EXPANDED
EIGHTH
EDITION

# APPLIED CALCULUS

For Business, Economics, and the Social and Life Sciences

LAURENCE D. HOFFMANN
GERALD L. BRADLEY
KENNETH H. ROSEN

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## For Business, Economics, and the Social and Life Sciences


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EXPANDED EIGHTH EDITION

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# PREFACE

## Overview of the Expanded Eighth Edition

This Expanded Eighth Edition of *Calculus for Business, Economics, and the Social and Life Sciences* (©2004) includes four additional chapters:

- Chapter 8, Differential Equations
- Chapter 9, Infinite Series and Taylor Approximations
- Chapter 10, Probability and Calculus
- Chapter 11, Trigonometric Functions

*Applied Calculus for Business, Economics, and the Social and Life Sciences* meets the needs of instructors who cover topics in one or more of these four chapters together with material from the initial seven chapters in their one-term course, or who teach a two-term course covering the material in all 11 chapters. (The word "Applied" in the title distinguishes this volume from the shorter edition.) Our book introduces calculus in real-world contexts; our primary goal is to provide a sound, intuitive understanding of basic concepts students need as they pursue careers in business, the life sciences, and the social sciences. We strive to teach techniques of differential and integral calculus without sacrificing mathematical accuracy. We carefully and completely state the main results, motivating and explaining them intuitively or geometrically when possible. We have attempted to make every concept and idea as easy to understand as possible. That we achieve these goals has been validated by the continued successful use of our books at a large number of colleges.

The previous edition of *Calculus for Business, Economics, and the Social and Life Sciences* was extremely successful. In the past five years, more than 75,000 students have studied from this book and more than a quarter of a million students have learned applied calculus from earlier editions. To help us build on that success many longtime users provided suggestions for improvement. An extensive panel of reviewers was assembled to provide guidance in the preparation of the Eighth Edition texts. The reviewers helped confirm the need for the additional topics covered in this expanded volume. They also have helped us focus on the most important topics and applications that make this book well suited for use by students in business, economics, and the life sciences. Furthermore, our reviewers have helped us to design the features of this book that make it an effective tool both for teaching and for learning applied calculus. Reviewers have also helped us keep this book up-to-date and relevant by providing suggestions for coverage of new applications.

Following the advice of our reviewers, we have built on these strengths while endeavoring to improve the presentation. We have added many new procedure boxes, definition boxes, and illustrations to clarify discussion. We have written new introductions for many topics and have added many new applied examples to motivate concepts. Reviewers also told us that our exercise sets were excellent and gave suggestions for improving them. To meet their needs, we have added a substantial number of routine problems to provide a more gradual building of skills in working with the procedures and concepts presented.

When asked, "What makes our text more teachable for your course?" reviewers pointed to our applied orientation to concepts, our problem-solving approach, and the straightforward writing style. Our goal has always been to assist the instructor by organizing the subject matter so that each section in the text corresponds to a single lecture and that each chapter contains the right amount of material for an examination. Students need to know how calculus applies to their particular area of interest. To assist the instructor in matching applications to a particular audience, we have organized applications sections according to their usefulness in business and economics or in the social and life sciences. Both Eighth Editions include many new applied exercises.

## Improvements to This Edition

We made a number of significant and important improvements within the Eighth Edition, many in response to user recommendations and to suggestions made by our reviewer panel.

### Enhanced Topic Coverage

Every section in the text underwent careful analysis and extensive review to ensure the most beneficial and clear presentation. For example, the treatment of rates and optimization has been improved by the addition of many new examples and exercises. The treatment of limits has been improved by the inclusion of limits at infinity in Chapter 1. A more detailed list of chapter-by-chapter changes can be found later in this preface.

### Effective Treatment of Integration

This edition features an improved treatment of integration. Definite integration is now introduced immediately after antidifferentiation and substitution in Chapter 5. A new section on modeling with the definite integral then follows. For maximum flexibility, applications of the definite integral are organized into separate sections for business and economics and for the life and social sciences. Chapter 6 extends the development of definite integration to include integration by parts as well as new coverage of integration using tables and numerical integration. The sections on integration by substitution and integration by parts have been extensively revised.

### Improved Exercise Sets

Several hundred new routine and applied exercises have been added to the already extensive problem sets. Based on feedback from users, routine exercises have been added where needed to ensure that students have more than enough practice to master basic skills. Furthermore, a wealth of new applied exercises has been added to help demonstrate the practicality of the material. These new exercises come from many fields of study, including economics, business, biology, medicine, and other areas of life science. A large number of new exercises involving integration have been added. Care has been taken to make sure that exercises use only updated data sets.

### Expanded Coverage of Applications to Economics

This edition expands on the already substantial treatment of economics in the previous edition. In particular, economic functions are introduced earlier in the text than before and are used more extensively in examples and exercises. There is also a new extended treatment of price elasticity of demand.

### Expanded Coverage of Applications to the Life Sciences

A key new feature is the expanded coverage of applications of calculus to the life sciences. Among the new topics covered are allometry, mathematical modeling of epidemics, diffusion of animal populations, and various applications from other branches of the biological sciences including physiology, medicine, nutrition, and ecology.

