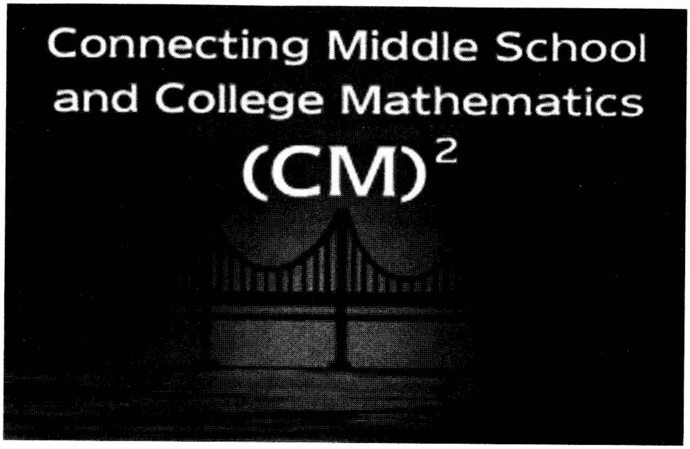
GEOMETRY CONNECTIONS

MATHEMATICS FOR MIDDLE SCHOOL TEACHERS

JOHN K. BEEM

NECTIONS IN MATHEMATICS COURSES FOR TEACHERS



Geometry Connections

Mathematics for Middle School Teachers

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Geometry Connections

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Preface

Improving the quality of mathematics education for middle school students is of critical importance, and increasing opportunities for students to learn important mathematics under the leadership of well-prepared and dedicated teachers is essential. New standards-based curriculum and instruction models, coupled with ongoing professional development and teacher preparation, are foundational to this change.

These sentiments are eloquently articulated in the Glenn Commission Report, *Before It's Too Late: A Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21st Century* (U.S. Department of Education, 2000). In fact, the principal message of the Glenn Commission Report is that America's students must improve their mathematics and science performance if they are to be successful in our rapidly changing technological world. To this end, the report recommends that we greatly intensify our focus on improving the quality of mathematics and science teaching in grades K–12 by bettering the quality of teacher preparation. The report also stresses the need to develop creative plans to attract and retain substantial numbers of future mathematics and science teachers.

Some fifteen years ago, mathematics teachers, mathematics educators, and mathematicians collaborated to develop the architecture for standards-based reform. Their recommendations for the improvement of school mathematics, instruction, and assessment were articulated in three seminal documents published by the National Council of Teachers of Mathematics: *Curriculum and Evaluation Standards for School Mathematics* (1989), *Professional Standards for School Mathematics* (1991), and *Assessment Standards in School Mathematics* (1995); more recently, these three documents were updated and combined into the single book *NCTM Principles and Standards for School Mathematics, a.k.a. PSSM* (2000).

The vision for school mathematics laid out in these three foundational documents was outstanding in spirit and content, yet abstract in practice. Concrete exemplary models reflecting the standards were needed, and implementing the "recommendations" would be unrealizable without significant commitment of resources. Recognizing the opportunity for stimulating improvement in student learning, the National Science Foundation (NSF) made a strong commitment to bring life to the documents' messages and supported several K–12 mathematics curriculum development projects (standards-based curriculum) and other related dissemination and implementation projects.

Standards-based middle school curricula are designed to engage students in a variety of mathematical experiences, including thoughtfully planned explorations that provide and reinforce fundamental skills while illuminating the power and utility of mathematics in our world. These materials integrate central concepts in algebra, geometry, data analysis and probability, and mathematics of change, and focus on important ideas such as proportional reasoning.

The mathematical content of standards-based middle school mathematics materials is challenging and relevant to our technological world. Its effective classroom implementation is dependent upon teachers having strong and appropriate mathematical preparation. *The Connecting Middle School and College Mathematics Project (CM)²* is a three-year (2001–2004) project funded by the National Science Foundation—this project addresses the need for improved teacher qualifications and viable recruitment plans for middle grade mathematics teachers through the development of four foundational mathematics courses, with accompanying support materials and the creation and implementation of effective teacher recruitment models.

The *(CM)²* materials are built upon a framework laid out in the *CBMS Mathematical Education of Teachers Book (MET)* (2001). This report outlines recommendations for the mathematical preparation of middle grade teachers that differ significantly from those for the preparation of elementary school teachers and provides guidance to those developing new programs. Our books are designed to provide middle grade mathematics teachers with a strong mathematical foundation and connect the mathematics they are learning with the mathematics they will be teaching. Their focus is on algebraic and geometric structures, data analysis and probability, and mathematics of change, and they employ standards-based middle grade mathematics curricular materials as a springboard to explore and learn mathematics in more depth. They have been extensively piloted in summer institutes, courses offered at school-based sites, in a variety of professional-development programs, and in semester courses offered at a number of universities throughout the nation.

