LINEAR ALGEBRA
AN INTRODUCTION WITH APPLICATIONS

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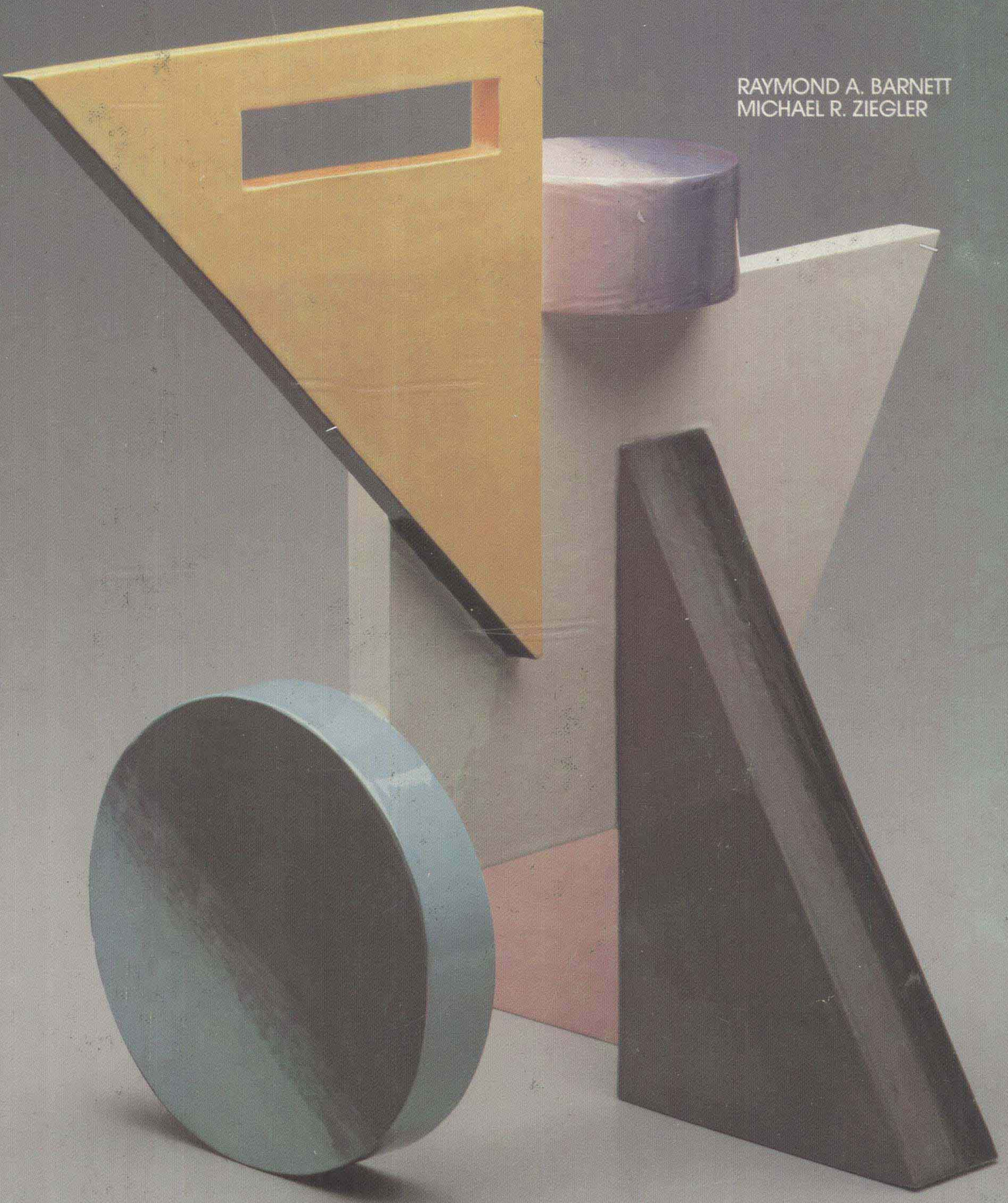
MICHAEL R. ZIEGLER