### Extensive Accuracy Checking

This new edition has benefited from an intensive effort to ensure its accuracy. The book has been reviewed by several technical reviewers who have checked examples, exercises, and computations. Not only has new material been extensively checked, but also material included from the previous edition has been double checked.

### Just-In-Time Reviews

A new feature of the Eighth Edition is the Just-in-Time Review, which is used to quickly remind students of important concepts from college algebra or precalculus as they are being used in examples and discussions. Each such review is placed in a box adjacent to the location in the text where the reviewed topic material is used. Extended discussion of these concepts can still be found within the Algebra Review appendix in the back of the text, but these quick reminders alert students to key facts without distracting from the material under discussion. Topics selected for these reviews have come from both classroom experience and reviewer suggestions.

### Mathematical Modeling

The role of mathematical modeling has been significantly expanded in the Eighth Edition. Modeling is introduced in Chapter 1 and appears as a recurrent theme throughout the text in applications involving both managerial and life science. The modeling theme is reinforced by many new applied examples and exercises dealing with rates, optimization, and the definite integral. To complement the coverage of modeling, data analysis is introduced early. In particular, least-squares approximation is previewed in Chapter 1 and then developed in more detail in Chapter 7. Chapter 7 also includes an introduction to nonlinear regression. Modeling with differential and difference equations is covered in Chapter 8. Using the trigonometric functions to model periodic phenomena is addressed in Chapter 11.

### Think About It Modeling Modules

Every chapter concludes with a new modeling-based Think About It essay. These essays are much more substantial than those found in the previous edition, and are designed to show students how material introduced in the chapter can be used to construct useful mathematical models. Taken together, the essays illustrate how models are constructed, how they give useful but only approximate solutions to problems, and how the process continues with the construction of more sophisticated models. The exercises following each essay are an excellent starting point for independent study and can be covered in the classroom or serve as the basis for group projects. Solutions to these exercises can be found at the text-specific website www.mhhe.com/hoffmann.

### Technology

Each Explore! box in the Eighth Edition has been updated to improve compatibility with current calculators, and rewritten to more clearly focus on specific examples within the text. Each chapter-ending Explore! Update section has been rewritten to provide more detailed hints and solutions to selected Explore! boxes.

## Graphing Calculator Orientation

New to the Eighth Edition is an extensive introduction to graphing calculators that appears on pages xxi through xxxii of this Preface. The material includes instructions regarding common calculator keystrokes, calculator terminology, and introductions to more advanced calculator applications that are developed in more detail at appropriate locations in the text. The Calculator Preface can serve as a primer for students unfamiliar with the use of graphing calculators, or as a guide to improved usage for others with some calculator experience.

## Expanded Author Team

To strengthen an already strong team, Kenneth H. Rosen contributed to the standard Eighth Edition and is a full co-author on this Expanded Eighth Edition. Ken has worked both in industry and in academia and is a noted author of computer books and textbooks, including McGraw-Hill's *Discrete Mathematics and its Applications,* Fifth Edition. Instructors particularly like his blend of concepts and relevant applications. His creativity, attention to detail, and industry experience make for a natural fit in expanding the applied focus of this text.

## Chapter-by-Chapter Changes

Users of the Seventh Edition of *Calculus for Business, Economics, and the Social and Life Sciences* will find the following detailed list of changes useful. This list highlights changes made to Chapters 1–7 of both Eighth Editions, and a few changes to Chapters 6 and 7 that are specific to this Expanded Eighth Edition. These improvements were made possible by reviewer and user feedback.

### Chapter 1: Functions, Graphs, and Limits

- Early introduction and use of economic functions.
- Preview of linear regression in Section 1.3 and a discussion of its use.
- Mathematical modeling introduced and discussed in Section 1.4.
- Limits involving infinity introduced with other limits in Section 1.5.
- One-sided limits introduced in Section 1.6 and used as part of a new graphical display illustrating the meaning of continuity and discontinuity.
- A new Think About It essay on Allometry.

### Chapter 2: Differentiation: Basic Concepts

- The chain rule introduced immediately after the product rule and quotient rule (in Section 2.4).
- Higher order derivatives introduced in a subsection of Section 2.3.
- Many new routine exercises and applied exercises involve rates.

### Chapter 3: Additional Applications of the Derivative

- Graphing procedure revised for easier use.
- Several graphing examples rewritten to improve clarity.
- A new list of procedures to assist students in solving optimization problems.
- An expanded discussion of price elasticity of demand.
- Many new routine and applied curve sketching and optimization problems.
- A new Think About It essay on Modeling AIDS Deaths (using regression).

### Chapter 4: Exponential and Logarithmic Functions

- Several new applied examples.
- Expanded discussion of logarithmic and logistic graphs.
- Many new routine and applied exercises, especially as they pertain to the life sciences.
- A New Think About It essay on An Arrhenius Model for Cricket Chirping.

### Chapter 5: Integration

- Reorganization of integration material. Chapter 5 begins with antidifferentiation and the method of substitution (as in the Seventh Edition), but now these topics are followed immediately by the definite integral and its applications. Integration by parts is now in Chapter 6.
- An extensively revised Section 5.2 on the method of substitution.
- Modeling with the definite integral is covered in Section 5.4, with applications of general interest such as area between curves, average value, and Lorentz curve analysis.
- The more specialized applications of the definite integral are split into two separate sections: Section 5.5 for business and economics and Section 5.6 for the life and social sciences.
- New applied examples, as well as extensively revised examples carried over from the Seventh Edition.
- Many routine and applied exercises added.
- A new Think About It essay on Just Noticeable Differences.

