DISK INCLUDED

Student Solutions Manual

Calculus

for Business, Economics, Life Sciences, and Social Sciences

Eighth Edition

Raymond A. Barnett / Michael R. Ziegler / Karl E. Byleen

Student Solutions Manual

Laurel Technical Services

Calculus

for Business, Economics, Life Sciences, and Social Sciences

Eighth Edition

Raymond A. Barnett

Michael R. Ziegler

Karl E. Byleen

PRENTICE HALL, Upper Saddle River, NJ 07458

Executive Editor: Sally Simpson
Managing & Supplements Editor: Gina M. Huck
Editorial Assistant: Sara Beth Newell
Special Projects Manager: Barbara A. Murray
Production Editor: Barbara A. Till
Supplement Cover Manager: Paul Gourhan
Supplement Cover Designer: Liz Nemeth
Manufacturing Buyer: Alan Fischer

Simon & Schuster / A Viacom Company
Upper Saddle River, NJ 07458

Printed in the United States of America

10 9 8 7 6 5 4 3

ISBN 0-13-961244-0

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada, Inc., *London*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brazil, Ltda., *Rio de Janeiro*

CONTENTS

1 A BEGINNING LIBRARY OF ELEMENTARY FUNCTIONS

EXERCISE 1-1

Things to remember:

1. A FUNCTION is a rule (process or method) that produces a correspondence between one set of elements, called a DOMAIN, and a second set of elements, called the RANGE, such that to each element in the domain there corresponds one and only one element in the range.

2. EQUATIONS AND FUNCTIONS:

   Given an equation in two variables. If there corresponds exactly one value of the dependent variable (output) to each value of the independent variable (input), then the equation defines a function. If there is more than one output for at least one input, then the equation does not define a function.

3. VERTICAL LINE TEST FOR A FUNCTION

   An equation defines a function if each vertical line in the coordinate system passes through at most one point on the graph of the equation. If any vertical line passes through two or more points on the graph of an equation, then the equation does not define a function.

4. AGREEMENT ON DOMAINS AND RANGES

   If a function is specified by an equation and the domain is not given explicitly, then assume that the domain is the set of all real number replacements of the independent variable (inputs) that produce real values for the dependent variable (outputs). The range is the set of all outputs corresponding to input values.

   In many applied problems, the domain is determined by practical considerations within the problem.

5. FUNCTION NOTATION—THE SYMBOL $f(x)$

   For any element x in the domain of the function f, the symbol $f(x)$ represents the element in the range of f corresponding to x in the domain of f. If x is an input value, then $f(x)$ is the corresponding output value. If x is an element which is not in the domain of f, then f is NOT DEFINED at x and $f(x)$ DOES NOT EXIST.

1. The table specifies a function, since for each domain value there corresponds one and only one range value.

3. The table does not specify a function, since more than one range value corresponds to a given domain value. (Range values 5, 6 correspond to domain value 3; range values 6, 7 correspond to domain value 4.)

5. This is a function.

7. The graph specifies a function; each vertical line in the plane intersects the graph in at most one point.

9. The graph does not specify a function. There are vertical lines which intersect the graph in more than one point. For example, the y-axis intersects the graph in three points.

11. The graph specifies a function.

13. $f(x) = 3x - 2$
$f(2) = 3(2) - 2 = 4$

15. $f(-1) = 3(-1) - 2$
$= -5$

17. $g(x) = x - x^2$
$g(3) = 3 - 3^2 = -6$

19. $f(0) = 3(0) - 2$
$= -2$

21. $g(-3) = -3 - (-3)^2$
$= -12$

23. $f(1) + g(2)$
$= [3(1) - 2] + (2 - 2^2) = -1$

25. $g(2) - f(2)$
$= (2 - 2^2) - [3(2) - 2]$
$= -2 - 4 = -6$

27. $g(3) \cdot f(0) = (3 - 3^2)[3(0) - 2]$
$= (-6)(-2)$
$= 12$

29. $\dfrac{g(-2)}{f(-2)} = \dfrac{-2 - (-2)^2}{3(-2) - 2} = \dfrac{-6}{-8} = \dfrac{3}{4}$

