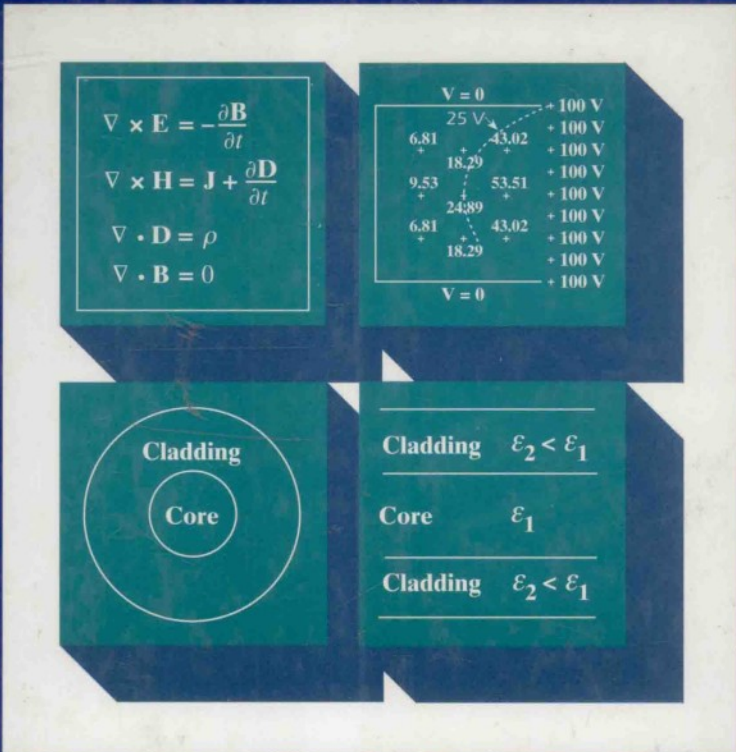
FOURTH EDITION

Elements of Engineering Electromagnetics



NANNAPANENI NARAYANA RAO

FOURTH EDITION

Elements of Engineering Electromagnetics

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From the Upanishads—
Matrudevo bhava: *Revere the mother as God!*
Pitrudevo bhava: *Revere the father as God!*
Acharyadevo bhava: *Revere the preceptor as God!*
Atidhidevo bhava: *Revere the guest as God!*
—as enunciated by Sri Satya Sai

The guiding *essence*, cherished and followed,
in communicating the *science* that follows.

Preface

Electromagnetics textbooks and this edition

The existing introductory textbooks on engineering electromagnetics can be classified broadly into three categories:

1. One-semester textbooks based on the historical approach, covering essentially electrostatics and magnetostatics and culminating in Maxwell's equations with some discussion of their applications.
2. Two-semester textbooks, with the first half or more covering electrostatics and magnetostatics as in category 1 and the remainder devoted to topics associated with electromagnetic waves.
3. One- or two-semester textbooks that have deviated from the historical approach with the degree and nature of deviation depending on the author.

This book belongs to category 3, and the deviation from the historical approach originating with the first edition, a one-semester textbook, has been preserved in the subsequent editions and expanded for one- or two-semester usage to provide flexibility and include PC programs, among other teaching aids. The substantial changes leading to the fourth edition have been prompted by the increasing need for coverage of material at the introductory level for application beyond the microwave region into the optical regime of the electromagnetic spectrum with the advent of the era of photonics overlapping with that of electronics. Thus Chapter 10 of the third edition on antennas has been converted to Chapter 11 and a new Chapter 10 has been created by shifting the material on total internal reflection and dielectric slab waveguides in the previous Chapter 9 and adding new material on topics of interest to photonics. Sections on cylindrical metallic waveguides and losses in waveguides and resonators have been added to Chapter 9, and a section on aperture antennas has been added to Chapter 11. The 18 PC programs written in BASIC for the IBM PC have been retained and one new program has been included.