### Chapter 6: Additional Topics in Integration

- Extensively rewritten material on integration by parts in Section 6.1.
- New material on integration using tables.
- Enhanced coverage of material on improper integrals in Section 6.2.
- A new section on numerical integration that illustrates how the trapezoidal rule and Simpson's rule can be used to estimate definite integrals representing quantities such as area, average value, and present value of an income flow.
- Many additional routine and applied exercises.
- A new Think About It essay on Cardiac Output.
- The coverage of continuous probability has been moved to Chapter 9.

### Chapter 7: Calculus of Several Variables

- A new discussion and summary of the second-partials test.
- Expansion of the material on least-squares approximation into a full section.
- New material on nonlinear (log-linear) regression.
- A new Think About It essay on Modeling Population Diffusion.
- Expanded coverage now includes double integrals over nonrectangular regions.
- The application of double integrals to computing volumes has been added, as has the use of double integrals to compute average values of functions of two variables.

## Appendix

- Evaluating Limits with L'Hôpital's Rule is now covered in Appendix A.

Instructors familiar with *Calculus for Business, Economics, and the Social and Life Sciences,* Eighth Edition, who are interested in the chapters added to this Expanded Edition will find the following details helpful:

### Chapter 8: Differential Equations

This chapter is designed to teach students how to use differential equations, focusing on how to use them to study important applications to business, economics, and the life sciences. The basic notion of a differential equation is introduced, as are techniques for solving separable and first-order differential equations. Among the applications covered are logistic growth, the spread of epidemics, price adjustment models, financial portfolio modeling, and Newton's law of cooling. The approximate solution of differential equations using Euler's method is also addressed. Finally, this chapter introduces the basics of difference equations with application to loan amortization, fishery management, and learning models. The interaction between supply and demand is studied using a cobweb model. The chapter also explains the first steps in modeling using both differential and difference equations.

### Chapter 9: Infinite Series and Taylor Approximation

The role of this chapter is to convey to students the idea of a convergent infinite series and to show how infinite series are used in applications. Applications of infinite geometric series to economics and to the biological sciences are studied. The harmonic series and its application to the frequency with which records are broken (as in athletics) are introduced. Section 9.2 examines the convergence of series with positive terms using the integral and comparison tests, motivated with geometric reasoning. Finally, the notion of the approximation of functions using Taylor series is covered.

### Chapter 10: Probability and Calculus

The goal of this chapter is to develop the most important aspects of probability for students in business and economics and in the life sciences. The chapter begins by introducing discrete random variables, covering discrete probability density functions, histograms, expected values, and variables. Geometric random variables, and their application to topics such as product reliability, are covered in detail. Continuous random variables are introduced, and uniform and exponential density functions are thoroughly studied. Joint probability density functions are studied using double integrals. The expected value and variance of continuous random variables are motivated and defined. The chapter shows how to apply the normal distribution to study questions in business and the life sciences. The Poisson distribution is introduced, and its broad applications are also covered.

### Chapter 11: Trigonometric Functions

This chapter covers the most important aspects of the calculus of the trigonometric functions. After a brief overview of the trigonometric functions and their properties, differentiation and integration of these functions is covered. Finally, applications of the trigonometric functions, including applications involving periodicity, are studied. The first steps of modeling with trigonometric functions are also introduced.

# KEY FEATURES OF THIS TEXT

4-41 **SECTION 4.3** Differentiation of Logarithmic and Exponential Functions 321

If $Q(x)$ is a differentiable function of $x$, note that

$$\frac{d}{dx}(\ln Q) = \frac{Q'(x)}{Q(x)}$$

where the ratio on the right is the relative rate of change of $Q(x)$. That is, *the relative rate of change of a quantity $Q(x)$ can be computed by finding the derivative of* $\ln Q$. This special kind of logarithmic differentiation can be used to simplify the computation of various growth rates, as illustrated in Example 4.3.14.

**EXAMPLE 4.3.14**

A country exports three goods, wheat $W$, steel $S$, and oil $O$. Suppose at a particular time $t = t_0$, the revenue (in billions of dollars) derived from each of these goods is

$$W(t_0) = 4 \qquad S(t_0) = 7 \qquad O(t_0) = 10$$

and that $S$ is growing at 8%, $O$ is growing at 15%, while $W$ is declining at 3%. At what relative rate is total export revenue growing at this time?

