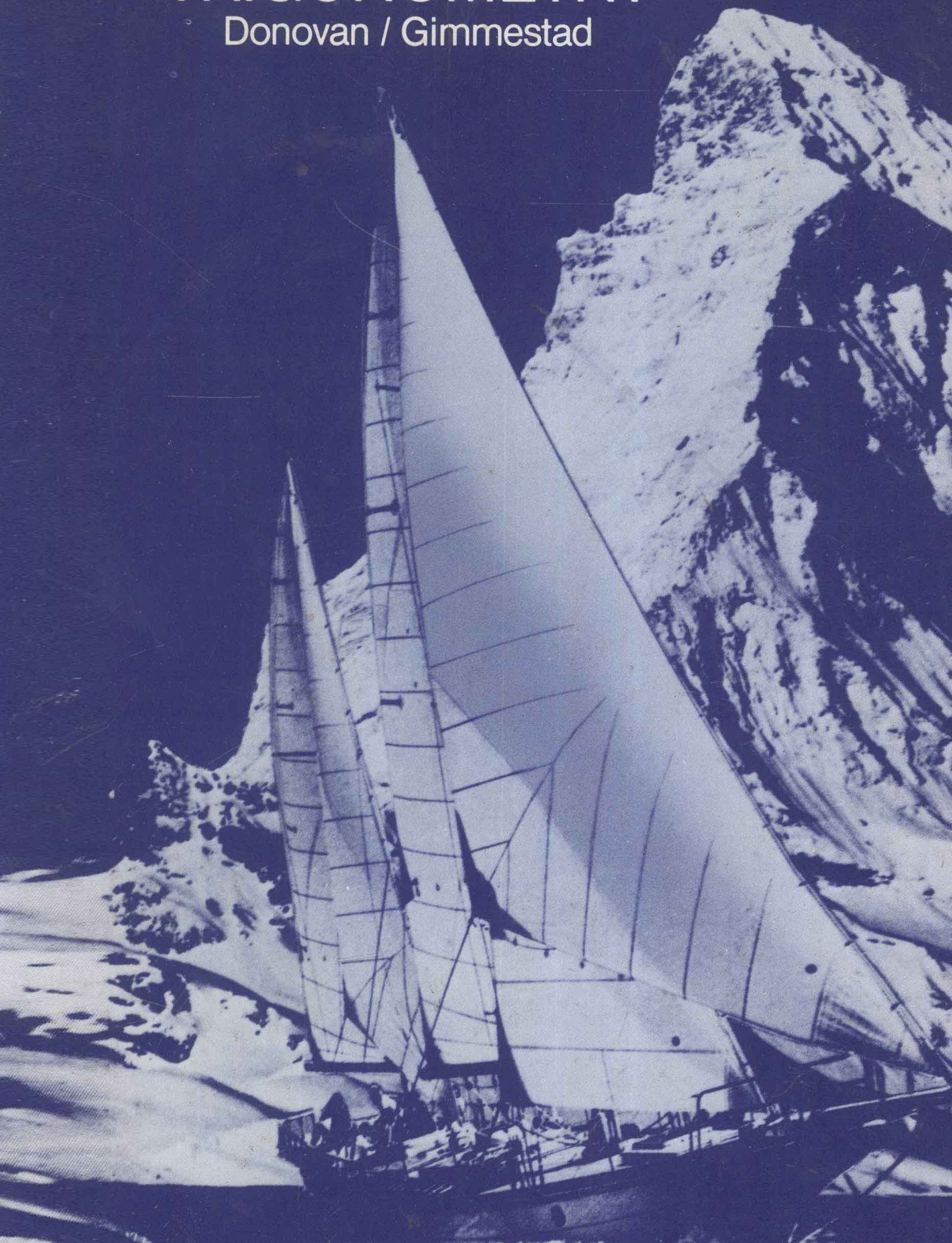
TRIGONOMETRY

Donovan / Gimmestad



TRIGONOMETRY

WITH CALCULATORS

George S. Donovan
Metropolitan State College

Beverly Beyreuther Gimmestad
Michigan Technological University

Prindle, Weber & Schmidt
Boston, Massachusetts

Prindle, Weber & Schmidt is a division of Wadsworth, Inc.

Library of Congress Cataloging in Publication Data

Donovan, George S
Trigonometry with calculators.