As in the second and third editions, this edition incorporates flexibility to facilitate its adoption according to the following options:

1. For a three-credit one-semester course or for a four-credit one-quarter course based on coverage of a combination of chapters, depending on the background preparation of the students and the needs of the curriculum. Some examples are
   (a) Chapters 1 through 6
   (b) Chapters 3 through 6 plus parts of Chapters 7, 8, 9, and 10
   (c) Chapters 6 through 11
2. For a two-semester or two-quarter sequence covering the entire book
3. As a text or supplementary text for a course emphasizing PC-assisted instruction

Thread of development of material

The thread of development of the material, evident from a reading of the table of contents, is essentially along the lines of the second and third editions. Some of the salient features are the following:

1. Discusses materials following the presentation of electric and magnetic field concepts and prior to the study of Maxwell's equations
2. Introduces collectively Maxwell's equations for time-varying fields, first in integral form and then in differential form
3. Considers boundary conditions following Maxwell's equations in integral form, and potential functions and associated equations following Maxwell's equations in differential form
4. Devotes a chapter to the development of selected topics in static and quasistatic fields in addition to the coverage of static fields in earlier chapters
5. Obtains uniform plane wave solutions by considering the infinite plane current sheet source first in free space and then in a material medium
6. Develops time-domain analysis of transmission lines in a progressive manner beginning with the case of a resistive load and culminating in the discussion of interconnections between logic gates
7. Presents sinusoidal steady-state analysis of transmission lines comprising the topics of standing waves, resonance, power transfer, and matching with emphasis on computer and graphical solutions
8. Discusses metallic waveguides by first introducing the parallel-plate waveguide by considering the superposition of two obliquely propagating uniform plane waves between two perfect conductors and then extending to rectangular and cylindrical waveguides
9. Devotes a chapter for electromagnetic principles for photonics, building up on the coverage of wave phenomena in earlier chapters
10. Introduces radiation by obtaining the complete field solution to the Hertzian dipole field through the magnetic vector potential, and then developing the basic concepts of antennas

Teaching and learning aids

All the teaching and learning aids employed in the previous editions have been retained: (1) examples distributed throughout the text, (2) discussion of practical applications of field concepts and phenomena interspersed among presentations of basic subject matter, (3) descriptions of brief experimental demonstrations suitable for presentation in the classroom, (4) summary of material and review questions for each chapter, (5) inclusion of drill problems (**D**) with answers at the end of each section, (6) marginal notes, and (7) key words (**K**) at the end of each section. Answers are provided for about 40 percent of the end-of-chapter problems. The comprehensive, user-interactive software package, extending the PC programs in the book and including additional topics made available with the third edition, has been updated for this edition and is again available free of charge. For information, write to the author, c/o Department of Electrical and Computer Engineering, University of Illinois at Urbana–Champaign, William L. Everitt Laboratory, 1406 West Green St., Urbana, IL 61801.

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Urbana, Illinois

Contents

1

Vectors and Fields

Electromagnetics deals with the study of electric and magnetic fields. It is at once apparent that we need to familiarize ourselves with the concept of a field, and in particular with electric and magnetic fields. These fields are vector quantities and their behavior is governed by a set of laws known as Maxwell's equations. The mathematical formulation of Maxwell's equations and their subsequent application in our study of the elements of engineering electromagnetics require that we first learn the basic rules pertinent to mathematical manipulations involving vector quantities. With this goal in mind, we devote this chapter to vectors and fields.

We first study certain simple rules of vector algebra without the implication of a coordinate system and then introduce the Cartesian, cylindrical, and spherical coordinate systems. After learning the vector algebraic rules, we turn our attention to a discussion of scalar and vector fields, static as well as time-varying, by means of some familiar examples. We devote particular attention to sinusoidally time-varying fields, scalar as well as vector, and to the phasor technique of dealing with sinusoidally time-varying quantities. With this general introduction to vectors and fields, in Chapter 2 we study the concepts of electric and magnetic fields, from considerations of the experimental laws of Coulomb and Ampère.

1.1 VECTOR ALGEBRA

Vectors versus scalars

In the study of elementary physics we come across several quantities such as mass, temperature, velocity, acceleration, force, and charge. Some of these quantities have associated with them not only a magnitude but also a direction in space whereas others are characterized by magnitude only. The former class of quantities are known as *vectors* and the latter class of quantities are known as *scalars*.

Mass, temperature, and charge are scalars, whereas velocity, acceleration, and force are vectors. Other examples are voltage and current for scalars and electric and magnetic fields for vectors.

