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A HISTORY OF THE Theory of Investments

MY ANNOTATED BIBLIOGRAPHY

MARK RUBINSTEIN

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Preface

Ideas are seldom born clothed, but are gradually dressed in an arduous process of accretion. In arriving at a deep knowledge of the state of the art in many fields, it seems necessary to appreciate how ideas have evolved: How do ideas originate? How do they mature? How does one idea give birth to another? How does the intellectual environment fertilize the growth of ideas? Why was there once confusion about ideas that now seem obvious?

Such an understanding has a special significance in the social sciences. In the humanities, there is little sense of chronological progress. For example, who would argue that in the past three centuries English poetry or drama has been written that surpasses the works of Shakespeare? In the natural sciences, knowledge accumulates by uncovering preexisting and permanent natural processes. Knowledge in the social sciences, however, can affect the social evolution that follows discovery, which through reciprocal causation largely determines the succeeding social theory.

In this spirit, I present a chronological, annotated bibliography of the financial *theory* of investments. It is not, however, a history of the *practice* of investing, and only occasionally refers to the real world outside of theoretical finance. To embed this "history of the theory of investments" in a broader context that includes the development of methodological and theoretical tools used to create this theory, including economics, mathematics, psychology, and the scientific method, I am writing companion volumes—a multiyear project—titled *My Outline of Western Intellectual History*, which also serves to carry this history back to ancient times.

Although this work can be used as a reference, to read it as a history one can read from the beginning to the end. For the most part, papers and books are not grouped by topic since I have tried to see the field as an integrated whole, and to emphasize how one strand of research impacts others that may initially have been thought to be quite separate. For this purpose a chronological ordering—though not slavishly adhered to—seems appropriate since a later idea cannot have influenced an earlier idea, only vice versa.

If I may indulge in the favorite pastime of historians, one can divide the history of financial economics into three periods: (1) the Ancient Period before 1950, (2) the Classical Period from about 1950 to 1980, and

(3) the Modern Period post-1980. Since about 1980, the foundations laid down during the Classical Period have come under increasing strain, and as this is written in 2005, it remains to be seen whether a new paradigm will emerge.

Of necessity, I have selected only a small portion of the full body of financial research that is available. Some papers are significant because they plant a seed, ask what turns out to be the right question, or develop important economic intuitions; others are extraordinarily effective in communicating ideas; yet others are important because they formalize earlier concepts, making all assumptions clear and proving results with mathematical rigor. Although I have tried to strike some balance between these three types of research, I have given more prominence to the first two. Unpublished working papers are included only if they either (1) are very widely cited or (2) appear many years before their ideas were published in papers by other authors. A few literature surveys are mentioned if they are particularly helpful in interpreting the primary sources. Mathematical statements or proofs of important and condensable results are also provided, usually set off by boxes, primarily to compensate for the ambiguity of words. However, the proofs are seldom necessary for an intuitive understanding.

The reader should also understand that this book, such as it is, is very much work in progress. Many important works are not mentioned, not because I don't think they are important, but simply because I just haven't gotten to them yet. So this history, even from my narrow vantage point, is quite partial and incomplete, and is very spotty after about 1980. In particular, though it traces intimations of nonrationalist ideas in both the ancient and classical periods, it contains very little of the newer results accumulating in the modern period that have come under the heading of "behavioral finance." Nonetheless, the publisher encouraged me to publish whatever I have since it was felt that even in such a raw form this work would prove useful. Hopefully, in the fullness of time, an updated version will appear making up this deficit.

The history of the theory of investments is studded with the works of famous economists. Twentieth-century economists such as Frank Knight, Irving Fisher, John Maynard Keynes, Friedrich Hayek, Kenneth Arrow, Paul Samuelson, Milton Friedman, Franco Modigliani, Jack Hirshleifer, James Tobin, Joseph Stiglitz, Robert Lucas, Daniel Kahneman, Amos Tversky, and George Akerlof have all left their imprint. Contributions to finance by significant noneconomists in this century include those by John von Neumann, Leonard Savage, John Nash, and Maurice Kendall. Looking back further, while the contributions of Daniel Bernoulli and Louis Bachelier are well known, much less understood but of comparable importance are works of Fibonacci, Blaise Pascal, Pierre de Fermat, Christiaan Huygens, Abraham de Moivre, and Edmund Halley.