31. $y = f(-5) = 0$

33. $y = f(5) = 4$

35. $f(x) = 0$ at $x = -5, 0, 4$

37. $f(x) = -4$ at $x = 6$

39. domain: all real numbers or $(-\infty, \infty)$

41. domain: all real numbers except -4

43. $x^2 + 3x - 4 = (x + 4)(x - 1)$; domain: all real numbers except -4 and 1.

45. $7 - x \geq 0$ for $x \leq 7$; domain: $x \leq 7$ or $(-\infty, 7]$

47. $7 - x > 0$ for $x < 7$; domain: $x < 7$ or $(-\infty, 7)$

49. f is not defined at the values of x where $x^2 - 9 = 0$, that is, at 3 and -3; f is defined at $x = 2$, $f(2) = \dfrac{0}{-5} = 0$.

51. $g(x) = 2x^3 - 5$

53. $G(x) = 2\sqrt{x} - x^2$

55. Function f multiplies the domain element by 2 and subtracts 3 from the result.

57. Function F multiplies the cube of the domain element by 3 and subtracts twice the square root of the domain element from the result.

59. Given $4x - 5y = 20$. Solving for y, we have:

$$-5y = -4x + 20$$
$$y = \frac{4}{5}x - 4$$

Since each input value x determines a unique output value y, the equation specifies a function. The domain is R, the set of real numbers.

61. Given $x^2 - y = 1$. Solving for y, we have:

$$-y = -x^2 + 1 \quad \text{or} \quad y = x^2 - 1$$

This equation specifies a function. The domain is R, the set of real numbers.

63. Given $x + y^2 = 10$. Solving for y, we have:

$$y^2 = 10 - x$$
$$y = \pm\sqrt{10 - x}$$

This equation does not specify a function since each value of x, $x \le 10$, determines two values of y. For example, corresponding to $x = 1$, we have $y = 3$ and $y = -3$; corresponding to $x = 6$, we have $y = 2$ and $y = -2$.

65. Given $xy - 4y = 1$. Solving for y, we have:

$$(x - 4)y = 1 \quad \text{or} \quad y = \frac{1}{x - 4}$$

This equation specifies a function. The domain is all real numbers except $x = 4$.

67. Given $x^2 + y^2 = 25$. Solving for y, we have:

$$y^2 = 25 - x^2 \quad \text{or} \quad y = \pm\sqrt{25 - x^2}$$

Thus, the equation does not specify a function since, for $x = 0$, we have $y = \pm 5$, when $x = 4$, $y = \pm 3$, and so on.

69. Given $F(t) = 4t + 7$. Then:

$$\frac{F(3 + h) - F(3)}{h} = \frac{4(3 + h) + 7 - (4\cdot 3 + 7)}{h}$$
$$= \frac{12 + 4h + 7 - 19}{h} = \frac{4h}{h} = 4$$

71. Given $Q(x) = x^2 - 5x + 1$. Then:

$$\frac{Q(2 + h) - Q(2)}{h} = \frac{(2 + h)^2 - 5(2 + h) + 1 - (2^2 - 5\cdot 2 + 1)}{h}$$
$$= \frac{4 + 4h + h^2 - 10 - 5h + 1 - (-5)}{h} = \frac{h^2 - h - 5 + 5}{h}$$
$$= \frac{h(h - 1)}{h} = h - 1$$

73. Given $f(x) = 4x - 3$. Then:

$$\frac{f(a + h) - f(a)}{h} = \frac{4(a + h) - 3 - (4a - 3)}{h}$$
$$= \frac{4a + 4h - 3 - 4a + 3}{h} = \frac{4h}{h} = 4$$

