Nonlinear Waves: Classical and Quantum Aspects

Edited by

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NATO Science Series

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Preface

After the discovery of a solitary wave by J.S. Russel almost one and half century ago and later implementation of the soliton conception into the modern physics by Zabusky and Kruskal in 1967, the topic continues to be intensively investigated. Initially the main interest in the soliton theory was related to the hydrodynamics, then to plasma physics and to nonlinear optics. Now the focus is shifted toward the condensed matter physics and biophysics. One of the latest intriguing subjects of the nonlinear science is a theory of nonlinear waves in the Bose-Einstein condensates (BEC's).

The soliton theory is an interdisciplinary topic, where many ideas from mathematical physics, statistical mechanics, nonlinear optics, solid state physics and quantum theory mutually benefit each others. Many of applications of the theory in different areas are based on similar model equations and thus allow unified theoretical approaches. Since the most of the systems are intrinsically quantum, like in linear physics, the nonlinear phenomena have two well pronounced levels of description: classical (or mean filed) one and quantum one. There are also other common features of practical applications of the soliton theory to different systems. They are inevitable presence of noise or thermal fluctuations, effect of disorder, interplay among different physical phenomena including nonlinearity, dispersion, and periodicity or discreteness, etc.

The Estoril Workshop "*Nonlinear Waves: Classical and Quantum Aspects*" (July 13-17, 2003) was focused on various aspects of the nonlinear wave theory and its particular applications. The topics discussed on the Workshop are related to the BEC theory, nonlinear optical phenomena, and discrete nonlinear systems.

The main objective was to explore analogies between the problems occurring in the different areas. Probably the most important example is given by the mean field Gross-Pitaevskii (GP) equation which has a form of the multidimensional nonlinear Schrödinger (NLS) equation with a trap potential, and as such is similar to standard models of the nonlinear optics of the Kerr media. On the other hand, in a number of cases (in the so-called tight-biding approximation) the GP equation is reduced to a discrete NLS equation (which is a differential-difference equation, also called lattice). Thus, experience acquired on the earlier stages of the development of the nonlinear optics and the lattice theory appears to be very useful for the analysis of the BEC problems.

The first part of the book contains contributions having interdisciplinary character or devoted mainly to mathematical aspects of the nonlinear evolution equations. It starts with the algebro-geometric integration of the GP equation (Enolski) and an analytical method for describing N-soliton interactions (Gerdjikov). Next, different mechanisms of the dynamical stabilization of nonlinear waves by periodic in time modulations of a trap potential, nonlinearity, and dispersion, as well their applications to the nonlinear optics and BEC problems, are analyzed (Abdullaev). A new concept of the nonlinear physics, referred to as dissipative solitons, is described by Akhmediev, Soto-Crespo and Ankiewicz. In the subsequent contributions a large variety of remarkable structures – solitons and vortices – which are relevant to the mean-filed theory of BEC's are presented for cases of a periodic (Baizakov, Malomed and Salerno) and harmonic (Crasovan *et. al.*) potentials. Białynicki-Birula and Sowiński describe a model, the logarithmic nonlinear Schrödinger equation, allowing a number of exact results, a rather rear event in the nonlinear science. An issue of recent interest, the effect of geometry on wave propagation with applications to the theory of photonic crystals is discussed by Gaididei *et al.* Next, the effect of nonlocality on the dynamics of breathers and kinks (Alfimov, Pierantozzi and Vázquez) and new phenomena described in terms of the fractional calculus (Vázquez) are presented. The dynamical behavior of matter coupled to gravity is considered in the contribution by Christodoulakis *et al.* De Lillo and Sanchini discuss various stochastic effects which can be obtained within the framework of the Eckhaus equation. Grecu and Visinescu study various aspects of the modulational instability of nonlinear systems. Collisions between nonlinear Schrödinger solitons and bound quantum states in localized one-dimensional potentials are investigated by Ludu. The study of oscillator-wave interaction is presented in the article by Damgov and Trenchev.

