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Lokenath Debnath (Editor)

Advances in nonlinear waves
VOLUME II

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PART III SOLITONS, INVERSE SCATTERING TRANSFORM, BÄCKLUND TRANSFORMATION AND NONLINEAR WAVES IN PHYSICS

Preface

The mathematical theory of nonlinear wave phenomena and their applications has grown considerably during the past two decades. With the advent of new ideas and methods, new results and applications, studies are continually being added to the central subjects of fluid mechanics, plasma physics, solid mechanics, nonlinear optics and nonlinear systems and circuits; these are themselves developing remarkably and coalescing. It is becoming more and more desirable for applied mathematicians, physicists and engineering scientists to study nonlinear phenomena as a whole. Yet it is increasingly difficult for them to do so since important articles often appear in journals of widely different character.

This difficulty may be alleviated by the publication in volume form of original research papers together with research-expository and survey articles selected from key areas. This is the second of two such volumes; the first has already been published. Like Volume I, this volume has also developed from manuscripts of diverse natures, submitted by renowned applied mathematicians and scientists who have made significant contributions to this important and rapidly growing field. It too brings together current developments in the theory and applications of nonlinear waves and solitons that seem likely to determine fruitful directions of future study and research.

This volume has been divided into three parts. Part I, entitled Nonlinear Waves in Fluids, consists of five chapters. Nonlinear Waves in Plasmas are treated in Part II, which has four chapters. Part III contains eight chapters on solitons, evolution equations and nonlinear interaction problems. Included also are recent developments in methods of asymptotic expansion and of finite-dimensional models in the theory of nonlinear waves, and nonlinear model equations in dispersive systems.

In the opening chapter, P.G. Saffman and H.C. Yuen describe the properties of three-dimensional water waves of permanent form in deep water. Special attention is given to spontaneous waves associated with the bifurcation of finite-amplitude Stokes waves which occurs when stationary disturbances exist. In Chapter 2, J. Vanden-Broeck considers a steady flow past a two-dimensional flat-bottomed body. Two different flow considerations are discussed. In the first, the flow separates at the corner of the body. In the second, the flow rises up along the rear face of the body to a stagnation point at which separation occurs. Numerical and semi-analytical methods are described to obtain solutions for these flow configurations.

Chapter 3, by P. Ripa, deals with pulses of sea-surface elevation that propagate across the Pacific Ocean along the equator (one fourth of the Earth's circumference). Both the meridional structure and the propagation speed of the pulses are found to be similar to those of first internal mode equatorial Kelvin waves. The celerity is, however, consistently greater than the value predicted by linear theory. A nonlinear explanation is given by studying the interaction between an equatorial Kelvin wave in the first internal vertical mode and itself, long Rossby waves and Kelvin waves in other vertical modes, and mean ocean currents. In Chapter 4, Y. Tsuchiya and T. Yasuda describe a cnoidal wave theory based upon the method of reductive perturbation. A set of higher-order wave equations for progressive monochromatic waves is derived from the governing hydrodynamic equations. The authors discuss a third-order solution for cnoidal and solitary waves and also the properties of these waves, including wave profile, wave celerity, water particle velocities and pressure, as well as the mass transport velocities, both in the Eulerian and Lagrangian descriptions. It is shown that results for wave celerity and mass transport velocity in the Lagrangian description are in excellent agreement with experimental findings.

In Chapter 5, R. Ravindran and P. Prasad present a brief review of the kinematics of a shock front and the role of nonlinearity in a hyperbolic caustic. It turns out that when the amplitude of the shock is not very small, the caustic is resolved, but a singularity in the shape of the shock front appears. Also, they present a theory which explains the existence of the third shock.

Chapter 6, by R. Bingham and C.N. Lashmore-Davies, is concerned with a general discussion of modulational instabilities in plasmas. The basic mechanism is a four-wave interaction and examples include Langmuir modulational instability, oscillating two-stream instability and the filamentation of laser light in plasmas. The single envelope model which usually leads to the nonlinear Schrödinger equation is contrasted with the more general multimode description of this chapter. General equations for the modulation of finite-amplitude high-frequency waves in unmagnetized plasmas are given. The stability properties of the linearized equations are briefly discussed and the conservation relations of the fully nonlinear equations are obtained. The filamentation of an electromagnetic wave in a plasma is also discussed in more detail. A physical argument is given for restricting the analysis to the initial wave and two sidebands. This is then put on a firmer footing with the aid of a recent theorem of Thyagaraja on the effective number of modes carrying the wave energy. Finally, exact analytic solutions of the fully nonlinear equations are obtained and the resulting filamentation length is compared with the experiment.

In Chapter 7, E. Infeld discusses theoretical, numerical and experimental investigations of cylindrical and spherical plasma soliton dynamics. He presents some current problems of interest. Some neglected areas are indicated briefly. Chapter 8, by Y. Inoue and N. Kimura, deals with weakly nonlinear hydromagnetic waves in a cold plasma. This study is based on the Whittaker method of analytical dynamics. The results of this analysis indicate that, in general, profiles of steady waves have no periodicity and oscillate irregularly.

N. Rudraiah and M. Venkatachalappa, in Chapter 9, present theories of linear and nonlinear internal gravity waves in a stratified conducting fluid. Special attention is given to the phase portrait technique in the study of nonlinear waves. It is shown that the waves are periodic only for small values of the porous parameter β, and are non-periodic for large values of β. This chapter also includes a study of nonlinear wave propagation in a compressible stratified rotating atmosphere, with a discussion on the effects of rotation.