**Solution**

Let $R = W + S + O$. At time $t = t_0$, we know that

$$R(t_0) = W(t_0) + S(t_0) + O(t_0) = 4 + 7 + 10 = 21$$

and

$$\frac{W'(t_0)}{W(t_0)} = -0.03 \qquad \frac{S'(t_0)}{S(t_0)} = 0.08 \qquad \frac{O'(t_0)}{O(t_0)} = 0.15$$

so that

$$W'(t_0) = -0.03W(t_0) \qquad S'(t_0) = 0.08S(t_0) \qquad O'(t_0) = 0.15\,O(t_0)$$

Thus, at $t = t_0$, the relative rate of growth of $R$ is

$$\frac{R'(t_0)}{R(t_0)} = \frac{d(\ln R)}{dt} = \frac{d}{dt}[\ln(W + S + O)]\Big|$$

## Applications

Throughout the text great effort is made to ensure that techniques are applied to practical problems soon after their introduction, providing methods for dealing with both routine computations and applied problems. These problem-solving methods and strategies are introduced in applied examples and practiced throughout in the exercise sets. Many new applied examples were added to this Eighth Edition from a wide variety of sources with careful attention to obsolete or outdated data.

2-69 **SECTION 2.6** Implicit Differentiation and Related Rates 163

**A Procedure for Solving Related Rates Problems**

1. Draw a figure (if appropriate) and assign variables.
2. Find a formula relating the variables.
3. Use implicit differentiation to find how the rates are related.
4. Substitute any given numerical information into the equation in step 3 to find the desired rate of change.

Here are four applied problems involving related rates.

**EXAMPLE 2.6.5**

The manager of a company determines that when $q$ hundred units of a particular commodity are produced, the total cost of production is $C$ thousand dollars, where $C^2 - 3q^3 = 4{,}275$. When 1,500 units are being produced, the level of production is increasing at the rate of 20 units per week. What is the total cost at this time and at what rate is it changing?

**Solution**

We want to find $\frac{dC}{dt}$ when $q = 15$ (1

Differentiating implicitly in the eq

we get

## Procedural Examples and Boxes

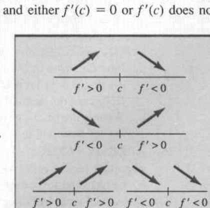
This text endeavors to approach each new topic with careful clarity by providing step-by-step problem-solving techniques. These techniques are demonstrated in the numerous procedural examples working through these problems, as well as in frequent procedural summary boxes highlighting the techniques demonstrated.

**The First Derivative Test for Relative Extrema** ■ Let $c$ be a critical number for $f(x)$ [that is, $f(c)$ is defined and either $f'(c) = 0$ or $f'(c)$ does not exist]. Then the critical point $P(c, f(c))$ is

**A relative maximum** if $f'(x) > 0$ to the left of $c$ and $f'(x) < 0$ to the right of $c$

**A relative minimum** if $f'(x) < 0$ to the left of $c$ and $f'(x) > 0$ to the right of $c$

**Not a relative extremum** if $f'(x)$ has the same sign on both sides of $c$




**EXAMPLE 3.1.3**

Find all critical numbers of the function

$$f(x) = 2x^4 - 4x^2 + 3$$

and classify each critical point as a relative maximum, a relative minimum, or neither.

## Definitions

Definitions and key concepts are set off in shaded boxes to provide easy referencing for the student.

## SECTION 3.3 | Curve Sketching

REVIEW

Just-In-Time

It is important to remember that $\infty$ is *not* a number. It is used only to represent a process of unbounded growth or the result of such growth.

So far in this chapter, you have seen how to use the derivative $f'(x)$ to determine where the graph of $f(x)$ is rising and falling and to use the second derivative $f''(x)$ to determine the concavity of the graph. While these tools are adequate for locating the high and low points of a graph and for sculpting its twists and turns, there are other graphical features that are best described using limits.

Recall from Section 1.5 that a limit of the form $\lim_{x\to+\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$, in which the independent variable $x$ either increases or decreases without bound, is called a *limit at infinity*. On the other hand, if the functional values $f(x)$ themselves grow without bound as $x$ approaches a number $c$, we say that $f(x)$ has an *infinite limit* at $x = c$ and write $\lim_{x\to c} f(x) = +\infty$ if $f(x)$ increases indefinitely as $x$ approaches $c$ or $\lim_{x\to c} f(x) = -\infty$ if $f(x)$ decreases indefinitely. Collectively, limits at infinity and infinite limits are referred to as **limits involving infinity.** Our first goal in this section is to see how limits involving infinity may be interpreted as graphical features. This information will then be combined with the derivative methods of Sections 3.1 and 3.2 to form a general procedure for sketching graphs.

## Just-In-Time Review

New to this edition, these references are used to quickly remind students of important concepts from college algebra or precalculus without distracting from the material under discussion. More detailed discussion can still be found in Appendix A: Algebra Review.

## Exercises

Always a strong feature of past editions, the Expanded Eighth Edition offers over 1,000 new exercises to increase the effectiveness of these exercise sets. Routine exercises have been added where needed to ensure students have enough practice to master basic skills, and a wealth of new applied exercises have been added to help demonstrate the practicality of the material. Answers for the odd-numbered end-of-section and Chapter Review problems can be found at the back of the text, with full solutions provided in the *Student Solutions Manual*.



