

A Geometry Toolbox

实用线性代数

(图解版)

Practical Linear Algebra

[美] Gerald Farin 著
Dianne Hansford



机械工业出版社
CHINA MACHINE PRESS

国外优秀数学教材系列

实用线性代数（图解版）

[美] Gerald Farin 著
Dianne Hansford

李红玲 注释



机械工业出版社

本书区别于以往线性代数的书籍,内容新颖,编排独特。作者以几何视角讲述线性代数,通过二维平面和三维空间中的例子解释线性代数中的各种概念和性质。本书强调直观性以及知识点的背景,结合计算机中各种图形的变换来理解线性变换,在注重可读性的同时突出数学的基本思想,将直观图形与数学证明进行了巧妙的结合。作者在书页侧边空白处手绘 200 余幅示意图并给出了相关概念的解释,以便更好地帮助读者理解。

本书可供非数学专业的学生及数学爱好者使用,亦可作为数学专业学生和教师的参考用书。

Practical Linear Algebra: A Geometry Toolbox/by Gerald Farin and Dianne Hansford /
ISBN:9781568812342

Copyright © 2005 by CRC Press.

Authorized licensed edition from English language edition published by CRC Press, part of Taylor & Francis Group LLC; All rights reserved.

本书英文注释影印版授权由机械工业出版社独家出版并限在中国大陆地区销售。未经出版者书面许可,不得以任何方式复制或发行本书的任何部分。

Copies of this book sold without a Taylor & Francis on the cover are unauthorized and illegal.

本书封面贴有Taylor & Francis公司防伪标签,无标签者不得销售。

北京市版权局著作权合同登记 图字:01-2013-5530号

图书在版编目(CIP)数据

实用线性代数:图解版:英文/(美)杰拉德(Farin, G.)
(美)戴安娜(Hansford, D.)著;李红玲注释. —北京:机械工业出版社, 2014.9

国外优秀数学教材系列

ISBN 978-7-111-47334-3

I. ①实… II. ①杰… ②戴… ③李… III. ①线性代数-高等学校-教材-英文 IV. ①O151.2

中国版本图书馆 CIP 数据核字(2014)第 152784 号

机械工业出版社(北京市百万庄大街 22 号 邮政编码 100037)

策划编辑:汤嘉 责任编辑:汤嘉 姜凤

封面设计:张静 责任校对:张薇

责任印制:刘岚

北京京丰印刷厂印刷

2014 年 10 月第 1 版第 1 次印刷

184mm×242mm·27.5 印张·491 千字

标准书号:ISBN 978-7-111-47334-3

定价:56.00 元

凡购本书,如有缺页、倒页、脱页,由本社发行部调换

电话服务

网络服务

社服务中心:(010) 88361066 教材网:<http://www.cmpedu.com>

销售一部:(010) 68326294 机工官网:<http://www.cmpbook.com>

销售二部:(010) 88379649 机工官博:<http://weibo.com/cmp1952>

读者购书热线:(010) 88379203 封面无防伪标均为盗版

注释者的话

对于工科与经济类的大学生而言，“线性代数”是一门公共基础课，也是必修的课程之一。常见的线性代数教科书大致有这样几个章节——行列式、线性方程组、矩阵、向量空间、特征值与特征向量、二次型。翻开教材，迎面而来的就是计算与证明，鲜有知识点产生的缘由及其在实际生活中的应用。诚然，数学是一门抽象的科学，它具有高度的概括性，但这不代表数学教材就应该这样“斩头去尾烧中段”，干巴巴得毫无吸引力，冷冰冰得让人生畏。数学教育研究者们一直在呼吁数学文化的渗透，那么对于具体的一本教材而言，渗透什么内容？如何进行渗透？渗透到什么程度？我想，最基本地来说，至少要将知识的来龙去脉说清楚。比如常见教科书开篇就是行列式的计算，那么学生肯定想要知道“行列式的本质是什么？为什么要学行列式？它在实际应用中的作用是什么？”在缺乏理解的基础上，就算学会了计算与证明，对知识的把握也是稀里糊涂的。这是件让人遗憾的事情。