Perhaps this field is like others, but I am nonetheless dismayed to see how little care is taken by many scholars to attribute ideas to their original sources. Academic articles and books, even many of those that purport to be historical surveys, occasionally of necessity but often out of ignorance oversimplify the sequence of contributors to a finally fully developed theory, attributing too much originality to too few scholars. No doubt that has inadvertently occurred in this work as well, but hopefully to a much lesser extent than earlier attempts. Even worse, an important work can lie buried in the forgotten past; occasionally, that work is even superior in some way to the later papers that are typically referenced.

For example, ask yourself who first discovered the following ideas:

Present value.

The Modigliani-Miller theorem.

Pratt-Arrow measures of risk aversion.

Markowitz mean-variance portfolio theory.

The Gordon growth formula.

The capital asset pricing model.

The Black zero-beta model.

The Cox-Ross-Rubinstein binomial option pricing model.

The Lucas exchange model.

The Milgrom-Stokey no trade theorem.

The derivation of expected utility from postulates of individual rationality.

The martingale pricing representation with risk-neutral probabilities.

Dynamic completeness.

The association of random walks with rational markets.

The use of nonstationary variance to describe the stochastic process of security prices.

The hypothesized relationship between upwardly biased stock prices, belief heterogeneity, and short-selling constraints.

The size effect.

The abnormal earnings growth model.

Prospect theory.

In most of these cases, the individuals commonly given bibliographical credit in academic papers were actually anticipated many years, occasionally decades or centuries, earlier. In some cases, there were others with independent and near-simultaneous discoveries who are seldom, if ever, mentioned, offering one of many proofs of Stephen Stigler's law of eponymy that scientific ideas are never named after their original discoverer! This includes Stigler's law itself, which was stated earlier by sociologist and philosopher of science Robert K. Merton. A prominent example in financial economics is the Modigliani-Miller theorem, which received possibly its most elegant exposition at its apparent inception in a single paragraph contained in a now rarely referenced but amazing book by John Burr Williams published in 1938, 20 years before Modigliani-Miller. Had his initial insight been well known and carefully considered, we might have been spared decades of confusion. A clear example of Merton's naming paradox is the "Gordon growth formula." Unfortunately, once this type of error takes hold, it is very difficult to shake loose. Indeed, the error becomes so ingrained that even prominent publicity is unlikely to change old habits.

Also, researchers occasionally do not realize that an important fundamental aspect of a theory was discovered many years earlier. To take a prominent example, although the Black-Scholes option pricing model developed in the early 1970s is surely one of the great discoveries of financial economics, fundamentally it derives its force from the idea that it may be possible to make up for missing securities in the market by the ability to revise a portfolio of the few securities that do exist over time. Kenneth Arrow, 20 years earlier in 1953, was the first to give form to a very similar idea. In turn, shades of the famous correspondence between Blaise Pascal and Pierre de Fermat three centuries earlier can be found in Arrow's idea. A field of science often progresses by drawing analogies from other fields or by borrowing methods, particularly mathematical tools, developed initially for another purpose. One of the delightful by-products of historical research is the connections that one often uncovers between apparently disparate and unrelated work—connections that may not have been consciously at work, but no doubt through undocumented byways must surely have exercised an influence.

One can speculate about how an academic field could so distort its own origins. Its history is largely rewritten, as it were, by the victors. New students too often rely on the version of scholarly history conveyed to them by their mentors, who themselves are too dependent on their mentors, and so forth. Seldom do students refuse to take existing citations at their word and instead dust off older books and journals that are gradually deteriorating on library shelves to check the true etiology of the ideas they are using. Scholars have the all-too-human tendency of biasing their attributions in the direction of those whom they know relatively well or those who have written several papers and spent years developing an idea, to the disadvantage of older and more original works by people who are not in the mainstream, either in their approach to the subject, by geography, or by timing. An excellent example of this is the sole paper on mean-variance portfolio selection by A.D. Roy, whom Harry Markowitz acknowledges deserves to share equal honor with himself as the co-father of portfolio theory.[1] Robert K. Merton has dubbed this the "Matthew effect" (particularly apt since it may serve as an example of itself) after the lines in the Gospel According to Matthew (25:29): "Unto everyone that hath shall be given, and he shall have abundance; but from him that hath not shall be taken away even that which he hath."