*In Problems 45 through 48, the derivative of a function f(x) is given. In each case, find the critical numbers of f(x) and classify each as corresponding to a relative maximum, a relative minimum, or neither.*

**45.** $f'(x) = x^2(4 - x^2)$

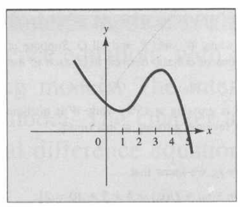
**46.** $f'(x) = \dfrac{x(2 - x)}{x^2 + x + 1}$

**47.** $f'(x) = \dfrac{(x + 1)^2(4 - 3x)^3}{(x^2 + 1)^2}$

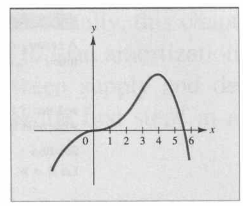
**48.** $f'(x) = x^3(2x - 7)^2(x + 5)$

*In Problems 49 through 52, the graph of a function f is given. In each case, sketch a possible graph for f'.*

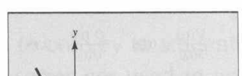
**49.**



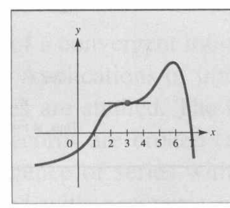

**50.**




**51.**




**52.**

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...n Model of Population ...melia," Ecological

**54. MARGINAL ANALYSIS** The total cost of producing $x$ units of a certain commodity is given by $C(x) = \sqrt{5x + 2} + 3$. Sketch the cost curve and find the marginal cost. Does marginal cost increase or decrease with increasing production?

**55. MARGINAL ANALYSIS** Let $p = (10 - 3x)^2$ for $0 \le x \le 3$ be the price at which $x$ hundred units of a certain commodity will be sold, and let $R(x) = xp(x)$ be the revenue obtained from the sale of the $x$ units. Find the marginal revenue $R'(x)$ and sketch the revenue and marginal revenue curves on the same graph. For what level of production is revenue maximized?



**43. ORNITHOLOGY** Recall from Problem 60 in Section 2.4 that according to the results* of Tucker and Schmidt-Koenig, the energy expended by a certain species of parakeet is given by

$$E(v) = \frac{1}{v}[0.074(v - 35)^2 + 22]$$

where $v$ is the bird's velocity (in km/hr).

a. What velocity minimizes energy expenditure?

b. Read an article on how mathematical methods can be used to study animal behavior and write a paragraph on whether you think such methods are valid. You may wish to begin with the reference cited in this problem.

**44. NATIONAL CONSUMPTION** Assume that total national consumption is given by a function $C(x)$, where $x$ is the total national income. The derivative $C'(x)$ is called the **marginal propensity to consume.** Then $S = x - C$ represents total national savings, and $S'(x)$ is called **marginal propensity to save.** Suppose the consumption function is $C(x) = 8 - 0.8x - 0.8\sqrt{x}$. Find the marginal propensity to consume and determine the value of $x$ that results in the smallest total savings.

**45. SENSITIVITY TO DRUGS** Body reaction to drugs is often modeled** by an equation of the form

$$R(D) = D^2\left(\frac{C}{2} - \frac{D}{3}\right)$$

where $D$ is the dosage and $C$ (a constant) is the maximum dosage that can be given. The rate of change of $R(D)$ with respect to $D$ is called the **sensitivity.**

a. Find the value of $D$ for which the sensitivity is the greatest. What is the greatest sensitivity? (Express your answer in terms of $C$.)

b. What is the reaction (in terms of $C$) when the dosage resulting in greatest sensitivity is used?

**46. AERODYNAMICS** In designing airplanes, an important feature is the so-called "drag factor"; that is, the retarding force exerted on the plane by the air. One model measures drag by a function of the form

$$F(v) = Av^2 + \frac{B}{v^2}$$

where $A$ and $B$ are positive constants. It is found experimentally that drag is minimized when $v =$ 160 mph. Use this information to find the ratio $\frac{B}{A}$.

**47. ELECTRICITY** When a resistor of $R$ ohms is connected across a battery with electromotive force $E$ volts and internal resistance $r$ ohms, a current of $I$ amperes will flow, generating $P$ watts of power, where

$$I = \frac{E}{r + R} \quad \text{and} \quad P = I^2R$$

Assuming $r$ is constant, what choice of $R$ results in maximum power?

**48. SURVIVAL OF AQUATIC LIFE** It is known that a quantity of water that occupies 1 liter at 0°C will occupy

$$V(T) = \left(\frac{-6.8}{10^8}\right)T^3 + \left(\frac{8.5}{10^6}\right)T^2 - \left(\frac{6.4}{10^5}\right)T + 1$$

liters when the temperature is $T$°C, for $0 \le T \le 30$.

a. Use a graphing utility to graph $V(T)$ for $0 \le T \le 10$. The density of water is maximized when $V(T)$ is minimized. At what temperature does this occur?

b. Does the answer to (a) surprise you? It should. Water is the only common liquid whose maximum density occurs *above* its freezing point (0°C for water). Read an article on the survival of aquatic life during the winter and then write a paragraph on how the property of water examined in this problem is related to such survival.

**49. BLOOD PRODUCTION** Recall from Problem 60 in Section 2.3 that a useful model for the production $p(x)$ of blood cells involves a function of the form

$$p(x) = \frac{Ax}{B + x^m}$$

where $x$ is the number of cells present; and $A$, $B$, and $m$ are positive constants.[†]

a. Find the rate of blood production $R(x) = p'(x)$ and determine where $R(x) = 0$.

b. Find the rate at which $R(x)$ is changing with respect to $x$ and determine where $R'(x) = 0$.

c. If $m > 1$, does the non-zero critical number you found in part (b) correspond to a relative maximum or a relative minimum. Explain.