Vector quantities are represented by symbols in boldface roman type (e.g., **A**), to distinguish them from scalar quantities, which are represented by symbols in lightface italic type (e.g., *A*). Graphically, a vector, say **A**, is represented by a straight line with an arrowhead pointing in the direction of **A** and having a length proportional to the magnitude of **A**, denoted $|\mathbf{A}|$ or simply A. Figure 1.1 shows four vectors drawn to the same scale. If the top of the page represents north, then vectors **A** and **B** are directed eastward, with the magnitude of **B** being twice that of **A**. Vector **C** is directed toward the northeast and has a magnitude three times that of **A**. Vector **D** is directed toward the southwest and has a magnitude equal to that of **C**. Since **C** and **D** are equal in magnitude but opposite in direction, one is the negative of the other.

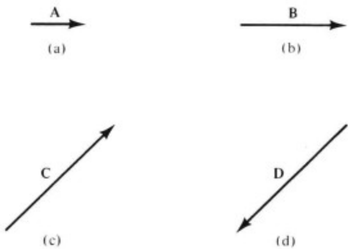

Figure 1.1. Graphical representation of vectors.

Unit vector defined

Since a vector may have in general an arbitrary orientation in three dimensions, we need to define a set of three reference directions at each and every point in space in terms of which we can describe vectors drawn at that point. It is convenient to choose these three reference directions to be mutually orthogonal, as, for example, east, north, and upward, or the three contiguous edges of a rectangular room. Thus let us consider three mutually orthogonal reference directions and direct *unit vectors* along the three directions as shown, for example, in Fig. 1.2(a). A unit vector has magnitude *unity*. We shall represent a unit vector by the symbol **i** and use a subscript to denote its direction. We shall denote the three directions by subscripts 1, 2, and 3. We note that for a fixed orientation of $\mathbf{i}_1$, two combinations are possible for the orientations of $\mathbf{i}_2$ and $\mathbf{i}_3$, as shown in Fig. 1.2(a) and (b). If we take a right-hand screw and turn it from $\mathbf{i}_1$ to $\mathbf{i}_2$ through the 90° angle, it progresses in the direction of $\mathbf{i}_3$ in Fig. 1.2(a) but opposite to the direction of $\mathbf{i}_3$ in Fig.1.2(b). Alternatively, a left-hand screw when turned from $\mathbf{i}_1$ to $\mathbf{i}_2$ in Fig. 1.2(b) will progress in the direction of $\mathbf{i}_3$. Hence the set of unit vectors in Fig. 1.2(a) corresponds to a right-handed system, whereas the set in Fig. 1.2(b) corresponds to a left-handed system. We shall work consistently with the right-handed system.

A vector of magnitude different from unity along any of the reference directions can be represented in terms of the unit vector along that direction. Thus $4\mathbf{i}_1$ represents a vector of magnitude 4 units in the direction of $\mathbf{i}_1$, $6\mathbf{i}_2$ represents a

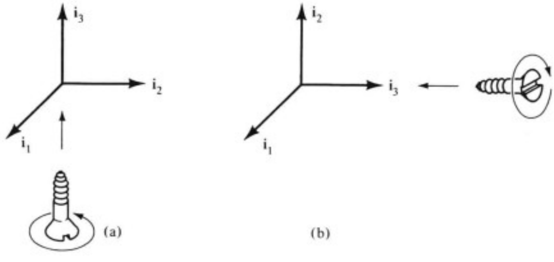

Figure 1.2. (a) Set of three orthogonal unit vectors in a right-handed system. (b) Set of three orthogonal unit vectors in a left-handed system.