Of course, financial economics is not alone in its tendency to oversimplify its origins. For example, consider the calculus, well known to have been invented by Isaac Newton and Gottfried Wilhelm Leibniz. Yet the invention of calculus can be traced back to the classical Greeks, in particular Antiphon, Eudoxus, and Archimedes, who anticipated the concept of limits and of integration in their use of the "method of exhaustion" to determine the area and volume of geometric objects (for example, to estimate the area of a circle, inscribe a regular polygon in the circle; as the number of sides of the polygon goes to infinity, the polygon provides an increasingly more accurate approximation of the area of the circle). Although Galileo Galilei did not write in algebraic formulas, his work on motion implies that velocity is the first derivative of distance with respect to time, and acceleration is the second derivative of distance with respect to time. Pierre de Fermat devised the method of tangents that in substance we use today to determine the maxima and minima of functions. Isaac Barrow used the notion of differential to find the tangent to a curve and described theorems for the differentiation of the product and quotient of two functions, the differentiation of powers of x, the change of variable in a definite integral, and the differentiation of implicit functions.

Unlike large swaths of history in general, much of the forgotten truth about the origins of ideas in financial economics is there for all to see, in older books residing on library shelves or in past journals now often available in electronic form. Much of the history of investments has only been *rewritten* by the victors, and can be *corrected* from primary sources. In this book, I have tried my best to do this. For each paper or book cited, my goal is to clarify its *marginal* contribution to the field.

Like the three witches in Shakespeare's *Macbeth* (and I hope the resemblance ends there), with hindsight, I can "look into the seeds of time, and say which grain will grow and which will not." Taking advantage of this, I will deemphasize research (such as the stable-Paretian hypothesis for stock prices) that, although once thought quite promising, ultimately proved to be a false path.

Nonetheless, I am certain that I also have omitted *many important* discoveries (in part because I just haven't gotten to them) or even attributed ideas to the wrong sources, unaware of even earlier work. Perhaps, on the other hand, I have succumbed to the historian's temptation to bias his interpretation of the written record in light of what subsequently is seen to be important or correct. I hope the reader will forgive me. I have already received some assistance from Morton Davis, and I wish to publicly thank him. I also ask the reader to take the constructive step of letting me know these errors so that future revisions of this history will not repeat them.

MARK RUBINSTEIN

Berkeley, California
January 2006

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The Ancient Period

Pre-1950

1202 **Fibonacci** or Leonardo of Pisa (1170–1240), *Liber Abaci* ("The Book of Calculation"); recently translated into English by Laurence E. Sigler in *Fibonacci's Liber Abaci: A Translation into Modern English of Leonardo Pisano's Book of Calculation* (New York: Springer-Verlag, 2002).

1478 **Unknown Author,** *The Treviso Arithmetic*; translated into English by David Eugene Smith, pp. 40–175, in Frank J. Swetz, *Capitalism and Arithmetic: The New Math of the 15th Century Including the Full Text of the Treviso Arithmetic of 1478* (LaSalle, IL: Open Court, 1987).

1761 **Edmond Halley** (November 8, 1656–January 14, 1742), "Of Compound Interest," in Henry Sherwin, *Sherwin's Mathematical Tables* (published posthumously after Halley's death in 1742, London: W. and J. Mount, T. Page and Son, 1761).

FIBONACCI SERIES, PRESENT VALUE, PARTNERSHIPS, FINITE-LIVED ANNUITIES, CAPITAL BUDGETING

Fibonacci (1202) is well-known as the most influential tract introducing positional numerical notation into Europe. Arabic numerals were first developed in India, perhaps in the mid-first millennium A.D. and were subsequently learned by Arab traders and scholars. In turn, Fibonacci learned about them while traveling through North Africa. He begins Chapter 1 with these words:

> *These are the nine figures of the Indians: 9, 8, 7, 6, 5, 4, 3, 2, 1. With these nine figures, and with this sign 0 which in Arabic is called zephirum, any number can be written, as will be demonstrated.*

After the publication of this tract, computation by Arabic numerals using pen and ink gradually replaced the use of the abacus. The book also develops the famous Fibonacci series, 1, 1, 2, 3, 5, 8, 13 . . . .