Linear Algebra

AN INTRODUCTION WITH APPLICATIONS

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Preface

This book presents a treatment of linear algebra suitable for students in a wide variety of disciplines, including mathematics, computer science, engineering, physics, economics, and the life sciences. The material is presented at a level appropriate for students in their freshman or sophomore year. **Calculus is not a prerequisite** (except for Section 9-3); however, optional examples and exercises are included for those students who have studied calculus. These examples and exercises are clearly marked and can be omitted by those students who have not had any previous calculus experience.

■ Emphasis

Teaching linear algebra requires a careful balance between theory, geometric intuition, computational skills, and applications. For most students, this is the first exposure to formal (abstract) mathematical structures (and these structures must be understood in order to perform the computations that arise naturally in the applications). To help students make the transition from the concrete to the abstract, most new concepts are introduced in terms of concrete examples and then generalized to a definition or a theorem. We have included those theorems that are vital to the subject and omitted those whose primary importance lies in their own abstraction. Proofs are included whenever they will lead to a better understanding of concepts under discussion or will contribute additional insight into the application of a concept. We have not hesitated to state theorems without proof if the proofs are too complicated or involve ideas that do not increase the student's understanding of the subject. In many instances, however, special cases of an omitted proof are discussed in the C-level exercises. We have avoided long and tedious arguments involving manipulation of summation notation and any proofs that require mathematical induction. Even with these omissions, there are a sufficient number of proved theorems and theoretical C-level exercises to provide a sound foundation in the theory of linear algebra.

■ Choice of Emphasis

The text provides an instructor or department with many choices in course design and emphasis—from a course strong in mechanics to a course strong in mathematical structure and theory. By deemphasizing proofs and restricting assigned problems to A and B levels in the exercise sets, a course emphasizing mechanics will evolve; by emphasizing proofs and problems in the C-level exercises, a fairly strong course emphasizing mathematical structure and theory will evolve; and, of course, a variety of courses combining these two emphases to varying degrees are possible. In short, an instructor can easily pitch a course to his or her own interests, class background, or department requirements.

■ Organization

The material is organized in three parts. The first four chapters present basic material concerning systems of linear equations, matrices, determinants, and vectors in the plane and in 3-space. The amount and depth of coverage here will depend on the background of the students. With the exception of Cramer's Rule (Section 3-4), students should be comfortable with all of this material before beginning Chapter 5.

Chapters 5 and 6 present a thorough coverage of the theory of finite-dimensional vector spaces and inner product spaces, culminating in a discussion of the Gram–Schmidt orthogonalization process. Eigenvalues and diagonalization techniques are covered in Chapter 7.

There are thirty-four sections in these first two parts of the book. Therefore, it should be possible to cover all of Chapters 1–7 comfortably in a one-quarter or one-semester course.

The third part of the text covers two topics: linear transformations (Chapter 8) and applications of diagonalization (Chapter 9). These topics are independent, so either chapter can directly follow Chapter 7.

Finally, a shorter course emphasizing applications can be formed by omitting Sections 3-4, 6-3 through 6-6, 7-3, 8-1 through 8-5, and 9-1.

■ Computers

One of the reasons for the growing importance of linear algebra is the rapid development of computers. Many problems that were impossible to solve by hand can now be solved routinely with the aid of a computer. Although we have not assumed that the students have had any previous experience with a computer or that they will be using a computer in this course, we have been careful to point out the relationship between linear algebra and computing. Whenever possible, procedures for solving problems have been presented in a manner that permits easy implementation on a computer. Procedures that are important for their theoretical implications and are not suited for use on a computer are clearly identified.