Includes index.
1. Trigonometry, Plane. 2. Calculating-machines. II. Gimmestad, Beverly Beyreuther, joint author. II. Title.
QA533.D59 516'.24 79-22110
ISBN 0-87150-284-4

Cover photograph used with permission of Claude L.P. Schmitt, Cannes, France.

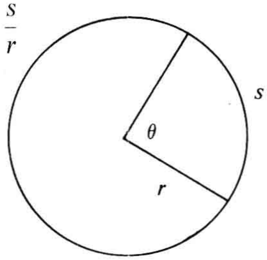
Designed by Elizabeth W. Thomson and the staff of Prindle, Weber & Schmidt. Composed in Monophoto Helvetica and Times Roman by Syntax International. Printed and bound by Halliday Lithograph Corp. Interior artwork by Julie Gecha.

Greek Alphabet

Alpha	A, α	Nu	N, ν
Beta	B, β	Xi	Ξ, ξ
Gamma	Γ, γ	Omicron	O, o
Delta	Δ, δ	Pi	Π, π
Epsilon	E, ε	Rho	P, ρ
Zeta	Z, ζ	Sigma	Σ, σ
Eta	H, η	Tau	T, τ
Theta	Θ, θ	Upsilon	Υ, υ
Iota	I, ι	Phi	Φ, ϕ
Kappa	K, κ	Chi	X, χ
Lambda	Λ, λ	Psi	Ψ, ψ
Mu	M, μ	Omega	Ω, ω

Subtended Central Angle

$\theta = \dfrac{s}{r}$



θ = central angle in radians
r = radius of the circle
s = length of the subtended arc

Trigonometric Functions

Let θ be in standard position. Let (x, y) be a point on the terminal side of θ such that $r = \sqrt{x^2 + y^2} \neq 0$. Then:

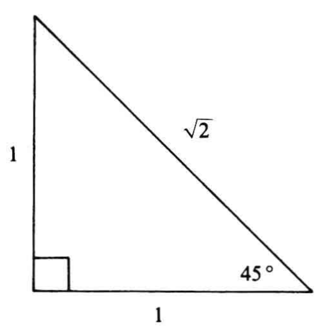
$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

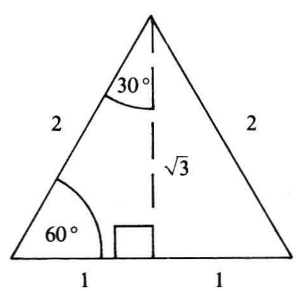
$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

(The denominator must not be zero.)

Exact Values

	0	$\pi/6 = 30°$	$\pi/4 = 45°$	$\pi/3 = 60°$	$\pi/2 = 90°$	$\pi = 180°$	$3\pi/2 = 270°$
$\sin\theta$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0	-1	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	—	0	—

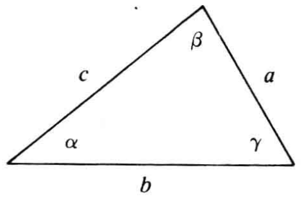




Degree–Radian Conversion

$$(\theta \text{ in radians}) \frac{180}{\pi} = (\theta \text{ in degrees})$$

$$(\theta \text{ in degrees}) \frac{\pi}{180} = (\theta \text{ in radians})$$



Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$
$$b^2 = a^2 + c^2 - 2ac \cos \beta$$
$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$
$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos \beta = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

International System of Units

Quantity	Unit	Symbol
electrical capacitance	farad	F
electrical current	ampere	A
electrical inductance	henry	H
electrical resistance	ohm	Ω
force	newton	N
frequency	hertz	Hz
length	meter	m
mass	kilogram	kg
temperature	kelvin	K
voltage	volt	V
work	Joule	J

Prefixes: kilo = 1000, centi = 1/100, milli = 1/1000

TRIGONOMETRY

WITH CALCULATORS

To our children, who must live in the age of calculators

George C. Donovan
Steven E. Donovan
Domenica A. Donovan
Maryann E. Gimmestad
Katherine D. Gimmestad

Preface

This book is a right-triangle trigonometry text. The text is designed to be used most conveniently by students who have calculators. The calculator, however, is not essential for successful completion of the text. After finishing a course taught from this book, the student should be expert at using a calculator to solve trigonometric problems. However, the authors wrote the book so as to encourage the student to learn the theory as well as the manipulations. In fact, use of the calculator should reduce the drudgery of calculation and leave the student more time to study the theory. Included in this text are certain problems that should be done without the aid of a calculator. In order to discourage the student from becoming a "mindless button pusher," the answers to many examples and exercises are given in fractional form (such as, $\sqrt{3/2}$, $\pi/3$, $\sqrt{18/8}$, etc.) when the student is supposed to "reason out" the answer without much help from the calculator. Since there is a wide variety of calculators available and a description of how to use each would be cumbersome, only a general description of which buttons to press is included.