75. Given $f(x) = 4x^2 - 7x + 6$. Then:

$$\frac{f(a+h) - f(a)}{h} = \frac{4(a+h)^2 - 7(a+h) + 6 - (4a^2 - 7a - 6)}{h}$$

$$= \frac{4(a^2 + 2ah + h^2) - 7a - 7h + 6 - 4a^2 + 7a - 6}{h}$$

$$= \frac{4a^2 + 8ah + 4h^2 - 7h - 4a^2}{h} = \frac{8ah + 4h^2 - 7h}{h}$$

$$= \frac{h(8a + 4h - 7)}{h} = 8a + 4h - 7$$

77. Given $f(x) = x^3$. Then:

$$\frac{f(a+h) - f(a)}{h} = \frac{(a+h)^3 - a^3}{h} = \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a^3}{h}$$

$$= \frac{h(3a^2 + 3ah + h^2)}{h} = 3a^2 + 3ah + h^2$$

79. Given $f(x) = \sqrt{x}$. Then:

$$\frac{f(a+h) - f(a)}{h} = \frac{\sqrt{a+h} - \sqrt{a}}{h}$$

$$= \frac{\sqrt{a+h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}} \quad \text{(rationalizing the numerator)}$$

$$= \frac{a + h - a}{h(\sqrt{a+h} + \sqrt{a})} = \frac{h}{h(\sqrt{a+h} + \sqrt{a})} = \frac{1}{\sqrt{a+h} + \sqrt{a}}$$

81. Given $A = \ell w = 25$.

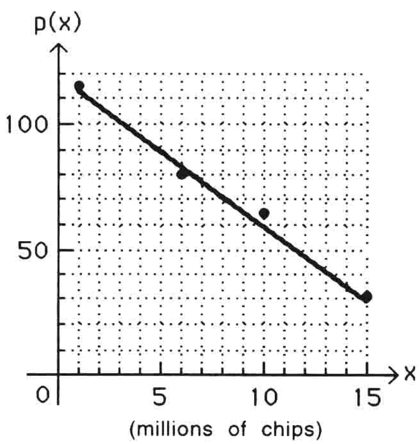
Thus, $\ell = \dfrac{25}{w}$. Now $P = 2\ell + 2w$

$$= 2\left(\frac{25}{w}\right) + 2w = \frac{50}{w} + 2w.$$

The domain is $w > 0$.

83. Given $P = 2\ell w + 2w = 100$ or $\ell + w = 50$ and $w = 50 - \ell$.
Now $A = \ell w = \ell(50 - \ell)$ and $A = 50\ell - \ell^2$.
The domain is $0 \le \ell \le 50$. [Note: $\ell \le 50$ since $\ell > 50$ implies $w < 0$.]

85.



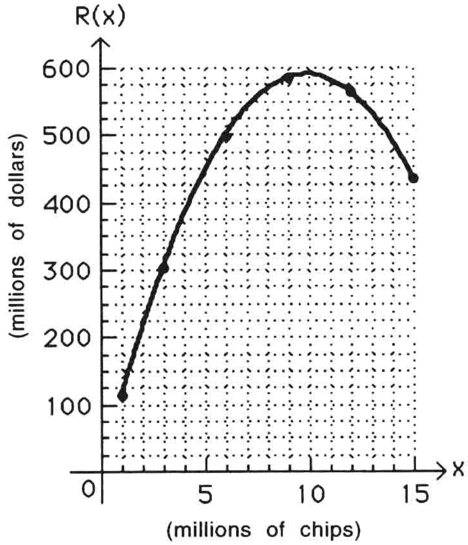
$p(8) = 71$ dollars per chip
$p(11) = 53$ dollars per chip

87. (A) $R(x) = xp(x) = x(119 - 6x)$
Domain: $1 \le x \le 15$

(B) Table 10 Revenue

x(millions)	$R(x)$ (millions)
1	\$113
3	303
6	498
9	585
12	564
15	435

(C)