A computer supplement and an APPLE II or IBM PC microcomputer program disk are available for those who wish to incorporate the use of a computer into this course. The programs on the disk implement many of the procedures presented in this book and can be used to reinforce concepts and to permit the consideration of problems involving calculations too complicated to be done by hand. These programs are interactive, easy to use, and require no previous computer experience on the part of the student. The manual contains examples that illustrate the use of these programs, exercises for the student, discussions of some additional applications that are particularly suited to computer solutions, and discussions of some numerical techniques in linear algebra. Programs corresponding to these additional topics are also included on the program disk.

▪ Examples and Matched Problems

We firmly believe that the best way to master this subject is to use it. To that end, the text contains over 350 completely worked examples. Each example is followed by a similar problem for the student to work while reading the material. The answers to these matched problems are included at the end of each section for easy reference.

▪ Exercises

This book contains over 2,300 exercises. Each exercise set is designed so that an average or below-average student will experience success and a very capable student will be challenged. They are divided into three parts, A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and theory).

▪ Applications

Enough applications are included in this book to convince even the most skeptical student that linear algebra is really useful. The first three chapters contain applications of elementary linear algebra to a wide variety of areas, including engineering, physics, biology, economics, graph theory, and geometry. The last chapter contains detailed discussions of three applications of diagonalization, utilizing most of the theory developed in the book. With the exception of the section on differential equations (Section 9-3), no specialized background is required to solve any of the applications in this book.

▪ Student Aids

Theorems, procedures for solving problems, and most important definitions are **displayed in boxes** for easy reference.

Examples and developments are often **annotated** to help students through critical stages.

A **second color** is used to indicate key steps, to delineate topics, and to increase the clarity of graphics.

Boldface type is used to introduce new terms and highlight important comments.

Answers to all odd-numbered, nontheoretical problems are included in the back of the book. If the answer to a theoretical problem is a proof, the proof is not included in the answer section (but some proofs are included in the student's solution manual).

Chapter review sections include a review of all important terms and symbols and a comprehensive review exercise. Answers to all (nontheoretical) review exercises are included in the back of the book.

A **solutions manual** by Robert Mullins is available through a book store. The manual includes solutions to all odd-numbered, nontheoretical problems, solutions to all nontheoretical review problems, and proofs (or outlines of proofs) for selected theoretical exercises. Each section of the solutions manual begins with a review of the important terms, theorems, and problem-solving procedures from the corresponding section in this book.

A **computer applications supplement** is available through a book store. This supplement is designed to be used with a **microcomputer program disk.** Copies of the program disk are available through instructors or departments using this book. (See the discussion earlier in this Preface.)

■ Instructor Aids

A unique **computer-generated random test system** is available to instructors without cost. The system, utilizing an IBM PC computer and a number of commonly used dot matrix printers, will generate an almost unlimited number of chapter tests and final examinations, each different from the other, quickly and easily. At the same time, the system produces an answer key and a student worksheet with an answer column that exactly matches the answer column on the answer key. Graphing grids are included on the answer key and on the student worksheet for problems requiring graphs.

A **printed and bound test battery** is also available to instructors without cost. The battery contains several chapter tests for each chapter, answer keys, and student worksheets with answer columns that exactly match the answer columns on the answer keys. Graphing grids are included on the answer key and on the student worksheet for problems requiring graphs.

An **instructor's answer manual** containing answers to the even-numbered problems not included in the text is available to instructors without charge.

A **student's solutions manual** by Robert Mullins (see Student Aids) is available to instructors without charge.

A **computer applications supplement** (see Student Aids) and an **APPLE II or**

IBM PC microcomputer program disk are available to instructors without charge. Copies of the program disk may be distributed to students at the discretion of the instructor.