Much less appreciated is the role *Liber Abaci* plays in the development of present value calculation, as has been quite recently discovered by William N. Goetzmann in [Goetzmann (2003)] "Fibonacci and the Financial Revolution," Yale ICF Working Paper No. 03-28 (October 23, 2003). Fibonacci illustrates his methods of calculation through several numerical examples. Among these are four types of applications to investments: (1) the fair allocation of profits to members of a partnership ("On Companies," pp. 172–173); (2) the calculation of profits from a sequence of investments, with intermediate withdrawals ("Problems of Travelers," pp. 372–373); (3) the calculation of future value ("A Noteworthy Problem on a Man Exchanging One Hundred Pounds at Some Banking House for Interest," pp. 384–386); and (4) the calculation of present value ("On a Soldier Receiving Three Hundred Bezants for His Fief," p. 392). His solution to (1) is simply to divide profits in proportion to contributed capital—a solution that is now obvious. As an example of (3) in Sigler's translation:

> *A man placed 100 pounds at a certain [banking] house for 4 denari per pound per month interest, and he took back each year a payment of 30 pounds; one must compute in each year the 30 pound reduction of capital and profit on the said 100 pounds. It is sought how many years, months, days and hours he will hold money in the house. (p. 384)*

Fibonacci calculates that the man will have some money with the bank for 6 years, 8 days, and "$(^1/_2)(^3/_9)5$" hours. This makes use of Fibonacci's notation whereby the denominator of each fraction is actually the product of its explicit denominator and all the denominators to the right, and the hours are the sum of these fractions. So the number of hours is $5 + (^3/_9)$hours + $(^1/_{18})$hours = 5 and $^7/_{18}$ hours, in modern notation. Note that as antiquated as Fibonacci's notation has become, it still remains very useful in situations where small units are measured in a different number of parts than larger units. For example, Fibonacci would have written 5 weeks, 3 days, 4 hours, 12 minutes, and 35 seconds as $(^{35}/_{60})(^{12}/_{60})(^4/_{24})(^3/_7)5$.

In problem (4), Fibonacci illustrates the use of present value by ranking the present values of two annuities, differing only in the periodicity of payment, where the interest rate that can be earned on the reinvestment of amounts received is 2 percent per quarter: Both pay 300 bezants per year, with one paying quarterly installments of 75 bezants and the other instead paying the entire 300 bezants at the end of each year.

Due to compounding, present value under a constant interest rate is the result of summing a weighted geometric series. Goetzmann speculates that Fibonacci's interest in finance may have provided the spark for his famous work on infinite series. Unfortunately, we know so little about Fibonacci that this cannot be verified.

After Fibonacci's work, Arabic numerals became widely used in Europe, particularly for commercial purposes. The *Treviso Arithmetic* (1478) published by an unknown author is the earliest known dated and printed book on arithmetic and serves as an early attempt to popularize the Arabic numeral system. The book starts by describing how to use Arabic numerals for enumeration, addition, subtraction, multiplication, and division—the same procedures in use today. By the *Treviso's* time, the numerals had just previously reached their modern forms. For example, the practice of writing 0 as Ø died out after 1275. This may be in part due to the *Treviso* itself, since printing technology may have forced standardization. However, notation for the operations of addition, subtraction, multiplication, and division was not introduced until later, "+" and "−" in print in 1489, "×" in 1631, and "÷" in 1659. While we are on the subject, "√" was introduced in 1525, "=" in 1557, "<" and ">" in 1631, "∫" in 1675 (by Gottfried Wilhelm Leibniz), "*f*(*x*)" in 1735 (by Leonhard Euler), and "*dx*/*dy*" in 1797 by Joseph-Louis Lagrange. Representation of fractions as decimals did not occur until 1585. Using letters for unknowns in equations waited until François Vieta's (1540–1603) formulation in about 1580. John Napier invented logarithms in 1614 and brought decimal notation for factions to Europe in 1617.