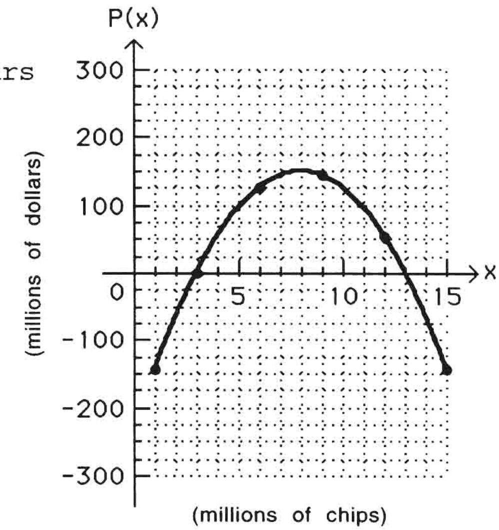


89. (A) $P(x) = R(x) - C(x)$
$= x(119 - 6x) - (234 + 23x)$
$= -6x^2 + 96x - 234$ million dollars
Domain: $1 \le x \le 15$

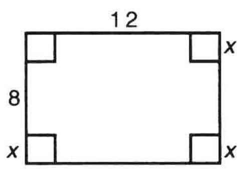
(B) Table 12 Profit

x(millions)	$P(x)$ (millions)
1	-\$144
3	0
6	126
9	144
12	54
15	-144

(C)



91.



(A) $V = $ (length)(width)(height)
$V(x) = (12 - 2x)(8 - 2x)x$
$= x(8 - 2x)(12 - 2x)$

(B) Domain: $0 \le x \le 4$

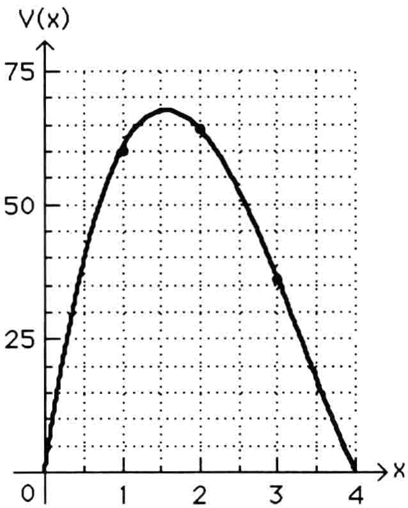
(C) $V(1) = (12 - 2)(8 - 2)(1)$
$= (10)(6)(1) = 60$
$V(2) = (12 - 4)(8 - 4)(2)$
$= (8)(4)(2) = 64$
$V(3) = (12 - 6)(8 - 6)(3)$
$= (6)(2)(3) = 36$

Thus,

Volume

x	$V(x)$
1	60
2	64
3	36

(D)



93. (A) The graph indicates that there is a value of x near 2, and slightly less than 2, such that $V(x) = 65$. The table is shown at the right.

Thus, $x = 1.9$ to one decimal place.

x	y_1
1.7	67.252
1.8	66.528
1.9	65.436
2.0	64.000

(B)

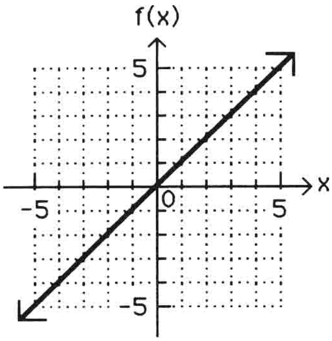
x	y_1
1.90	65.436
1.91	65.307
1.92	65.176
1.93	65.040
1.94	64.902
1.95	64.760
1.96	64.614

Thus, $x = 1.93$ to two decimal places.