In Chapter 10, R. V. Dooren introduces an alternative method to classical perturbation techniques for the description of the interaction of N-solitons. The method is then applied to the three-soliton interaction occurring in the two-dimensional KdV equation. As a novelty, it is proposed to expand the unknown function of the governing equation in a series which involves all possible interaction terms as well as the three basic solitary waves. The unknown coefficients in this expansion are readily determined by applying a certain balance principle method to the relevant equation.

H. Cornille, in Chapter 11, defines a class of nonintegrable partial differential equations in 1 + 1 dimensions, with solution k = K(x,t): (i) K^n and K^nK_x are nonlinearities, (ii) the linear operators, associated with the linear part of the equations, are factorized differential operators, (iii) there exist powers of the bi-solitons which are rational functions and (iv) a direct method exists which simultaneously allows construction of both the linear operator and the solutions. In the K^n case, for a large class of bi-solitons, the origin of the factorization property is explained with a conjecture which states that all possible bi-solitons belong to the class defined above. In the K^nK_x case, the linear operator factorizes the corresponding one of the Burgers equations for N = 2 and a generalization of it for N > 2.

In Chapter 12, N. Saitoh and M. Toda derive a certain Helmholtz-like equation for a discrete time nonlinear lattice. It is shown that the equation admits multi-soliton solutions and certain periodic solutions. Chen et al. in Chapter 13, present a method to derive a new hierarchy of symmetries for integrable nonlinear evolution equations including the Korteweg-de Vries equation, the modified KdV equation, the nonlinear Schrödinger equation, the sine-Gordon equation, the Benjamin-Ono equation and the Kadomtsev-Petviashvili equation. These new symmetries, which depend explicitly on the space variables and the time variable, do not have conservation laws associated with them. Together with the previously known classical symmetries, they constitute an infinite-dimensional Lie algebra.

Chapter 14, by Y.H. Ichikawa, deals with construction of an alternative set of Lax-pair operators for the sine-Gordon equation. As in the case of the modified Korteweg-de Vries equation, the eigenfunctions of these Lax-pair operators are sums of squared eigenfunctions of the Ablowitz-Kaup-Newell-Segur scheme, and their temporal evolution is determined simply by an infinitesimal transformation around a solution of the sine-Gordon equation. In Chapter 15, Z. Peradzynski discusses geometry of interactions of Riemann waves. Special attention is given to the study of the nonlinear interaction problem to gain understanding of physical phenomena related to it. Also, this study provides a deeper insight into purely mathematical aspects. Although not all results in this chapter are directly concerned with

Riemann waves, they can be generalized without difficulty to complex characteristic elements. The complex elements may be introduced not only for elliptic systems but also for hyperbolic systems.

In Chapter 16, V.M.E. Eleonsky et al. describe methods of asymptotic expansion and of finite-dimensional models in the theory of nonlinear waves. In a number of problems it is supposed that formal solutions constructed by the method of asymptotic power expansion give a model for exact self-localized field states. They discuss the above methods and general properties of the self-localized state problems for both nonlinear hyperbolic and elliptic equations. They examine the difference between finding self-localized solutions for equations of hyperbolic and of elliptic types. For elliptic equations, the self-localized solutions obtained by the asymptotic expansion method are in satisfactory agreement with solutions for finite-dimensional models. The authors emphasize that the analysis of finite-dimensional models indicates the existence of self-localized solutions of more complicated structure than that of solutions obtained by the asymptotic expansion method.

The final chapter, by L. Debnath and B.K. Shivamoggi, is devoted to recent developments in the mathematical theory of nonlinear waves and solitons, and their applications to various physical problems in fluid dynamics, plasma physics and nonlinear optics. Special attention is given to the discussion and derivation of the varied forms of the nonlinear model equations from the theory of water waves. These model equations include the Boussinesq equation, the Korteweg-de Vries (or KdV) equation, nonlinear Schrödinger (or NLS) equation, the Kadomtsev and Petviashvili (or KP) equation, and the Benjamin, Bona and Mahony (or BBM) equation. Recent progress on these equations, with special emphasis on their solitary wave solutions, is discussed in some detail. The Boussinesq equation for ion-acoustic waves is derived and its solitary wave solutions are discussed.

I wish to express my grateful thanks to the authors for their cooperation and contributions, and hope that this volume also brings together all of the important recent developments in the mathematical theory and applications of nonlinear waves and solitons in fluids and plasmas. I want the reader to share in the excitement of present-day research in this rapidly growing subject and to become stimulated to explore nonlinear phenomena. I hope that the two volumes of ADVANCES IN NONLINEAR WAVES will not only generate

new useful leads for those engaged in research but, in view of its tremendous potential, will attract new researchers into the field.

Herewith my gratitude to Dr Carroll A. Webber for his help in improving the readability of several chapters, and to my son, Jayanta, for his assistance during preparation of the index. I am thankful to my wife for her constant help and encouragement during the preparation of the monograph. I wish to thank Ms Bridget Buckley for her constant and expert assistance with the final preparation of the manuscript. In conclusion, my sincere thanks also go to the publishers of this monograph, Pitman Publishing in England and in the United States of America.

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