These operations are illustrated by a number of problems. Partnerships can be traced as far back as 2,000 B.C. in Babylonia. This form of business organization provided a way to finance investments requiring large amounts of capital over extended periods of time. In Christian Europe, partnerships also provided a way to circumvent usury prohibitions against charging interest. Here is the first partnership problem posed in the *Treviso* (p. 138):

> *Three merchants have invested their money in a partnership, whom to make the problem clearer I will mention by name. The first was called Piero, the second Polo, and the third Zuanne. Piero put in 112 ducats, Polo 200 ducats, and Zuanne 142 ducats. At the end of a certain period they found they had gained 563 ducats. Required is to know how much falls to each man so that no one shall be cheated.*

The recommended solution, following the same principle as already set forth by Fibonacci in his problem "On Companies," is to divide the profits among the investors in proportion to their respective investments. The second partnership problem is much more interesting (p. 138):

> *Two merchants, Sebastiano and Jacomo, have invested their money for gain in a partnership. Sebastiano put in 350 ducats on the first day in January, 1472, and Jacomo 500 ducats, 14 grossi on the first day of July, 1472; and on the first day of January, 1474 they found they had gained 622 ducats. Required is the share of each.*

After converting both investments to a common unit, 8,400 grossi for Sebastiano and 12,014 grossi for Jacomo, the *Treviso* adjusts for the timing of the investments by the number of months of the respective investments:

Sebastiano: 8,400 × 24 = 201,600 Jacomo: 12,014 × 18 = 216,252

The profits are then divided according to these proportions. The sum 201,600 + 216,252 = 417,852. Sebastiano receives 622 × (201,600/417,852) = 300 ducats and Jacomo 622 × (216,252/417,852) = 322 ducats.

The modern analyst would approach this allocation in one of two ways, depending on whether Jacomo's delayed contribution were contracted in advance or whether the terms of his contribution were determined near the time of his contribution. In the former case, he would then need to know the interest rate to work out the fair division of profits, and in the second he would need to know the value of a share in the partnership on July 1, 1472. Although the author of the *Treviso* has posed an interesting problem and probably learned much from Fibonacci, his answer suggests he does not yet understand Fibonacci's more sophisticated present value analysis.

But by the 1500s, Fibonacci's work on present value had become better known, despite usury laws. Consider, for example, a problem from Jean Trenchant [Trenchant (1558)], *L'Arithmétique*, 2nd edition, 1637, Lyons (p. 307): Which has the higher present value, a perpetual annuity of 4 percent per quarter or a fixed-life annuity of 5 percent per quarter for 41 quarters? Trenchant solves the problem by comparing the future value at the end of 41 quarters of a 1 percent annuity per quarter, with the present value in the 41st quarter of a perpetual annuity at 5 percent starting then. Trenchant's book also contains the first known table of present value discount factors.

In the forgotten age before computers, once it was desired to determine the effects of interest rates on contracts, much work was devoted to developing fast means of computation. These include the use of logarithms, precalculated tables, and closed-form algebraic solutions to present value problems. Edmond Halley, cataloger of stars in the Southern Hemisphere from telescopic observation, creator of the first meteorological charts, publisher of early population mortality tables, is, of course, best known as the first to calculate the orbits of comets. Not the least of his achievements includes results in financial economics. Halley (1761) derives (probably not for the first time) the formula for the present value of an annual annuity beginning at the end of year 1 with a final payment at the end of year T: $[X/(r-1)][1-(1/r^T)]$, where r is 1 plus the annual discrete interest rate of the annuity and X is the annual cash receipt from the annuity. Another relatively early derivation of this formula can be found in Fisher (1906).

Although valuation by present value, as we have seen, had appeared much earlier, Fisher (1907) may have been the first to propose that *any* capital project should be evaluated in terms of its present value. Using an arbitrage argument, he compared the stream of cash flows from the project to the cash flows from a portfolio of securities constructed to match the project. Despite this, according to Faulhaber-Baumol (1988), neither the *Harvard Business Review* from its founding in 1922 to World War II, nor widely used textbooks in corporate finance as late as 1948, made any reference to present value in capital budgeting. It was not until Joel Dean in his book [Dean (1951)] *Capital Budgeting: Top Management Policy on Plant, Equipment, and Product Development* (New York: Columbia University Press, 1951) that the use of present value was popularized. More recently, according to John R. Graham and Campbell Harvey in [Graham-Harvey (2001)] "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics* 60, Nos. 2–3 (May 2001), pp. 187–243, most large firms use some form of present value calculation to guide their capital budgeting decisions.

1494 Luca Pacioli (circa 1445-1517), *Summa de arithmetica, geometria, proportioni et proportionalita* ("Everything about Arithmetic, Geometry and Proportions"); the section on accounting, "Particularis de computis et scripturus," translated into English by A. von Gebstattel, *Luca Pacioli's Exposition of Double-Entry Bookkeeping: Venice 1494* (Venice: Albrizzi Editore, 1994).