Chapter 1 reviews the basic concepts of trigonometry. The time spent on this chapter will vary with student preparation. In order to make the book compatible with the calculator, decimals are used for degree measure instead of minutes and seconds. However, the concept of minutes and seconds is explained in Chapter 1 so the student will not be at a loss if he comes across the terms elsewhere. The International System of Units is used throughout the book. Section 2.4, which is optional, is included because it is often assumed that a student who has taken a trigonometry course can read tables and interpolate. Rounding conventions for the trigonometric functions are fairly complicated. The rounding conventions used in this book are given in Appendix A. For convenience, all decimal answers are given in two forms: (1) They are rounded to six digits and placed in brackets, and (2) they are rounded to the appropriate number of places (for example, 3.89 × 2.617 = [10.1801] = 10.18). Thus, coverage of rounding conventions is optional. Chapter 7 contains some very interesting applications of trigonometry. The remainder of the book contains the traditional topics of trigonometry, adapted for the calculator.

Pre-exercises are used at the beginning of some sections, where appropriate. There are psychological advantages to be gained when a student has a calculator. One tends to trust a calculator, to believe that its answers are irrefutable. The student can use a calculator to do a number of simple exercises quickly without exhausting his motivation. The *optional* Pre-exercises can be done quickly and give the student a hint of the content of the section they accompany. In essence, the student gets a chance to discover for himself some of the basic theory.

The odd and even exercises are paired. For each odd exercise, there is usually an even one of the same type. Thus, the instructor may use the even exercises as examples in class and assign the odd exercises as homework. (This pairing does not apply to the Chapter Tests.)

We would like to thank the following people for their valuable comments and suggestions:

Robert A. Chaffer, Central Michigan University; William E. Coppage, Wright State University; Edward Franklin, Long Beach Community College; Frieda Holley, Metropolitan State College; John Kuisti, Michigan Technological University; John Lamb, East Texas State University; Crepin M. Mahop, University of New Mexico; Richard Marshal, East Michigan University; Vern Nelson, Metropolitan State College; James F. Porter, University of Arkansas; Carlos Rodriguez, San Antonio College; Bruce F. Sloan, University of Nebraska; John White, Texas Technological University; Ronald Whittekin, Metropolitan State College; Howard L. Wilson, Oregon State University; Albert W. Zechmann, University of Nebraska.

Special thanks are due John Kimmel, the initial editor of this text. His hard work and dedication are much appreciated by the authors.

Special thanks also to Carol Donovan for her long hours correcting mistakes, typing the manuscript, and solving the exercises. Without her help, the book might never have been completed.

George S. Donovan   Beverly B. Gimmestad

Addition Formulas

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \pm \tan\theta\tan\phi}$$

Double Angle Formulas

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \sin\theta}}$$

Formulas for $\sin^2\theta$ and $\cos^2\theta$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

Sum and Product Formulas

$$2\cos\theta\cos\phi = \cos(\theta - \phi) + \cos(\theta + \phi)$$

$$2\sin\theta\sin\phi = \cos(\theta - \phi) - \cos(\theta + \phi)$$

$$2\sin\theta\cos\phi = \sin(\theta + \phi) + \sin(\theta - \phi)$$

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$

$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$

$$\sin\theta \pm \sin\phi = 2\sin\left(\frac{\theta \pm \phi}{2}\right)\cos\left(\frac{\theta \mp \phi}{2}\right)$$

Fundamental Relationships

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cos^2\theta + \sin^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

Trigonometric Rounding Conventions (in terms of decimal places)

Angle in Degrees

Trigonometric Function	Accuracy	Inverse Trigonometric Function	Accuracy
sin	1 more place	arcsin	2 less places
cos	1 more place	arccos	2 less places
tan	1 more place	arctan	1 less place

Angle in Radians

Trigonometric Function	Accuracy	Inverse Trigonometric Function	Accuracy
sin	equal places	arcsin	1 less place
cos	equal places	arccos	1 less place
tan	1 less place	arctan	equal places

Table of Contents