vector of magnitude 6 units in the direction of $\mathbf{i}_2$, and $-2\mathbf{i}_3$ represents a vector of magnitude 2 units in the direction opposite to that of $\mathbf{i}_3$, as shown in Fig. 1.3. Two vectors are added by placing the beginning of the second vector at the tip of the first vector and then drawing the sum vector from the beginning of the first vector to the tip of the second vector. Thus to add $4\mathbf{i}_1$ and $6\mathbf{i}_2$, we simply slide $6\mathbf{i}_2$ without changing its direction until its beginning coincides with the tip of $4\mathbf{i}_1$ and then draw the vector $(4\mathbf{i}_1 + 6\mathbf{i}_2)$ from the beginning of $4\mathbf{i}_1$ to the tip of $6\mathbf{i}_2$, as shown in Fig. 1.3. To see this, imagine that on the floor of an empty rectangular room you are going from one corner to the opposite corner. Then to reach the destination, you can first walk along one edge and then along the second edge. Alternatively, you can go straight to the destination along the diagonal. By adding $-2\mathbf{i}_3$ to the vector $(4\mathbf{i}_1 + 6\mathbf{i}_2)$ in a similar manner, we obtain the vector $(4\mathbf{i}_1 + 6\mathbf{i}_2 - 2\mathbf{i}_3)$, as shown in Fig. 1.3. We note that the magnitude of $(4\mathbf{i}_1 + 6\mathbf{i}_2)$ is $\sqrt{4^2 + 6^2}$ or 7.211 and that the magnitude of $(4\mathbf{i}_1 + 6\mathbf{i}_2 - 2\mathbf{i}_3)$ is $\sqrt{4^2 + 6^2 + 2^2}$ or 7.483. Conversely to the foregoing discussion, a vector $\mathbf{A}$ at a given point is simply the superposition of three vectors $A_1\mathbf{i}_1$, $A_2\mathbf{i}_2$, and $A_3\mathbf{i}_3$ which are the projections of $\mathbf{A}$ onto the reference directions at that point. A_1, A_2, and A_3 are known as the *components* of $\mathbf{A}$ along the 1, 2, and 3 directions, respectively. Thus

$$\boxed{\mathbf{A} = A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3} \tag{1.1}$$

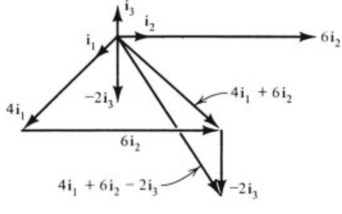

Figure 1.3. Graphical addition of vectors.

We now consider three vectors, **A**, **B**, and **C** given by

$$\mathbf{A} = A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3 \tag{1.2a}$$

$$\mathbf{B} = B_1\mathbf{i}_1 + B_2\mathbf{i}_2 + B_3\mathbf{i}_3 \tag{1.2b}$$

$$\mathbf{C} = C_1\mathbf{i}_1 + C_2\mathbf{i}_2 + C_3\mathbf{i}_3 \tag{1.2c}$$

at a point and discuss several algebraic operations involving vectors as follows:

Vector addition and subtraction. Since a given pair of like components of two vectors are parallel, addition of two vectors consists simply of adding the three pairs of like components of the vectors. Thus

$$\begin{aligned}\mathbf{A} + \mathbf{B} &= (A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3) + (B_1\mathbf{i}_1 + B_2\mathbf{i}_2 + B_3\mathbf{i}_3) \\ &= (A_1 + B_1)\mathbf{i}_1 + (A_2 + B_2)\mathbf{i}_2 + (A_3 + B_3)\mathbf{i}_3\end{aligned} \tag{1.3}$$

Vector subtraction is a special case of addition. Thus

$$\begin{aligned}\mathbf{B} - \mathbf{C} &= \mathbf{B} + (-\mathbf{C}) \\ &= (B_1\mathbf{i}_1 + B_2\mathbf{i}_2 + B_3\mathbf{i}_3) + (-C_1\mathbf{i}_1 - C_2\mathbf{i}_2 - C_3\mathbf{i}_3) \\ &= (B_1 - C_1)\mathbf{i}_1 + (B_2 - C_2)\mathbf{i}_2 + (B_3 - C_3)\mathbf{i}_3\end{aligned} \tag{1.4}$$

Multiplication and division by a scalar. Multiplication of a vector **A** by a scalar m is the same as repeated addition of the vector. Thus

$$m\mathbf{A} = m(A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3) = mA_1\mathbf{i}_1 + mA_2\mathbf{i}_2 + mA_3\mathbf{i}_3 \tag{1.5}$$

