1. Certain portions of the book were completely rewritten: Chapters 1, 2, and 9, and Sections 19.3, 20.3, 22.6, 22.7, 23.5, 24.1, 26.6, 28.1, and 28.4. Many other sections have been extensively revised.
2. Several new sections have been introduced: scientific calculator operations (1.4), BASIC programming (Appendix C), arithmetic progressions (18.1), subsections on the order of operations (1.2) and addition of vectors by components (9.3), a separate section on formulas (2.3), and a review of geometry (Appendix D).
3. A new section on integration of rational functions has been introduced in Chapter 25 (Section 25.8). To increase the book's flexibility, however, the discussion of partial fraction expansions in Chapter 29 has been retained. Thus Chapter 29 is independent of Section 25.8.
4. The student-oriented features of the first edition were enhanced through additional diagrams and marginal annotations, and by providing greater detail in the presentation.

Some additional new features are:

5. More space is given to calculator operations throughout the book.
6. BASIC notations are introduced in several places in the text.
7. Most of the exercise sets have been expanded and, in some cases, rearranged. All exercise sets containing algebra word problems have been completely revised.
8. Greater use is made of decimal degrees.

9. The polar form of a complex number is now denoted by r/θ .
10. The number of examples and figures has been greatly increased, particularly in the calculus part of the book. More figures have been added to the answer section.
11. All answers to the review and cumulative review exercises are given in the answer section.
12. More technical applications have been included.

Continuing Features

The presentation in this book is exceptionally student-oriented and fills the needs of today's technology student in many ways:

1. The approach is concrete and intuitive.
2. Drill exercises make use of notations commonly encountered in technical areas.
3. Most sections contain exercises that illustrate how mathematics is applied to technical problems.
4. Color ink is not used in a merely decorative way. Its main function is to help explain difficult steps.
5. The most important concepts are boxed and labeled for easy reference; other concepts are identified by marginal labels.
6. Important procedures are summarized; step-by-step procedures are provided whenever appropriate.
7. Calculator operations are discussed throughout the book.
8. Examples are worked out in great detail. Marginal notes are used to help explain the steps.
9. Common pitfalls are pointed out in special segments called *Common Errors*.
10. A large number of drill exercises is included to help reinforce basic concepts.

The use of realistic notations, even in drill exercises, is particularly important, since students tend to have great difficulty in transferring skills to technical problems that use a different notation. No prior knowledge of the different technical areas is assumed, however.

Some other features are:

11. Throughout the book, the use of graphs has been given particular attention.
12. Both metric and English units are employed, with SI notation generally used for the former.
13. Every chapter ends with a set of review exercises. Cumulative Review Exercises are given at the end of every third chapter.
14. A background in algebra and geometry is assumed for most of the material in this book. However, the discussion begins at a sufficiently low level to help overcome deficiencies in some areas.

Flexibility

While this book allows considerable flexibility in the order of presentation of topics, careful attention has been paid to the fact that most technical curricula require certain topics at definite times. For example, although scientific notation is introduced in Chapter 1 and basic trigonometry in Chapter 4, a discussion of these topics may be postponed. On the other hand, trigonometric identities (Chapter 16) can be taken up after Chapter 10. Since determinants up to the third order are discussed in Chapter 3, higher-order determinants and matrices, which are first discussed in Chapter 15, can be introduced earlier or omitted altogether.

The basic plan of the second part of the book is to introduce the differential and integral calculus in Chapters 20–23, with two-year technology programs in mind. The later chapters treat various topics from more advanced areas and may be used according to the requirements of individual programs.

Coverage and Scope

Chapter 1 discusses basic algebra. The topics covered range from signed numbers and order of operations to operations with polynomials. Zero and negative exponents are discussed briefly in case the teacher wants to cover scientific notation (Section 1.9) early in the course, but these topics may be postponed to Chapter 10. Because of the large number of applied problems in this book, calculator operations are also introduced early (Section 1.4). (Calculations with approximate numbers and conversion of units are presented in Appendices A and B, respectively.)

Chapter 2 begins with the solution of linear equations and formulas, followed by a section on ratio and proportion and a section on variation. These topics are a natural continuation of the discussion of equations and formulas and provide a concrete basis for the definition of a function in Section 2.8.

Chapter 3 deals with systems of linear equations and their applications. The methods of solution discussed are graphing, addition or subtraction, and substitution. Determinants up to the third order are also introduced. (Higher-order determinants are covered in Chapter 15.)