本书，是一本弥补遗憾的书，是一本不同视角的书，是一本呈现知识点来龙去脉的书。本书按照先二维后三维的顺序呈现知识，使得知识形象化，便于理解。全书共分18章，第1章到第9章是二维情形，以独特的顺序与适宜的方式介绍了线性代数的基本知识点；第10章到第13章是三维情形，因此这四章是前九章的推广，但并不重复，各有侧重；第14章到第18章是四维情形，呈现了许多实际生活中的应用，同时也有助于读者抽象思维的发展。本书采用了非常规却符合认知的知识呈现顺序，以直观的、几何的叙述方式呈现内容，以大量的实际例证呈现应用。如果你不曾学过线性代数，阅读本书，会让你兴趣盎然地沉浸在其中，顺理成章地掌握所有应掌握的知识点；如果你曾经学过线性代数，阅读本书，会让你不断地恍然大悟：“哦！原来这个知识点是这么来的！原来这个知识点是这么用的！原来这两个知识点之间竟然有这层关系！”如果你想要看一本有趣且有用的线性代数书籍，那么本书就是一个不可错过的上好选择！

作为注释者，必须要兼顾到两个方面：第一是作者方面——注释者必须要掌握书中的内容，理解作者的意图，跟随作者的思路，体会各种安排的妙处；第二是读者方面——注释者必须站在读者的角度去学习、去思考、去询问、去解答，才会知道哪些点是需要注释的。通过与编辑的交流，我们统一了标准：既要给读者有疑惑的地方给予注释，同时又要注意到不可以越线——注释不应该代替读者的学习，所以注释中尽量只提出解决疑惑的方法与工具，具体的过程仍然留给读者自行完成。如有读者想要讨论该书的注释或内容，请发邮件至 sxslshl@126.com，愿与君同思考共探索！

李红玲

Preface

We assume just about everyone has watched animated movies, such as *Toy Story* or *Shrek*, or is familiar with the latest three-dimensional computer games. Enjoying 3D entertainment sounds like more fun than studying a Linear Algebra book, right? But it is because of Linear Algebra that those movies and games can be brought to a TV or computer screen. When you see a character move on the screen, it's animated using some equation straight out of this book. In this sense, Linear Algebra is a driving force of our new digital world: it is powering the software behind modern visual entertainment and communication.

But this is not a book on entertainment. We start with the fundamentals of Linear Algebra and proceed to various applications. So it doesn't become too dry, we replaced mathematical proofs with motivations, examples, or graphics. For a beginning student, this will result in a deeper level of understanding than standard theorem-proof approaches. The book covers all of undergraduate-level linear algebra in the classical sense—except it is not delivered in a classical way. Since it relies heavily on examples and pointers to applications, we chose the title *Practical Linear Algebra*, or *PLA* for short.

The subtitle of this book is *A Geometry Toolbox*; this is meant to emphasize that we approach linear algebra in a geometric and algorithmic way. This book grew out of a previous one, namely *The Geometry Toolbox for Graphics and Modeling*. Our goal was to bring the material of that book to a broader audience, motivated in a large part by our observations of how little engineers and scientists (non-math majors) retain from classical linear algebra classes. Thus, we set out to fill a void in the linear algebra textbook market. We feel

that we have achieved this, as well as maintaining the spirit of our first effort: present the material in an intuitive, geometric manner that will lend itself to retention of the ideas and methods.

Review of Contents

As stated previously, one clear motivation we had for writing PLA was to present the material so that the reader would retain the information. In our experience, approaching the material first in two and then in three dimensions lends itself to visualizing and then to understanding. Incorporating many illustrations, Chapters 1–7 introduce the fundamentals of linear algebra in a 2D setting. These same concepts are revisited in Chapters 10–13 in a 3D setting. The 3D world lends itself to concepts that do not exist in 2D, and these are explored there too.