*V. A. Tucker and K. Schmidt-Koenig, "Flight Speeds of Birds in Relation to Energetics and Wind Directions," *The Auk*, Vol. 88 (1971), pages 97–107.

**R. M. Thrall et al., *Some Mathematical Models in Biology*, U. of Michigan, 1967.

[†]M. C. Mackey and L. Glass, "Oscillations and Chaos in Physiological Control Systems," *Science*, Vol. 197, pages 287–289.

## Writing Exercises

Every exercise set includes writing problems that are related to issues raised in the examples and exercises, designated by the writing icon. These problems challenge a student's critical thinking skills and invite students to research topics on their own, exploring the language of mathematics.

## Calculator Exercises

Each section includes numerous problems that can be completed only through the use of a graphing utility. These exercises are clearly labeled by the calculator icon.

## Chapter Review

Essential to a student's understanding is the Chapter Review material that aids in synthesizing the important concepts discussed within the chapter. The Important Terms, Symbols, and Formulas provides a master list of all key technical terms and mathematics discussed throughout the chapter.



### Important Terms, Symbols, and Formulas

CHAPTER SUMMARY

Function (2)
Functional notation: $f(x)$ (2)
Domain and range of a function (2)
Domain convention (4)
Independent and dependent variables (3)
Functions used in economics:
- Demand (5)
- Supply (47)
- Cost (5)
- Revenue (5)
- Profit (5)

Composition of functions: $g(h(x))$ (6)
Graph of a function: the points $(x, f(x))$ (15)
$x$ and $y$ intercepts (17)
Piecewise-defined functions (4)
Power function (20)
Polynomial (20)

Joint proportionality: $Q = kxy$ (45)
Market equilibrium; law of supply and demand (47)
Shortage and surplus (47)
Break-even analysis (48)
Limit of a function: $\lim_{x \to c} f(x)$ (58)
Limits at infinity:
$$\lim_{x \to +\infty} f(x) \text{ and } \lim_{x \to -\infty} f(x) \quad (64)$$
Reciprocal power rules:
$$\lim_{x \to +\infty} \frac{A}{x^k} = 0 \text{ and } \lim_{x \to -\infty} \frac{A}{x^k} = 0 \quad k > 0 \quad (65)$$
Limits at infinity of a rational function $f(x) = \frac{p(x)}{q(x)}$:
Divide all terms in $f(x)$ by the highest power $x^k$ in the denominator $q(x)$ and use the reciprocal power rules. (66)
Horizontal asympt
Infinite limit: $\lim_{x \to c}$

## Chapter Checkup

A new Chapter Checkup provides a quick quiz for students to test their understanding of the concepts introduced in the chapter. All answers to this quiz are provided in the end-of-text Answer Key.

### Checkup for Chapter 2

1. In each case, find the derivative $\frac{dy}{dx}$.
   a. $y = 3x^4 - 4\sqrt{x} + \frac{5}{x^2} - 7$
   b. $y = (3x^3 - x + 1)(4 - x^2)$
   c. $y = \frac{5x^2 - 3x + 2}{1 - 2x}$
   d. $y = (3 - 4x + 3x^2)^{3/2}$
2. Find the second derivative of the function $f(t) = t(2t + 1)^2$.
3. Find an equation for the tangent line to the curve $y = x^2 - 2x + 1$ at the point where $x = -1$.
4. Find the rate of change of the function $f(x) = \frac{x + 1}{1 - 5x}$ with respect to $x$ when $x = 1$.

   b. When is the object stationary? When is it advancing? Retreating?
   c. What is the total distance traveled by the object for $0 \le t \le 2$?
7. **PRODUCTION COST** Suppose the cost of producing $x$ units of a particular commodity is $C(x) = 0.04x^2 + 5x + 73$ hundred dollars.
   a. Use marginal cost to estimate the cost of producing the sixth unit.
   b. What is the actual cost of producing the sixth unit?
8. **INDUSTRIAL OUTPUT** At a certain factory, the daily output is $Q = 500L^{3/4}$ units, where $L$ denotes the size of the labor force in worker-hours. Currently, 2,401 worker-hours of labor are used each day. Use calculus (increments) to estimate the effect on output of increasing the size of the labor force by 200 worker-hours from its current level.
9. **PEDIATRIC MEASUREMENT** Pediatricians use the formula $S = 0.2029w^{0.425}$ to estimate the surface area $S$ ($m^2$) of a child 1 meter tall who weighs $w$ kilograms (kg). A particular child weighs 30 kg and is gaining weight at the rate of 0.13 kg per week while remaining 1 meter tall. At what rate is this child's surface area changing?
10. **GROWTH OF A TUMOR** A cancerous tumor is roughly spherical in shape. Estimate the percentage error that can be allowed in the measurement of the radius $r$ to ensure there will be no more than an 8% error in the calculation of volume using the formula $V = \frac{4}{3}\pi r^3$.