Chapter 4 is an introduction to right-triangle trigonometry. The topics covered are basic definitions, values of trigonometric functions, and right-triangle applications.

Chapter 5 on factoring and fractions is organized to allow a gradual mastery of factoring: Basic special products, introduced in Section 5.1, are followed by the corresponding factoring cases in Section 5.2. Section 5.3 introduces more special products, and Section 5.4 the corresponding factoring cases. Factoring by grouping is then discussed in Section 5.5. The rest of the chapter is devoted to operations with fractions, complex fractions, and fractional equations.

Chapter 6 covers quadratic equations and their applications. The methods of solution discussed are factoring, completing the square, and the use of the quadratic formula.

Chapter 7 expands the discussion of trigonometric functions begun in Chapter 4. The topics covered are the functions of angles in any quadrant, functions of special angles, radian measure, and applications of radian measure, including linear and angular velocity.

Chapter 8 covers the graphs of trigonometric functions, as well as graphing by addition of coordinates. Considerable space is given to applications of sinusoidal functions.

Vectors and applications of vectors are treated in Chapter 9. The sections on the sine and cosine laws contain additional vector applications.

Chapter 10 comprises a detailed treatment of exponents and radicals: zero, negative, and fractional exponents, and fundamental operations with radicals.

Chapter 11 covers complex numbers: rectangular, polar, and exponential forms of complex numbers and powers and roots by De Moivre's theorem. The chapter ends with a brief discussion of phasors.

Chapter 12 contains a thorough treatment of logarithmic and exponential functions with special emphasis on applications. Also included are discussions of natural logarithms, properties of logarithms, exponential and logarithmic equations, and graphing on logarithmic paper. As in the case of trigonometric functions, the emphasis is on the use of calculators rather than of tables. (Computations with logarithms are introduced only briefly.)

Chapter 13 begins with a brief discussion of conic sections to provide a basis for solving systems of two quadratic equations by means of graphs. Algebraic methods are discussed next, followed by equations in quadratic form and fractional equations.

Chapter 14 contains a detailed treatment of higher-order equations. Approximation of irrational roots by linear interpolation is also discussed in detail.

Higher-order determinants and a detailed treatment of matrices can be found in Chapter 15.

Chapter 16 is devoted to trigonometric identities, trigonometric equations, and inverse trigonometric functions.

Chapter 17 on inequalities covers both graphical and algebraic methods of solution. The chapter ends with an optional section on linear programming.

Arithmetic progressions, geometric series, the binomial theorem, and binomial series are treated in Chapter 18.

Chapter 19 covers the traditional topics of analytic geometry. Although intended mainly for use in calculus, a number of applications of conic sections are also discussed. The derivative is introduced in Chapter 20 and applications of the derivative in Chapter 21. Chapters 22 and 23 cover basic integration, including the trapezoidal rule and Simpson's rule. Although based on Riemann sums, the emphasis in setting up integrals is on a shortcut using a singly typical element, referred to as a "sloppy Riemann sum." This shortcut is used extensively in Chapter 23 on applications of integration. The purpose is to enable the student to gain the necessary insight for setting up integrals in many different situations, rather than relying on memorized formulas.

Chapter 24 covers transcendental functions and their applications to various technical fields. Also included is a brief discussion of L'Hospital's rule. Chapter 25 develops different integration techniques. These techniques are covered in considerable detail and may be selected according to individual needs. For example, if time constraints do not allow a detailed discussion of trigonometric substitution, integrals of this form can be obtained by use of tables,

discussed in Section 25.9. As noted earlier, the discussion of partial fractions (Section 25.8) can be postponed to Chapter 29.

Chapter 26 emphasizes power-series expansions, but it contains an optional section on tests of convergence. A section on Fourier series is included for use in electrical technology curricula.

Chapters 27, 28, and 29 are all devoted to differential equations since these provide particularly interesting and powerful applications to numerous technical fields. Chapter 29 on Laplace transforms includes a section on partial fractions. Although time-consuming, partial fractions are essential to the Laplace transform technique.

A brief discussion of Newton's method is given in Chapter 30. This method of solving equations is particularly useful for technology students.

Appendices A–D discuss, respectively, approximation and measurement, reduction and conversion of units, BASIC programming, and review of geometry.

Supplements

The answers to the odd-numbered exercises and all answers to the review and cumulative review exercises are given in the answer section; the answers to the remaining exercises are published in a separate answer book. A Student Solutions Manual, with approximately every fourth problem worked in detail, is also available.

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Peter Kuhfittig

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