College students using this book will have learned a fair amount of Euclidean geometry before entering college; consequently, this book is not intended as a first introduction to the subject. Nevertheless, many students will need a substantial review of the subject and will need help in understanding how the content they are learning relates to the mathematics they will eventually be teaching. Throughout the book, the reader will find illustrations of problems and activities from four of the standards-based middle school curricula. These demonstrate the presentation and use of geometry in the middle school. They serve to connect many of the concepts covered in this book to future teaching experiences the college students will eventually encounter and thus provide future teachers with real motivation to learn more mathematical content. Teachers understand they must learn mathematics on a much deeper level than they will be presenting in their future classrooms, but they need to see that what they are learning is related to what they will be teaching.

This book includes a number of Classroom Connections and Classroom Discussions. In both cases, these are activities or discussions that are designed to deepen the connections between the geometry that students are studying now and the geometry they will teach. The **Classroom Connections** are simple activities that teachers might someday use in the middle school classroom. They may be left as part of the readings for the course. The **Classroom Discussions** are explorations that are more central to the book's purpose. These are suggestions that are intended for both the instructor and the students. Some of them may be assigned as readings for the course, and others may be covered as homework assignments. Any of them may be used to shape actual guided discussions in the college classroom. Furthermore, the instructor may wish to assign a few of them as projects to be presented in the classroom by individual students or teams of students.

This book is designed to accommodate both instructors who wish to go slow and spend more time on things such as Classroom Connections and Classroom Discussions and instructors who wish to move more quickly and cover more topics. The former may wish to concentrate on covering the first four chapters. In a course where the chapters are covered more quickly, there should be time to cover all of the first five chapters and most of Chapter 6. A reasonable compromise is to cover Section 1.1 through Section 5.4.

The author would like to thank all of the many faculty members at various schools for piloting preliminary versions of this book and making very helpful suggestions.The author would also like to thank the following reviewers for their helpful suggestions:

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Euclid's Geometry

CHAPTER 1

One of the first mathematicians was Thales (*thay leez*) (c. 624 BC–546 BC). He is often credited with being the first person to make extensive use of mathematical arguments that used logical steps. To a large extent he began the tradition of using proofs in mathematics. Many people contributed to mathematics in the years following Thales. One of these was Euclid (born about 325 BC). Among other things, he wrote *The Elements*, which is a collection of thirteen books on mathematics. These books contain more than just geometry, they include several other topics such as ratio and proportion and number theory. Some of the results in these books were due to Euclid, such as his proof that there are an infinite number of primes. On the other hand, most of the mathematics in Euclid's *Elements* represents contributions by others. Euclid took manuscripts of earlier mathematicians and combined them in a very organized fashion.

In this chapter, Euclid's postulates and axioms are given in Section 1.1, and there is a short introduction to logic in Section 1.2. In Section 1.3, some notation is given, and in Section 1.4 there is an introduction to solids. Although the major theme of this book is Euclidean plane geometry, you will also find several results in this book involving solids. At a number of places, spherical geometry is compared and contrasted with Euclidean plane geometry. Section 1.5 describes how Eratosthenes measured the radius of the Earth in about 240 BC. His result was extremely accurate.

Exercises are provided at the end of each section throughout this book. This book also includes Classroom Connections and Classroom Discussions. The

Classroom Connections are simple activities for eventual use in the middle school classroom. Classroom Discussions are explorations that are intended to be more central to the college course. The college instructor may wish to cover Classroom Discussions in various ways. Some might be assigned as readings, others might be covered in classroom discussions, and still others might be assigned as projects that individuals or teams of students will present in the classroom.

1.1 EUCLID'S POSTULATES AND COMMON NOTIONS

Many students find it surprising that Euclid did not prove everything. He started with ten assumptions consisting of five postulates and five common notions. These assumptions are used to prove later results in *The Elements*, but these original ten assumptions are not proven. It is important for teachers to understand and to be able to explain why Euclid did not prove everything. One way to help prepare future mathematics teachers is to have a college classroom discussion such as the one outlined here.

Classroom Discussion 1.1.1

A fundamental concept in mathematics that Euclid evidently understood is that not everything in mathematics is to be proven. In other words, one needs to start with some unproven assumptions and then prove results based on these assumptions. A good classroom discussion can be centered on the question of why one needs to base things on unproven assumptions. ◆

At the start of Book 1 of Euclid's *Elements*, one finds a list of definitions followed by a list of five **postulates** and then a list of five **common notions**. Although these definitions, postulates, and common notions are not up to modern mathematical standards, we list the postulates and common notions to give an understanding of the historical development of geometry. Also, most middle school and high school treatments of geometry make use of these assumptions. Often Euclid's common notions are called *axioms*. Euclid's original postulates and common notions are listed in Appendix II in translated form. The following are slightly reformulated versions of Euclid's postulates and common notions. You can compare these statements to corresponding statements given in Appendix II.

Postulates:

1. Two distinct points lie on exactly one line.
2. One may extend a line segment indefinitely in each direction.
3. Given any two distinct points, one may construct a circle with one point as center and the segment joining the second point as a radius.
4. Any two right angles are equal in measure to each other.
5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, then the two straight lines, if extended indefinitely, meet on that side on which the angles are less than the two right angles.