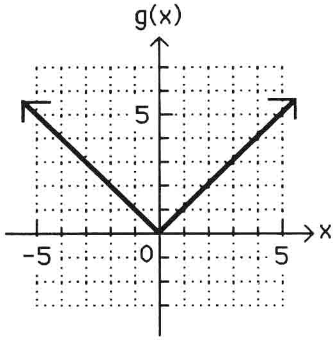
95. Given $(w + a)(v + b) = c$. Let $a = 15$, $b = 1$, and $c = 90$. Then:
$(w + 15)(v + 1) = 90$
Solving for v, we have
$v + 1 = \frac{90}{w + 15}$ and $v = \frac{90}{w + 15} - 1 = \frac{90 - (w + 15)}{w + 15}$, so that $v = \frac{75 - w}{w + 15}$.
If $w = 16$, then $v = \frac{75 - 16}{16 + 15} = \frac{59}{31} \approx 1.9032$ cm/sec.

Things to remember:

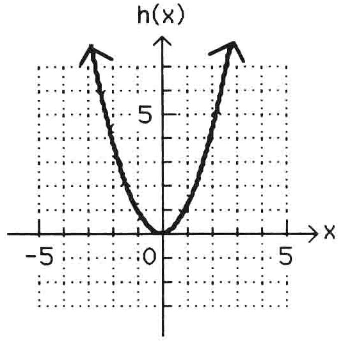
1. LIBRARY OF ELEMENTARY FUNCTIONS



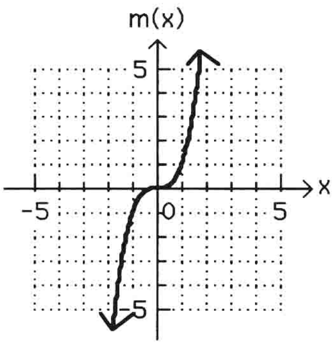
<u>Identity Function</u>
$f(x) = x$
Domain: All real numbers
Range: All real numbers
(a)



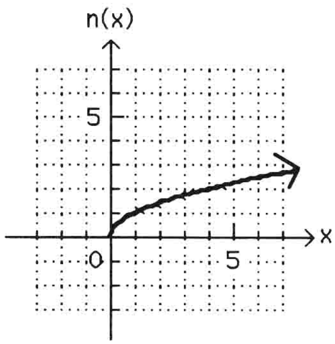
<u>Absolute Value Function</u>
$g(x) = |x|$
Domain: All real numbers
Range: $[0, \infty)$
(b)



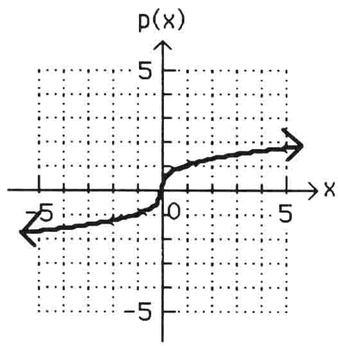
<u>Square Function</u>
$h(x) = x^2$
Domain: All real numbers
Range: $[0, \infty)$
(c)



<u>Cube Function</u>
$m(x) = x^3$
Domain: All real numbers
Range: All real numbers
(d)



<u>Square-Root Function</u>
$n(x) = \sqrt{x}$
Domain: $[0, \infty)$
Range: $[0, \infty)$
(e)



<u>Cube-Root Function</u>
$p(x) = \sqrt[3]{x}$
Domain: All real numbers
Range: All real numbers
(f)

NOTE: Letters used to designate the above functions may vary from context to context.

2. GRAPH TRANSFORMATIONS SUMMARY

<u>Vertical Translation:</u>

$$y = f(x) + k \quad \begin{cases} k > 0 & \text{Shift graph of } y = f(x) \text{ up } k \text{ units} \\ k < 0 & \text{Shift graph of } y = f(x) \text{ down } |k| \text{ units} \end{cases}$$

<u>Horizontal Translation:</u>

$$y = f(x + h) \quad \begin{cases} h > 0 & \text{Shift graph of } y = f(x) \text{ left } h \text{ units} \\ h < 0 & \text{Shift graph of } y = f(x) \text{ right } |h| \text{ units} \end{cases}$$

<u>Reflection:</u>

$y = -f(x)$ Reflect the graph of $y = f(x)$ in the x axis

<u>Vertical Expansion and Contraction:</u>

$$y = Af(x) \quad \begin{cases} A > 1 & \text{Vertically expand graph of } y = f(x) \\ & \text{by multiplying each ordinate value by } A \\ 0 < A < 1 & \text{Vertically contract graph of } y = f(x) \\ & \text{by multiplying each ordinate value by } A \end{cases}$$

3. PIECEWISE-DEFINED FUNCTIONS

Functions whose definitions involve more than one rule are called PIECEWISE-DEFINED FUNCTIONS.