■ Acknowledgments

In addition to the authors, many others are involved in the successful publication of a book. We wish to thank personally: C. Bandy, Southwest Texas State University; Gary Brown, College of St. Benedict; Bruce Edwards, University of Florida; Paul Eenigenburg, Western Michigan University; Garry Etgen, University of Houston; Frederic Gooding, Jr., Goucher College; Thomas Kearns, Northern Kentucky University; Stanley Lukawecki, Clemson University; Stephen Merrill, Marquette University; Robert Mullins, Marquette University; and John Spellmann, Southwest Texas State University.

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Producing this new edition with the help of all these extremely competent people has been a most satisfying experience.

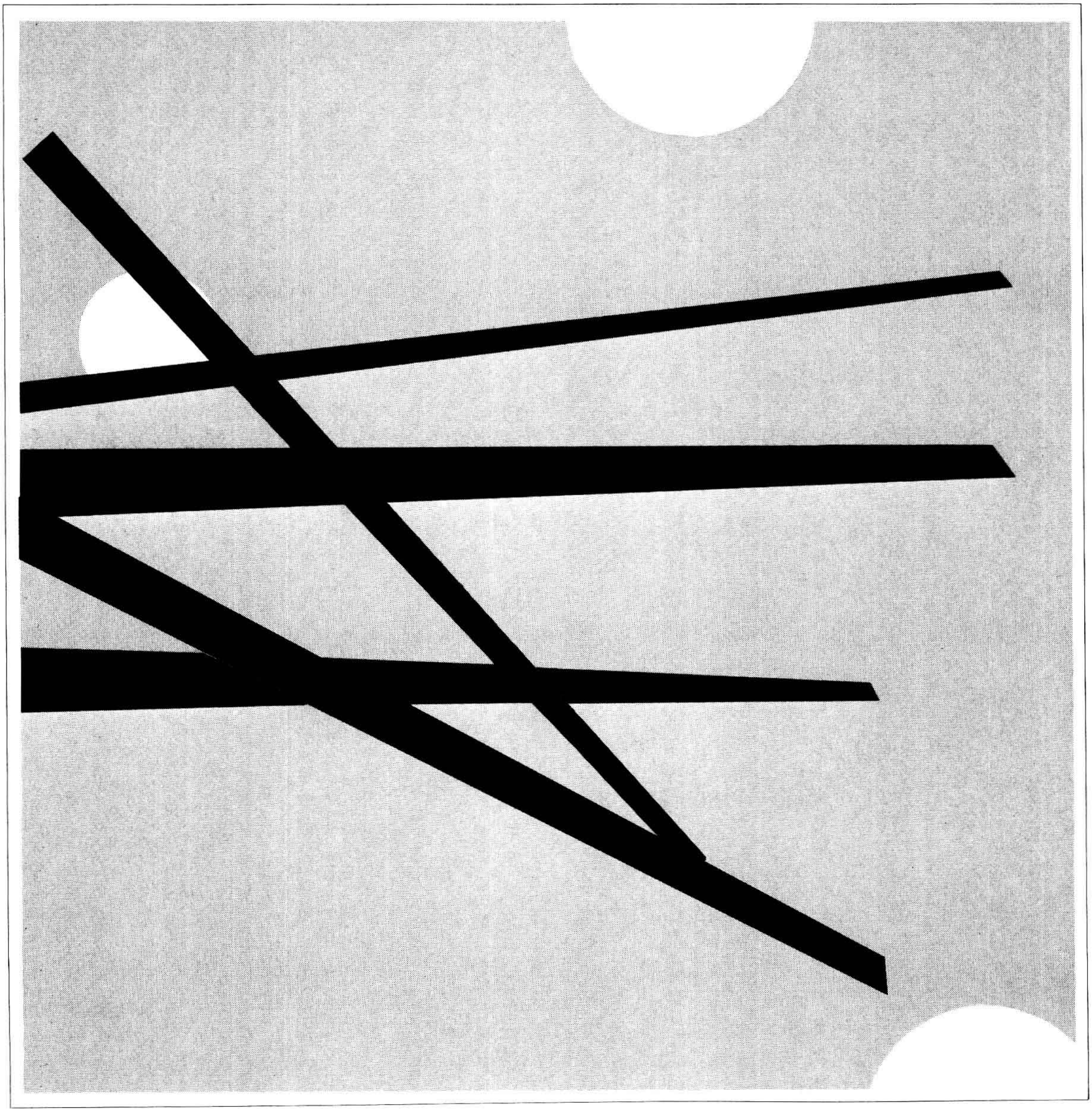
R. A. Barnett
M. R. Ziegler

Linear Algebra

AN INTRODUCTION WITH APPLICATIONS

Contents

1 Systems of Linear Equations



1 Contents

Linear algebra began as a study of systems of linear equations which occur naturally in a variety of areas, including mathematics, engineering, economics, and the physical, life, and social sciences. In some of these areas, it is quite common to find applications involving systems where the equations and the variables number in the thousands. Of course, these large-scale systems require computers for their solution. In this chapter we will introduce systematic procedures for solving systems of linear equations. These procedures have two important features. First, they can be applied to any linear system however large. Second, a digital computer can be programmed to carry out the calculations, making the solution of large systems practical as well as feasible. As a pleasant bonus, we will see that the techniques learned here have many other important applications. Thus, this chapter builds the foundation for much of what follows in this text and the material presented here deserves your most careful attention.