The second part is devoted to a number of latest experimental and theoretical results in the physics of BEC's, a field which received a great deal of attention during the last years. The chapter begins with a brief review on unified approach to BEC from the point of view of the nonlinear quantum mechanics with main emphasis on fundamental problems of superfluidity and BEC in optical lattices (Pitaevskii), and with basics of statistical mechanics of quantum integrable systems within the framework of the approximation of a hard core Bose-gas (Wadati, Kato and Iida). Next, a number of exciting experimental results on BEC's in optical lattices obtained in the Florence (Fort and Fallani) and Pisa (Morsch and Arimondo) experimental groups are reported. Superfluid-insulator transition, expansion of BEC's in moving optical lattices, Landau-Zener tunneling and instabilities are some of them. Theoretical investigations of BEC's embedded in optical lattices, presented in this chapter, deal with the problem of quantum bound states and matter waves delocalization (Salerno), with the use of Feschbach resonance for generation of soliton trains (Brazhnyi and Konotop), and with nonlinear periodic waves in two-component BEC's (Kostov *et al*). Finally, a study of creation and evolution of shock waves in BEC's is reported (Kamchatnov, Gammal and Kraenkel).

The third part is devoted to propagation of electromagnetic waves in different physical systems. As it was already mentioned, the related problems have many similarities with the dynamics of matter waves in BEC systems. Here a waveguide structure plays the role of a trap, while the Kerr nonlinearity plays the role of the effective nonlinearity induced by elastic interactions between two atoms. Nonlinear photonics and BEC in optical lattices also share many common properties. The chapter starts with the study of light propagation in 2D nonlinear photonic structures with defects (Aceves and Dohnal), in quantum dot ensembles (Maimistov), and in surface polaritonic crystals (Darmanyan, Nevière and Zayats). Light propagation in the media with cubic-quintic nonlinearities, including generation of sophisticated coherent structures like pulsed beams, vortices and vortex solitons, as well as instabilities, are presented by Michinel, Paz-Alonso and Salgueiro and by Skarka, Aleksic and Berezhiani. This part of the book contains also reports on propagation of extremely short pulses on the basis of Maxwell-Duffing equations (Kazantzeva), various aspects of statistics of soliton propagation in optical fibers (Boscolo, Derevyanko and Turitsyn; Villarroel), some exactly solvable models for the nonlinear electrodynamics (Shvartsburg).

In the last part of the book classical and quantum aspects of dynamics of anharmonic lattices are are discussed. Consideration is focused on the topics as follows: the quantum breathers in a Hubbard model (Eilbeck and Palmero); the relation between the energy localization in solids and molecules and the first order phase transition (Takeno and Suzuki); propagation of discrete solitons in long disordered Ablowitz-Ladik and Toda chains (Garnier); ultrafast electron transfer in different systems (Aubry). Also the chapter contains recent experimental results demonstrating that self-trapping is a common feature in hydrogen bonded systems (Edler and Hamm); discussion of a concept of quantum anharmonic phonons and their properties for the Fermi-Pasta-Ulam chain (Szeftel); new results on the perturbation theory for discrete solitons of the perturbed Ablowitz-Ladik model (Doktorov, Matsuka and Rothos); analysis of possibility of the existence of traveling waves in the discrete sine-Gordon and Klein-Gordon equations (Rothos and Feckan; Zolotaryuk); solitary wave dynamics in perturbed ladder lattices (Vakhnenko); application of discrete solitons to the array of BEC's with varying atomic scattering length (Tsoy *et al*); new exact results on functional presentation of the Volterra hierarchy (Vekslerchik) and on Heisenberg chains (Serikbaev *et al*; Rahimov *et al*).

In conclusion the book contains a panoramic view of rapidly developing areas of the nonlinear science – physics of matter waves, nonlinear optics and photonics, and nonlinear discrete systems. The review character of a number of contributions make this book to be interesting for postgraduate students starting a research while the original results reported will be of interest for experts working in the respective fields.

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