Higher dimensions, necessary for many real-life applications and the development of abstract thought, are visited in Chapters 14–16. The focus of these three chapters includes linear system solvers (Gauss elimination, LU decomposition, Householder’s method, and iterative methods), determinants, inverse matrices, revisiting “eigen things,” linear spaces, inner products, and the Gram-Schmidt process.

Conics, discussed in Chapter 9, are such a fundamental geometric entity, and since their development provides a wonderful application for affine maps, “eigen things,” and symmetric matrices, they really shouldn’t be missed. Triangles in Chapter 8 and polygons in Chapter 17 are discussed because they are fundamental geometric entities and are important in generating computer images. The basics of generating curves are presented in Chapter 18; this, too, is a nice example of how linear algebra may be applied.

The illustrations in the book come in two forms: figures and sketches. The figures are computer generated and tend to be complex. The sketches are hand-drawn and illustrate the core of a concept. Both are great teaching and learning tools! We made all of them available on the book’s website (<http://vidya.prism.asu.edu/~farin/pla>). Many of the figures were generated using PostScript, an easy-to-use geometric language. We have provided a tutorial to this language in Appendix A. This brief tutorial gives enough information for the reader to modify the figures in this book, as well as create their own. However, this book can certainly be used without getting involved with PostScript.

At the end of each chapter, we have included a list of topics, *What You Should Know* (WYSK), marked by the icon on the left. This list



is intended to encapsulate the main points of each chapter. It is not uncommon for a topic to appear in more than one chapter. We have made an effort to revisit some key ideas more than once. Repetition is useful for retention!

Exercises are listed at the end of each chapter. Solutions to selected exercises are given in Appendix B. More solutions may be found on the book's website. PostScript figures are also used as a basis for some exercises—PostScript offers an easy way to get hands-on experience with many geometry concepts. Often these exercises are labeled *PS* to indicate that they involve PostScript.

Classroom Use

PLA is meant to be used at the freshman/sophomore undergraduate level. It serves as an introduction to Linear Algebra for engineers or computer scientists, as well as a general introduction to geometry. It is also an ideal preparation for Computer Graphics and Geometric Modeling.

As a one-semester course, we recommend choosing a subset of the material that meets the needs of the students. In the table below, LA refers to an introductory Linear Algebra course and CG refers to a course tailored to those planning to work in Computer Graphics or Geometric Modeling. We have created shortened chapter titles to make the table more readable.

	Chapter	LA	CG
1	2D Coordinate Systems	•	•
2	2D Points & Vectors	•	•
3	2D Lines		•
4	2D Linear Maps	•	•
5	2×2 Linear Systems	•	•
6	2D Affine Maps	•	•
7	2D Eigen Things	•	
8	Triangles		•
9	Conics	•	
10	3D Geometry	•	•
11	3D Interactions		•
12	3D Linear Maps	•	•

	Chapter	LA	CG
13	3D Affine Maps	•	•
14	Linear Systems	•	•
15	Linear Spaces	•	
16	Numerical Methods	•	
17	Polygons		•
18	Curves		•

Website

Practical Linear Algebra, A Geometry Toolbox has a website:
<http://vidya.prism.asu.edu/~farin/pla>.

This website provides:

- teaching materials,
- additional solutions to exercises,
- the PostScript files illustrated in the book,
- data files referred to in the text,
- errata,
- and more!

A K Peters, Ltd. also maintains a website:
<http://www.akpeters.com>.

Acknowledgements

We certainly appreciate the positive feedback that we received on *The Geometry Toolbox for Graphics and Modeling*. Some of that feedback may be found on ‘The Toolbox’s’ website.

A few 3D images were generated using GEOMVIEW from The Geometry Center at the University of Minnesota, <http://www.geom.uiuc.edu/>. (It is now closed.)

Lastly, thanks to the great team at A K Peters! They are a pleasure to work with.