1-1 Systems of Linear Equations—Introduction

- Linear Equations in Two Variables
- Systems of Linear Equations in Two Variables
- Nature of Solutions
- Systems of Linear Equations in Three Variables
- Application

Linear Equations in Two Variables

An equation of the form

$$ax + by = c \tag{1}$$

where a, b, and c are real constants (a and b not both zero), is called a **linear equation in *x* and *y*.** The **solution set** of a linear equation is the set of all ordered pairs of real numbers that satisfy the equation. Using set notation, the solution set can be expressed as

$$S = \{(x, y) | ax + by = c\} \qquad \text{Solution set}$$

The graph of the solution set (a and b not both zero) is a straight line. It may seem strange, but it is necessary to consider equations of the form (1) where a

and b are both zero. All possible types of solution sets for equation (1) are listed in Table 1.

Table 1
Solution Sets for $ax + by = c$

Equation	Solution Set	Graph
$ax + by = c, \quad a \neq 0$ or $b \neq 0$	$\{(x, y) \mid ax + by = c\}$	A straight line
$0x + 0y = c, \quad c \neq 0$	$\varnothing$	The empty set
$0x + 0y = 0$	$\{(x, y) \mid x$ and y are real numbers$\}$	The entire xy plane

Example 1 (A) Use set notation to describe the solution set of the linear equation

$$2x + 3y = 12$$

(B) List two explicit points in the solution set.
(C) Graph the solution set.

Solution (A) $S = \{(x, y) \mid 2x + 3y = 12\}$ Solution set

(B) Let $x = 0$. Then

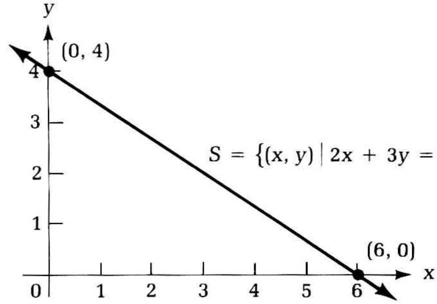
$$2 \cdot 0 + 3y = 12$$
$$3y = 12$$
$$y = 4$$

Let $y = 0$. Then

$$2x + 3 \cdot 0 = 12$$
$$2x = 12$$
$$x = 6$$

Thus, (0, 4) and (6, 0) are two of the infinite number of points in the solution set S. Other points in S can be determined by assigning any value to x (or to y), substituting this value in the equation $2x + 3y = 12$, and solving for y (or for x).

(C) Since two points determine a line, we complete the graph of $2x + 3y = 12$ by drawing a line through the two points found in part (B).