### Review Problems

CHAPTER SUMMARY

*In Problems 1 and 2, use the definition of the derivative to find $f'(x)$.*

1. $f(x) = x^2 - 3x + 1$
2. $f(x) = \frac{1}{x - 2}$

*In Problems 3 through 13, find the derivative of the given function.*

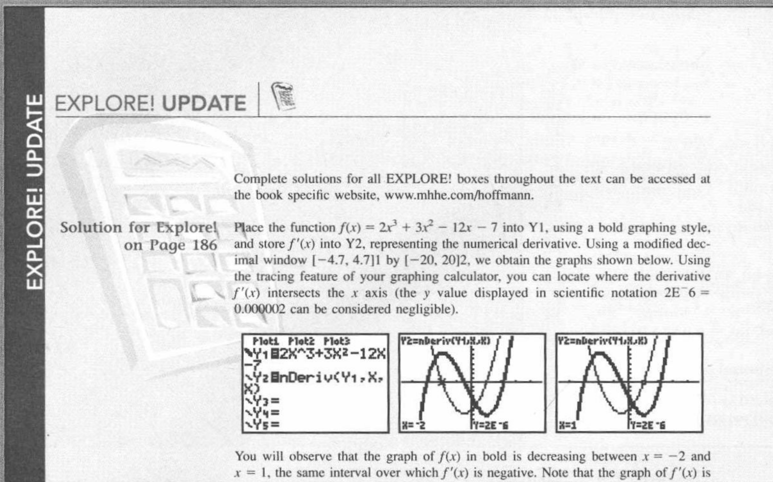
3. $f(x) = 6x^4 - 7x^3 + 2x + \sqrt{2}$
4. $f(x) = x^3 - \frac{1}{3x^5} + 2\sqrt{x} - \frac{3}{x} + \frac{1 - 2x}{x^3}$
5. $y = \frac{2 - x^2}{3x^2 + 1}$
6. $y = (x^3 + 2x - 7)(3 + x - x^2)$
7. $f(x) = (5x^4 - 3x^2 + 2x + 1)^{10}$
8. $f(x) = \sqrt{x^2 + 1}$
9. $y = \left(x + \frac{1}{x}\right)^2 - \frac{5}{\sqrt{3x}}$
10. $y = \left(\frac{x + 1}{1 - x}\right)^2$
11. $f(x) = (3x + 1)\sqrt{6x + 5}$
12. $f(x) = \frac{(3x + 1)^3}{(1 - 3x)^4}$
13. $y = \sqrt{\frac{1 - 2x}{3x + 2}}$

*In Problems 14 through 17, find an equation for the tangent line to the graph of the given function at the specified point.*

14. $f(x) = x^2 - 3x + 2; x = 1$
15. $f(x) = \frac{4}{x - 3}; x = 1$
16. $f(x) = \frac{x}{x^2 + 1}; x = 0$
17. $f(x) = \sqrt{x^2 + 5}; x = -2$
18. In each of these cases, find the rate of change of $f(t)$ with respect to $t$ at the given value of $t$.
    a. $f(t) = t^3 - 4t^2 + 5t\sqrt{t} - 5$ at $t = 4$
    b. $f(t) = \frac{2t^2 - 5}{1 - 3t}$ at $t = -1$
    c. $f(t) = t^3(t^2 - 1)$ at $t = 0$
    d. $f(t) = (t^2 - 3t + 6)^{1/2}$ at $t = 1$
19. In each of these cases, find the percentage rate of change of the function $f(t)$ with respect to $t$ at the given value of $t$.
    a. $f(t) = t^2 - 3t + \sqrt{t}$ at $t = 4$
    b. $f(t) = t^2(3 - 2t)^3$ at $t = 1$
    c. $f(t) = \frac{1}{t + 1}$ at $t = 0$
20. Use the chain rule to find $\frac{dy}{dx}$.
    a. $y = 5u^2 + u - 1; u = 3x + 1$
    b. $y = \frac{1}{u^2}; u = 2x + 3$
21. Use the chain rule to find $\frac{dy}{dx}$ for the given value of $x$.
    a. $y = u^3 - 4u^2 + 5u + 2; u = x^2 + 1$; for $x = 1$
    b. $y = \sqrt{u}, u = x^2 + 2x - 4$; for $x = 2$
22. Find the second derivative of each of these functions:
    a. $f(x) = 6x^5 - 4x^3 + 5x^2 - 2x + \frac{1}{x}$
    b. $z = \frac{2}{1 + x^2}$
    c. $y = (3x^2 + 2)^4$
    d. $f(x) = 2x(x + 4)^3$
    e. $f(x) = \frac{x - 1}{(x + 1)^2}$
23. Find $\frac{dy}{dx}$ by implicit differentiation.
    a. $5x + 3y = 12$
    b. $x^2y = 1$
    c. $(2x + 3y)^5 = x + 1$
    d. $(1 - 2xy^3)^5 = x + 4y$
24. Use implicit differentiation to find the slope of the line that is tangent to the given curve at the specified point.
    a. $xy^3 = 8$; (1, 2)
    b. $x^2y - 2xy^3 + 6 = 2x + 2y$; (0, 3)

## Review Problems

A wealth of additional routine and applied problems are provided within the end-of-chapter exercise sets, offering further opportunities for practice.