Division by a scalar is a special case of multiplication by a scalar. Thus

$$\frac{\mathbf{B}}{n} = \frac{1}{n}(\mathbf{B}) = \frac{B_1}{n}\mathbf{i}_1 + \frac{B_2}{n}\mathbf{i}_2 + \frac{B_3}{n}\mathbf{i}_3 \tag{1.6}$$

Magnitude of a vector. From the construction of Fig. 1.3 and the associated discussion, we have

$$\boxed{|\mathbf{A}| = |A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3| = \sqrt{A_1^2 + A_2^2 + A_3^2}} \tag{1.7}$$

Unit vector along A. The unit vector $\mathbf{i}_A$ has a magnitude equal to unity, but its direction is the same as that of **A**. Hence

$$\boxed{\mathbf{i}_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{A_1}{|\mathbf{A}|}\mathbf{i}_1 + \frac{A_2}{|\mathbf{A}|}\mathbf{i}_2 + \frac{A_3}{|\mathbf{A}|}\mathbf{i}_3} \tag{1.8}$$

Dot product

Scalar or dot product of two vectors. The scalar or dot product of two vectors **A** and **B** is a scalar quantity equal to the product of the magnitudes of **A** and **B** and the cosine of the angle between **A** and **B**. It is represented by a boldface dot between **A** and **B**. Thus if α is the angle between **A** and **B**, then

$$\boxed{\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos \alpha = AB \cos \alpha} \tag{1.9}$$

For the unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$, we have

$$\mathbf{i}_1 \cdot \mathbf{i}_1 = 1 \qquad \mathbf{i}_1 \cdot \mathbf{i}_2 = 0 \qquad \mathbf{i}_1 \cdot \mathbf{i}_3 = 0 \tag{1.10a}$$

$$\mathbf{i}_2 \cdot \mathbf{i}_1 = 0 \qquad \mathbf{i}_2 \cdot \mathbf{i}_2 = 1 \qquad \mathbf{i}_2 \cdot \mathbf{i}_3 = 0 \tag{1.10b}$$

$$\mathbf{i}_3 \cdot \mathbf{i}_1 = 0 \qquad \mathbf{i}_3 \cdot \mathbf{i}_2 = 0 \qquad \mathbf{i}_3 \cdot \mathbf{i}_3 = 1 \tag{1.10c}$$

By noting that $\mathbf{A} \cdot \mathbf{B} = A(B \cos \alpha) = B(A \cos \alpha)$, we observe that the dot product operation consists of multiplying the magnitude of one vector by the scalar obtained by projecting the second vector onto the first vector as shown in Fig. 1.4(a) and (b). The dot product operation is commutative since

$$\mathbf{B} \cdot \mathbf{A} = BA \cos \alpha = AB \cos \alpha = \mathbf{A} \cdot \mathbf{B} \tag{1.11}$$

The distributive property also holds for the dot product as can be seen from the construction of Fig. 1.4(c), which illustrates that the projection of $\mathbf{B} + \mathbf{C}$ onto $\mathbf{A}$ is equal to the sum of the projections of $\mathbf{B}$ and $\mathbf{C}$ onto $\mathbf{A}$. Thus

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C} \tag{1.12}$$

Using this property, we have

$$\begin{aligned}
\mathbf{A} \cdot \mathbf{B} &= (A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3) \cdot (B_1\mathbf{i}_1 + B_2\mathbf{i}_2 + B_3\mathbf{i}_3) \\
&= A_1\mathbf{i}_1 \cdot B_1\mathbf{i}_1 + A_1\mathbf{i}_1 \cdot B_2\mathbf{i}_2 + A_1\mathbf{i}_1 \cdot B_3\mathbf{i}_3 \\
&\quad + A_2\mathbf{i}_2 \cdot B_1\mathbf{i}_1 + A_2\mathbf{i}_2 \cdot B_2\mathbf{i}_2 + A_2\mathbf{i}_2 \cdot B_3\mathbf{i}_3 \\
&\quad + A_3\mathbf{i}_3 \cdot B_1\mathbf{i}_1 + A_3\mathbf{i}_3 \cdot B_2\mathbf{i}_2 + A_3\mathbf{i}_3 \cdot B_3\mathbf{i}_3
\end{aligned}$$

Then using the relationships (1.10a)–(1.10c), we obtain

$$\boxed{\mathbf{A} \cdot \mathbf{B} = A_1B_1 + A_2B_2 + A_3B_3} \tag{1.13}$$

Thus the dot product of two vectors is the sum of the products of the like components of the two vectors.