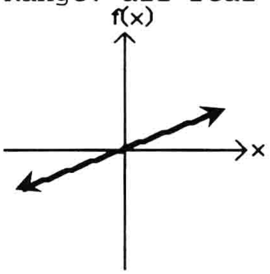
For example,

$$f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

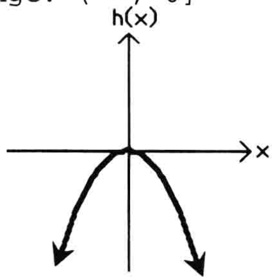
is a piecewise-defined function.

---

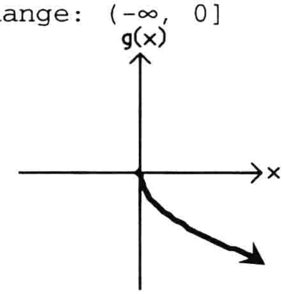
1. Domain: all real numbers; Range: all real numbers



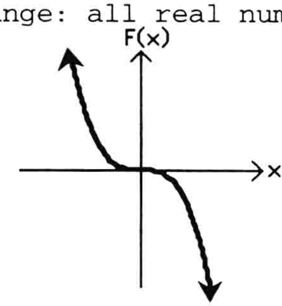
3. Domain: all real numbers; Range: $(-\infty, 0]$



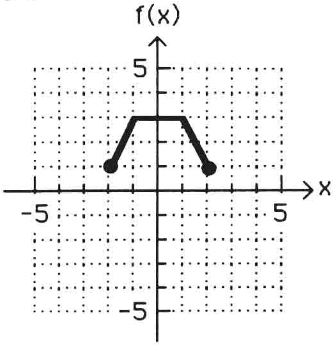
5. Domain: $[0, \infty)$; Range: $(-\infty, 0]$



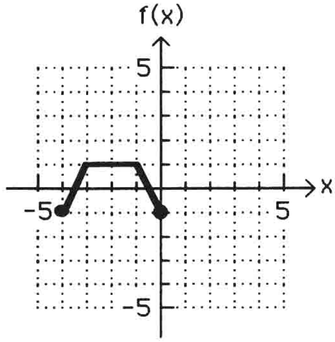
7. Domain: all real numbers; Range: all real numbers



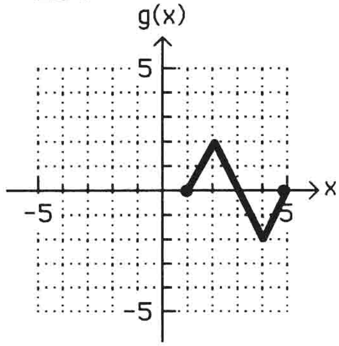
9.



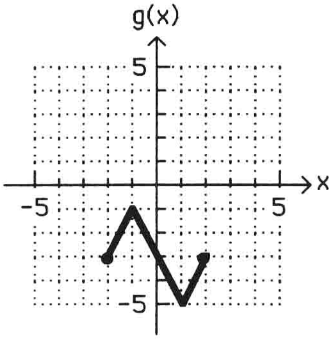
11.



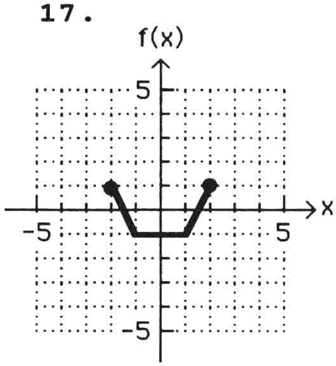
13.