EXPLORE! UPDATE

Complete solutions for all EXPLORE! boxes throughout the text can be accessed at the book specific website, www.mhhe.com/hoffmann.

Solution for Explore! on Page 186

Place the function $f(x) = 2x^3 + 3x^2 - 12x - 7$ into Y1, using a bold graphing style, and store $f'(x)$ into Y2, representing the numerical derivative. Using a modified decimal window $[-4.7, 4.7]1$ by $[-20, 20]2$, we obtain the graphs shown below. Using the tracing feature of your graphing calculator, you can locate where the derivative $f'(x)$ intersects the $x$ axis (the $y$ value displayed in scientific notation $2E^{-}6 = 0.000002$ can be considered negligible).

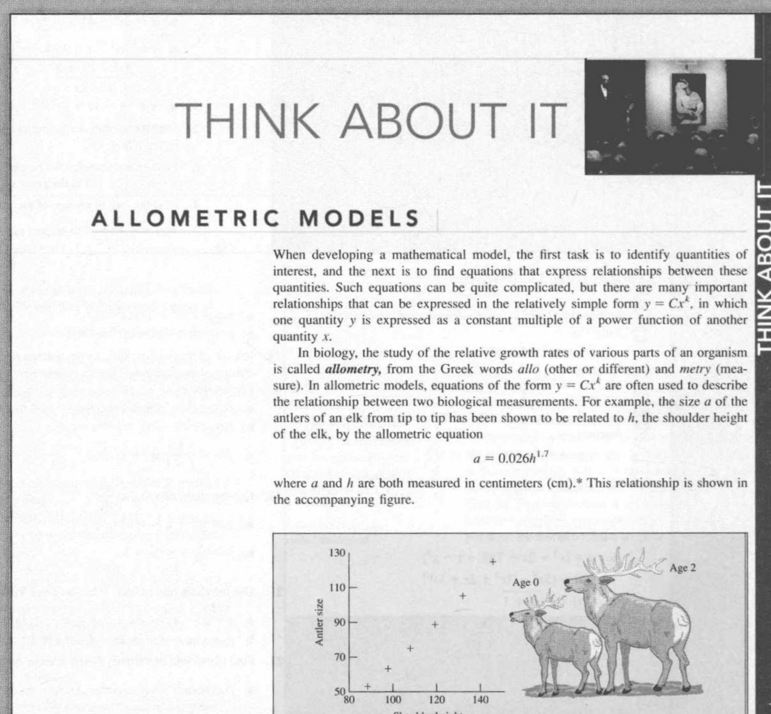
You will observe that the graph of $f(x)$ in bold is decreasing between $x = -2$ and $x = 1$, the same interval over which $f'(x)$ is negative. Note that the graph of $f'(x)$ is

## Technology

For those choosing to include a graphing focus in their course, the Explore! boxes provide an optional path through the material. These explorations, tied to specific examples, continue to guide students in the use of graphing calculators and challenge their understanding of the topics presented. Each chapter concludes with an Explore! Update section that provides solutions and hints to selected boxes throughout the chapter.

## Think About It

The modeling-based Think About It essays show students how material introduced in the chapter can be used to construct useful mathematical models while explaining the modeling process. The exercises following each essay are an excellent starting point for student projects and can be covered in the classroom or serve as the basis for group projects.


THINK ABOUT IT

ALLOMETRIC MODELS

When developing a mathematical model, the first task is to identify quantities of interest, and the next is to find equations that express relationships between these quantities. Such equations can be quite complicated, but there are many important relationships that can be expressed in the relatively simple form $y = Cx^k$, in which one quantity $y$ is expressed as a constant multiple of a power function of another quantity $x$.

In biology, the study of the relative growth rates of various parts of an organism is called ***allometry***, from the Greek words *allo* (other or different) and *metry* (measure). In allometric models, equations of the form $y = Cx^k$ are often used to describe the relationship between two biological measurements. For example, the size $a$ of the antlers of an elk from tip to tip has been shown to be related to $h$, the shoulder height of the elk, by the allometric equation

$$a = 0.026h^{1.7}$$

where $a$ and $h$ are both measured in centimeters (cm).* This relationship is shown in the accompanying figure.

## Supplements

### Student Solutions Manual

The *Student Solutions Manual* accompanying this eighth edition has been fully revised with all new art and more detailed explanations behind the solutions. This manual contains comprehensive, worked-out solutions for all odd-numbered problems in the text (ISBN 0-07-301951-8).

### Instructor's Solutions Manual

The *Instructor's Solutions Manual* accompanying this eighth edition has been fully revised with all new art and contains comprehensive, worked-out solutions to all even-numbered problems in the text (ISBN 0-07-301952-6).

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### Course Management Systems

- *WebCT and Blackboard*—With help from our partners, WebCT and Blackboard, professors can take complete control over their course content. These course cartridges also provide online testing and powerful student-tracking features.

## Acknowledgments

As in past editions, we have enlisted the feedback of professors teaching from our text as well as those using other texts to point out possible areas for improvement. Our extensive reviewer panel provided a wealth of detailed information on both our content and the changing needs of their course. Many changes made in this eighth edition are a direct result of a consensus among this panel. We thank every individual involved in this process.

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