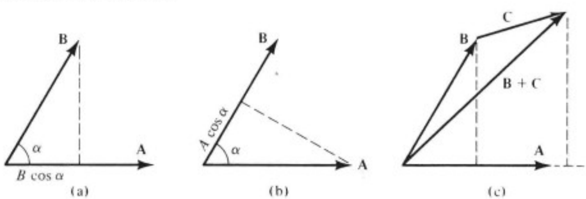

Figure 1.4. (a) and (b) For showing that the dot product of two vectors **A** and **B** is the product of the magnitude of one vector and the projection of the second vector onto the first vector. (c) For proving the distributive property of the dot product operation.

Finding angle between two vectors

From (1.9) and (1.13), we note that the angle between the vectors **A** and **B** is given by

$$\alpha = \cos^{-1}\frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \cos^{-1}\frac{A_1B_1 + A_2B_2 + A_3B_3}{AB} \tag{1.14}$$

Thus the dot product operation is useful for finding the angle between two vectors. In particular, the two vectors are perpendicular if $\mathbf{A}\cdot\mathbf{B} = A_1B_1 + A_2B_2 + A_3B_3 = 0$.

Cross product

Vector or cross product of two vectors. The vector or cross product of two vectors **A** and **B** is a vector quantity whose magnitude is equal to the product of the magnitudes of **A** and **B** and the sine of the smaller angle α between **A** and **B** and whose direction is normal to the plane containing **A** and **B** and toward the side of advance of a right-hand screw as it is turned from **A** to **B** through the angle α, as shown in Fig. 1.5. It is represented by a boldface cross between **A** and **B**. Thus if $\mathbf{i}_N$ is the unit vector in the direction of advance of the right-hand screw, then

$$\boxed{\mathbf{A}\times\mathbf{B} = |\mathbf{A}||\mathbf{B}|\sin\alpha\;\mathbf{i}_N = AB\sin\alpha\;\mathbf{i}_N} \tag{1.15}$$

For the unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$, we have

$$\mathbf{i}_1\times\mathbf{i}_1 = \mathbf{0} \qquad \mathbf{i}_1\times\mathbf{i}_2 = \mathbf{i}_3 \qquad \mathbf{i}_1\times\mathbf{i}_3 = -\mathbf{i}_2 \tag{1.16a}$$

$$\mathbf{i}_2\times\mathbf{i}_1 = -\mathbf{i}_3 \qquad \mathbf{i}_2\times\mathbf{i}_2 = \mathbf{0} \qquad \mathbf{i}_2\times\mathbf{i}_3 = \mathbf{i}_1 \tag{1.16b}$$

$$\mathbf{i}_3\times\mathbf{i}_1 = \mathbf{i}_2 \qquad \mathbf{i}_3\times\mathbf{i}_2 = -\mathbf{i}_1 \qquad \mathbf{i}_3\times\mathbf{i}_3 = \mathbf{0} \tag{1.16c}$$

Note that the cross product of two identical unit vectors is the null vector **0**, that is, a vector whose components are all zero. If we arrange the unit vectors in the manner $\mathbf{i}_1\mathbf{i}_2\mathbf{i}_3\mathbf{i}_1\mathbf{i}_2$, then going to the right the cross product of any two successive unit vectors is the following unit vector, whereas going to the left the cross product of any two successive unit vectors is the negative of the following unit vector.

The cross product operation is not commutative since

$$\mathbf{B}\times\mathbf{A} = |\mathbf{B}||\mathbf{A}|\sin\alpha\,(-\mathbf{i}_N) = -AB\sin\alpha\;\mathbf{i}_N = -\mathbf{A}\times\mathbf{B} \tag{1.17}$$

The distributive property holds for the cross product (we shall prove this later in this section) so that

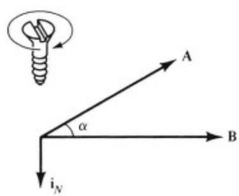

Figure 1.5. Cross product operation $\mathbf{A}\times\mathbf{B}$.