Gerald Farin
Dianne Hansford

August 2004
Arizona State University

目 录

注释者的话

前言

笛卡儿的发现	1	第 1 章
1.1 二维平面中局部坐标与整体坐标的互化	2	
1.2 整体坐标到局部坐标的转化	6	
1.3 三维空间中局部坐标与整体坐标的互化	8	
1.4 单位框外一点坐标的转化	9	
1.5 建立坐标系	10	
1.6 习题	12	
无处不在：二维平面中的点与向量	13	第 2 章
2.1 点与向量的坐标及运算	14	
2.2 点与向量的区别	16	
2.3 向量场	17	
2.4 向量的长度	18	
2.5 点的组合	21	
2.6 线性无关	24	
2.7 标量积	24	
2.8 正交投影	28	
2.9 不等式	29	
2.10 习题	30	
排列起来：二维平面上的直线	33	第 3 章
3.1 直线的定义	34	
3.2 直线的参数方程	35	
3.3 直线的隐式方程	37	

Contents

Preface

Descartes' Discovery	1
1.1 Local and Global Coordinates: 2D	2
1.2 Going from Global to Local	6
1.3 Local and Global Coordinates: 3D	8
1.4 Stepping Outside the Box	9
1.5 Creating Coordinates	10
1.6 Exercises	12

Chapter 1

Here and There: Points and Vectors in 2D	13
2.1 Points and Vectors	14
2.2 What's the Difference?	16
2.3 Vector Fields	17
2.4 Length of a Vector	18
2.5 Combining Points	21
2.6 Independence	24
2.7 Dot Product	24
2.8 Orthogonal Projections	28
2.9 Inequalities	29
2.10 Exercises	30

Chapter 2

Lining Up: 2D Lines	33
3.1 Defining a Line	34
3.2 Parametric Equation of a Line	35
3.3 Implicit Equation of a Line	37

Chapter 3

3.4 直线的显式方程	40	
3.5 参数方程与隐式方程的互化	41	
3.6 点到直线的距离	43	
3.7 点在直线上的投影	47	
3.8 相遇的地方：直线相交的计算	48	
3.9 习题	54	
改变形状：二维平面上的线性映射	57	第 4 章
4.1 倾斜的目标框	58	
4.2 矩阵形式	59	
4.3 矩阵的计算性质	61	
4.4 图形放缩	63	
4.5 图形反射	65	
4.6 图形旋转	68	
4.7 图形切变	69	
4.8 图形投影	71	
4.9 投影的核	73	
4.10 面积与线性映射：行列式	74	
4.11 线性映射的复合	77	
4.12 矩阵乘法的更多性质	81	
4.13 矩阵运算的更多性质	83	
4.14 习题	84	
2×2 线性方程组	87	第 5 章
5.1 再议倾斜的目标框	88	
5.2 矩阵形式	89	
5.3 直接求解法：克拉默法则	90	
5.4 高斯消元法	91	
5.5 取消映射：逆矩阵	93	
5.6 无解方程组	99	
5.7 欠定方程组	100	
5.8 齐次方程组	100	
5.9 数值应用：主元法	102	
5.10 用矩阵定义映射	104	
5.11 习题	104	
在周围移动：二维平面上的仿射映射	107	第 6 章
6.1 坐标变换	108	
6.2 仿射映射与线性映射	110	
6.3 平移	111	
6.4 更多常见的仿射映射	112	
6.5 从三角形映射到三角形	114	