Problem 1 Repeat Example 1 for the linear equation

$$x - y = 1$$

Systems of Linear Equations in Two Variables

To establish some basic concepts, consider the following simple example. A school has $13,000 to purchase ten microcomputers. Each available computer has either one or two built-in disk drives. If a computer with one disk drive costs $1,000 and one with two drives costs $1,500, how many of each type should the school purchase in order to use all of the $13,000?

Let

x = Number of computers with one drive

y = Number of computers with two drives

Then

$x + y = 10$	Number of computers
$1{,}000x + 1{,}500y = 13{,}000$	Purchase cost

We now have a **system of two linear equations in two variables.** The **solution set** of such a system is the set of all ordered pairs of real numbers that satisfy both equations at the same time. In elementary algebra, you may have learned to solve systems of this type by graphing or by substitution. Since these methods are not suitable for larger systems, we will present a new method here, called **elimination with back substitution.** This method involves replacement of systems of equations with simpler *equivalent systems* (by performing appropriate operations) until we obtain a system that is easy to solve. Equivalent systems are defined as follows:

Equivalent Systems of Linear Equations

Two systems of linear equations are **equivalent** if they have the same solution set.

Theorem 1 lists the operations that can be performed on a system of linear equations to produce an equivalent system. (The proof is omitted.)

Theorem 1

Operations on Systems of Equations

Equivalent systems of equations result if:

(A) Two equations are interchanged.
(B) An equation is multiplied by a nonzero constant.
(C) A constant multiple of one equation is added to another equation.

Returning to our example, we use Theorem 1 to eliminate one of the variables and obtain an equivalent system:

$$x + y = 10$$
$$1{,}000x + 1{,}500y = 13{,}000$$

If we multiply the top equation by −1,000 and add it to the bottom equation, we can eliminate x.

$$\begin{aligned} -1{,}000x - 1{,}000y &= -10{,}000 && \text{−1,000 times top equation} \\ 1{,}000x + 1{,}500y &= 13{,}000 && \text{Bottom equation} \\ \hline 500y &= 3{,}000 && \text{Sum} \end{aligned}$$

If we replace the original bottom equation with this new equation, we obtain a system that is equivalent to the original system and that is easy to solve:

$$\begin{aligned} x + y &= 10 \qquad \text{Equivalent system} \\ 500y &= 3{,}000 \end{aligned}$$

Now, solve this system for y and then x:

$$\begin{aligned} 500y &= 3{,}000 && \text{Solve the bottom equation for } y. \\ y &= 6 \\ x + 6 &= 10 && \text{Substitute } y = 6 \text{ in the top equation and solve for } x. \\ x &= 4 \end{aligned}$$

Thus, the school should purchase 4 computers with one disk drive and 6 with two disk drives.

Check

$$\begin{aligned} x + y &= 10 & 1{,}000x + 1{,}500y &= 13{,}000 \\ 4 + 6 &\stackrel{?}{=} 10 & 1{,}000(4) + 1{,}500(6) &\stackrel{?}{=} 13{,}000 \\ 10 &\stackrel{\checkmark}{=} 10 & 13{,}000 &\stackrel{\checkmark}{=} 13{,}000 \end{aligned}$$

The first step in this process is called *elimination* and the second is called *back substitution*. The following example further illustrates the solution of a system by elimination with back substitution.

Example 2 Solve the system:

$$2x + 3y = 2$$
$$5x + 6y = 11$$

Solution First, eliminate x from the bottom equation:

$$\begin{aligned} -5x - \tfrac{15}{2}y &= -5 \\ 5x + 6y &= 11 \\ \hline -\tfrac{3}{2}y &= 6 \end{aligned}$$

Multiply the top equation by $-\frac{5}{2}$ and add to the bottom equation.

This produces the following equivalent system:

$$2x + 3y = 2$$
$$-\tfrac{3}{2}y = 6$$