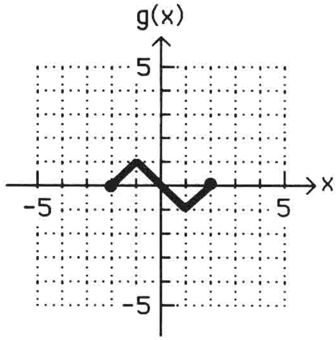
15.



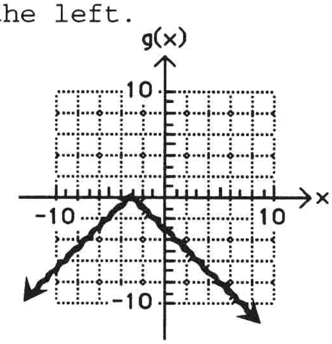
17.



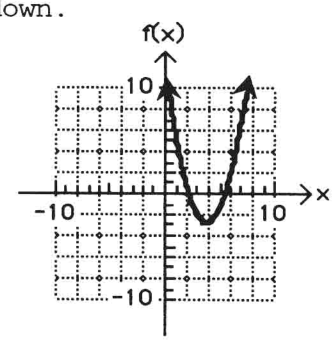
19.



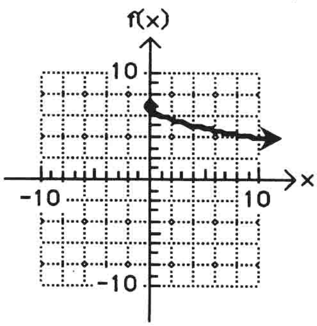
21. The graph of $g(x) = -|x + 3|$ is the graph of $y = |x|$ reflected in the x axis and shifted 3 units to the left.



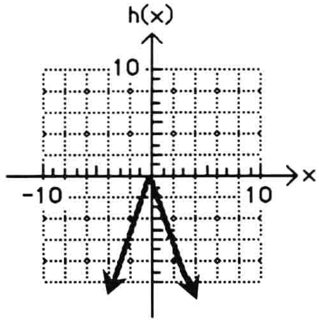
23. The graph of $f(x) = (x - 4)^2 - 3$ is the graph of $y = x^2$ shifted 4 units to the right and 3 units down.



25. The graph of $f(x) = 7 - \sqrt{x}$ is the graph of $y = \sqrt{x}$ reflected in the x axis and shifted 7 units up.



27. The graph of $h(x) = -3|x|$ is the graph of $y = |x|$ reflected in the x axis and vertically expanded by a factor of 3.



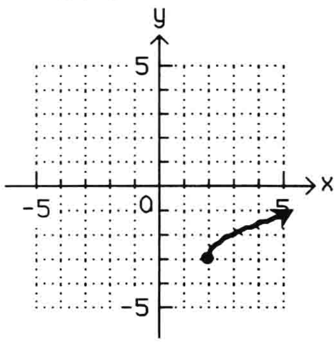
29. The graph of the basic function $y = x^2$ is shifted 2 units to the left and 3 units down. Equation: $y = (x + 2)^2 - 3$.

31. The graph of the basic function $y = x^2$ is reflected in the x axis, shifted 3 units to the right and 2 units up. Equation: $y = 2 - (x - 3)^2$.

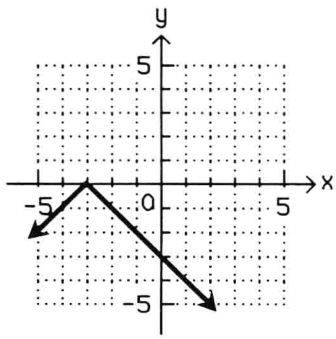
33. The graph of the basic function $y = \sqrt{x}$ is reflected in the x axis and shifted 4 units up. Equation: $y = 4 - \sqrt{x}$.

35. The graph of the basic function $y = x^3$ is shifted 2 units to the left and 1 unit down. Equation: $y = (x + 2)^3 - 1$.