3.4 Explicit Equation of a Line	40	
3.5 Converting Between Parametric and Implicit Equations	41	
3.6 Distance of a Point to a Line	43	
3.7 The Foot of a Point	47	
3.8 A Meeting Place: Computing Intersections	48	
3.9 Exercises	54	
Changing Shapes: Linear Maps in 2D	57	Chapter 4
4.1 Skew Target Boxes	58	
4.2 The Matrix Form	59	
4.3 More about Matrices	61	
4.4 Scalings	63	
4.5 Reflections	65	
4.6 Rotations	68	
4.7 Shears	69	
4.8 Projections	71	
4.9 The Kernel of a Projection	73	
4.10 Areas and Linear Maps: Determinants	74	
4.11 Composing Linear Maps	77	
4.12 More on Matrix Multiplication	81	
4.13 Working with Matrices	83	
4.14 Exercises	84	
2×2 Linear Systems	87	Chapter 5
5.1 Skew Target Boxes Revisited	88	
5.2 The Matrix Form	89	
5.3 A Direct Approach: Cramer's Rule	90	
5.4 Gauss Elimination	91	
5.5 Undoing Maps: Inverse Matrices	93	
5.6 Unsolvable Systems	99	
5.7 Underdetermined Systems	100	
5.8 Homogeneous Systems	100	
5.9 Numerical Strategies: Pivoting	102	
5.10 Defining a Map	103	
5.11 Exercises	104	
Moving Things Around: Affine Maps in 2D	107	Chapter 6
6.1 Coordinate Transformations	108	
6.2 Affine and Linear Maps	110	
6.3 Translations	111	
6.4 More General Affine Maps	112	
6.5 Mapping Triangles to Triangles	114	

6.6 仿射映射的复合	116	
6.7 习题	120	
特征	123	第 7 章
7.1 固定方向	124	
7.2 特征值	125	
7.3 特征向量	127	
7.4 特殊情形	129	
7.5 对称矩阵的几何图形	132	
7.6 重复映射	135	
7.7 映射的条件数	137	
7.8 习题	138	
剖分：三角	141	第 8 章
8.1 重心坐标	142	
8.2 仿射不变性	144	
8.3 几个特殊点	145	
8.4 二维平面上的三角剖分	148	
8.5 数据结构	149	
8.6 点的位置	150	
8.7 三维空间中的三角剖分	151	
8.8 习题	153	
圆锥曲线	155	第 9 章
9.1 常见的圆锥曲线	156	
9.2 圆锥曲线类型的判定	160	
9.3 圆锥曲线位置的判定	162	
9.4 习题	163	
三维空间中的几何	165	第 10 章
10.1 从二维到三维	166	
10.2 向量积	168	
10.3 直线	172	
10.4 平面	173	
10.5 应用：光与影	177	
10.6 标量三重积	180	
10.7 线性空间	181	
10.8 习题	183	

6.6 Composing Affine Maps	116	
6.7 Exercises	120	
Eigen Things	123	Chapter 7
7.1 Fixed Directions	124	
7.2 Eigenvalues	125	
7.3 Eigenvectors	127	
7.4 Special Cases	129	
7.5 The Geometry of Symmetric Matrices	132	
7.6 Repeating Maps	135	
7.7 The Condition of a Map	137	
7.8 Exercises	138	
Breaking It Up: Triangles	141	Chapter 8
8.1 Barycentric Coordinates	142	
8.2 Affine Invariance	144	
8.3 Some Special Points	145	
8.4 2D Triangulations	148	
8.5 A Data Structure	149	
8.6 Point Location	150	
8.7 3D Triangulations	151	
8.8 Exercises	153	
Conics	155	Chapter 9
9.1 The General Conic	156	
9.2 Analyzing Conics	160	
9.3 The Position of a Conic	162	
9.4 Exercises	163	
3D Geometry	165	Chapter 10
10.1 From 2D to 3D	166	
10.2 Cross Product	168	
10.3 Lines	172	
10.4 Planes	173	
10.5 Application: Lighting and Shading	177	
10.6 Scalar Triple Product	180	
10.7 Linear Spaces	181	
10.8 Exercises	183	