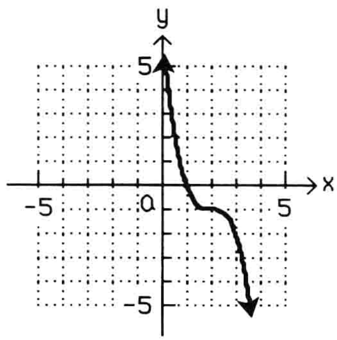
37. $g(x) = \sqrt{x - 2} - 3$



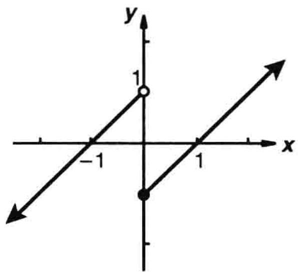
39. $g(x) = -|x + 3|$



41. $g(x) = -(x - 2)^3 - 1$

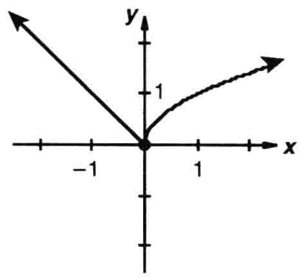


43. $f(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ x - 1 & \text{if } x \geq 0 \end{cases}$



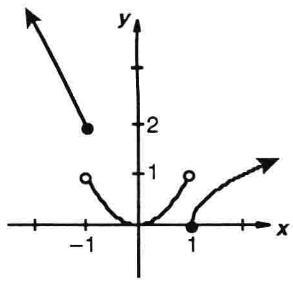
f is discontinuous at $x = 0$.

45. $h(x) = \begin{cases} -x & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$



There are no points of discontinuity.

47. $p(x) = \begin{cases} -2x & \text{if } x \le -1 \\ x^2 & \text{if } -1 < x < 1 \\ \sqrt{x-1} & \text{if } 1 \le x \end{cases}$



p is discontinuous at $x = -1$ and $x = 1$.

49. The graph of the basic function: $y = |x|$ is reflected in the x axis and has a vertical contraction by the factor 0.5. Equation: $y = -0.5|x|$.

51. The graph of the basic function $y = x^2$ is reflected in the x axis and is vertically expanded by the factor 2. Equation: $y = -2x^2$.

53. The graph of the basic function $y = \sqrt[3]{x}$ is reflected in the x axis and is vertically expanded by the factor 3. Equation: $y = -3\sqrt[3]{x}$.

55. Vertical shift, horizontal shift.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A vertical shift of k units followed by a horizontal shift of h units moves (a, b) to $(a, b + k)$ and then to $(a + h, b + k)$.

In the reverse order, a horizontal shift of h units followed by a vertical shift of k units moves (a, b) to $(a + h, b)$ and then to $(a + h, b + k)$. The results are the same.

57. Vertical shift, reflection in the x axis.
Reversing the order can change the result. For example, let (a, b) be a point in the plane with $b > 0$. A vertical shift of k units, $k \ne 0$, followed by a reflection in the x axis moves (a, b) to $(a, b + k)$ and then to $(a, -[b + k]) = (a, -b - k)$.

In the reverse order, a reflection in the x axis followed by the vertical shift of k units moves (a, b) to $(a, -b)$ and then to $(a, -b + k)$; $(a, -b - k) \ne (a, -b + k)$ when $k \ne 0$.

59. Horizontal shift, reflection in y axis.
Reversing the order can change the result. For example, let (a, b) be a point in the plane with $a > 0$. A horizontal shift of h units followed by a reflection in the y axis moves (a, b) to the point $(a + h, b)$ and then to $(-[a + h], b) = (-a - h, b)$.

In the reverse order, a reflection in the y axis followed by the horizontal sift of h units moves (a, b) to $(-a, b)$ and then the $(-a + h, b)$, $(-a - h, b) \ne (-a + h, b)$ when $h \ne 0$.