三维空间中的相交	185	第 11 章
11.1 点与平面的距离	186	
11.2 两直线间的距离	187	
11.3 直线与平面相交	189	
11.4 直线与三角形相交	191	
11.5 光在平面上的反射	191	
11.6 三个平面相交	193	
11.7 两个平面相交	194	
11.8 建立正交坐标系	195	
11.9 习题	197	
三维空间中的线性映射	199	第 12 章
12.1 矩阵与线性映射	200	
12.2 图形放缩	202	
12.3 图形反射	204	
12.4 图形切变	204	
12.5 图形投影	207	
12.6 图形旋转	209	
12.7 体积与线性映射: 行列式	213	
12.8 线性映射的组合	216	
12.9 更多的矩阵性质	218	
12.10 逆矩阵	219	
12.11 习题	221	
三维空间中的仿射映射	223	第 13 章
13.1 仿射映射	224	
13.2 平移	225	
13.3 四面体的映射	225	
13.4 投影	229	
13.5 齐次坐标与透视映射	232	
13.6 习题	238	
一般线性方程组	241	第 14 章
14.1 问题的引入	242	
14.2 高斯消元求解法	244	
14.3 行列式	250	
14.4 超定方程组	253	
14.5 逆矩阵	256	
14.6 矩阵的 LU 分解	258	
14.7 习题	262	

Interactions in 3D	185	Chapter 11
11.1 Distance Between a Point and a Plane	186	
11.2 Distance Between Two Lines	187	
11.3 Lines and Planes: Intersections	189	
11.4 Intersecting a Triangle and a Line	191	
11.5 Lines and Planes: Reflections	191	
11.6 Intersecting Three Planes	193	
11.7 Intersecting Two Planes	194	
11.8 Creating Orthonormal Coordinate Systems	195	
11.9 Exercises	197	
 Linear Maps in 3D	 199	 Chapter 12
12.1 Matrices and Linear Maps	200	
12.2 Scalings	202	
12.3 Reflections	204	
12.4 Shears	204	
12.5 Projections	207	
12.6 Rotations	209	
12.7 Volumes and Linear Maps: Determinants	213	
12.8 Combining Linear Maps	216	
12.9 More on Matrices	218	
12.10 Inverse Matrices	219	
12.11 Exercises	221	
 Affine Maps in 3D	 223	 Chapter 13
13.1 Affine Maps	224	
13.2 Translations	225	
13.3 Mapping Tetrahedra	225	
13.4 Projections	229	
13.5 Homogeneous Coordinates and Perspective Maps	232	
13.6 Exercises	238	
 General Linear Systems	 241	 Chapter 14
14.1 The Problem	242	
14.2 The Solution via Gauss Elimination	244	
14.3 Determinants	250	
14.4 Overdetermined Systems	253	
14.5 Inverse Matrices	256	
14.6 LU Decomposition	258	
14.7 Exercises	262	

一般线性空间	265	第 15 章
15.1 基本性质	266	
15.2 线性映射	268	
15.3 内积	271	
15.4 格拉姆 - 施密特正交化方法	271	
15.5 高维特征问题	272	
15.6 空间一览	274	
15.7 习题	276	
数值方法	279	第 16 章
16.1 线性方程组的另一种解法：豪斯霍尔德法	280	
16.2 向量的范数与序列	285	
16.3 方程组的迭代解法：高斯 - 雅可比法与高斯 - 赛德尔法	287	
16.4 求特征值：幂法	290	
16.5 习题	294	
直线组团来袭：折线和多边形	297	第 17 章
17.1 折线	298	
17.2 多边形	299	
17.3 凸性	300	
17.4 多边形的类别	301	
17.5 不常见的多边形	302	
17.6 转向角与分支数	304	
17.7 面积	305	
17.8 验证共面问题	309	
17.9 验证点与多边形的位置问题	310	
17.10 习题	313	
曲线	315	第 18 章
18.1 应用：参数曲线	316	
18.2 贝齐尔曲线的性质	321	
18.3 矩阵形式	323	
18.4 导数	324	
18.5 合成曲线	326	
18.6 平面曲线的几何	327	
18.7 沿曲线移动	329	
18.8 习题	331	