# *Introduction to the mechanics of plastic forming of metals*

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# Preface

The plastic forming of metals is a branch of technology, in which great progress has been achieved mainly by collecting practical data resulting from long-time experience. The contribution of theoretical analysis to this progress is not significant. The common argument was, that such an analysis is too difficult and often even impossible. However, the constantly increasing significance of coldwork processing in industrial practice and the progress in the automatic control of these processes, demanding a broad knowledge of the influence of various parameters, and also the need to improve the mechanical behaviour of coldworked elements have proved beyond doubt that better understanding of the theory of plastic forming processes is necessary. This general trend has become evident in several recent publications in recent years, even though these represen only the engineering, approximate approach to theoretical analysis.

Recent significant progress in the mathematical theory of plasticity allows to build an analysis of plastic forming processes on a more sound basis. In particular there have been developed graphical methods by means of which slip-line and hodograph meshes are constructed for plane strain and axially symmetric problems. These graphical methods yield not only the required forces but also the mode of plastic deformation, even for very complex problems. Numerical procedures also give good results if the digital computer technique is employed.

The aim of the book is to present as simply as possible those methods of the mathematical theory of plasticity which may be applied to the analysis of plastic forming processes. Recent achievements in this analysis are presented, including some original theoretical results and their experimental verification.

Since a large range of problems has been covered in the book, there remained no space to include tables or diagrams which would be directly applicable in engineering practice. Our attempt was to give a systematic presentation of the modern methods used in the analysis of the plastic forming processes, and we hope that the book will be useful for graduate students of mechanical engineering and for mechanical engineers concerned with theoretical knowledge.

In order to give the book a uniform character we shall not deal here with the approximate methods of analysis, although such approximate methods are at the present used broadly in the engineering practice due to their simplicity. Their description may be found in a number of well-known monographs quoted at the end of the book as Supplementary references.

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# 1

# *Mechanical properties of metals*

## 1.1 The plastic behaviour of metals

Most of the commercial metals subject to plastic forming processes have a polycrystalline structure. This means that they are composed of a large number of relatively small crystalline grains randomly oriented in space. Each grain displays anisotropy of mechanical properties depending on crystallographic directions. Polycrystalline metals in the annealed condition display, however, approximately isotropic properties, resulting from random orientation of a large number of fine grains. They generally offer more resistance to deformation than single crystals. As an example, in Fig. 1 are presented stress-strain curves for zinc, for which this effect is

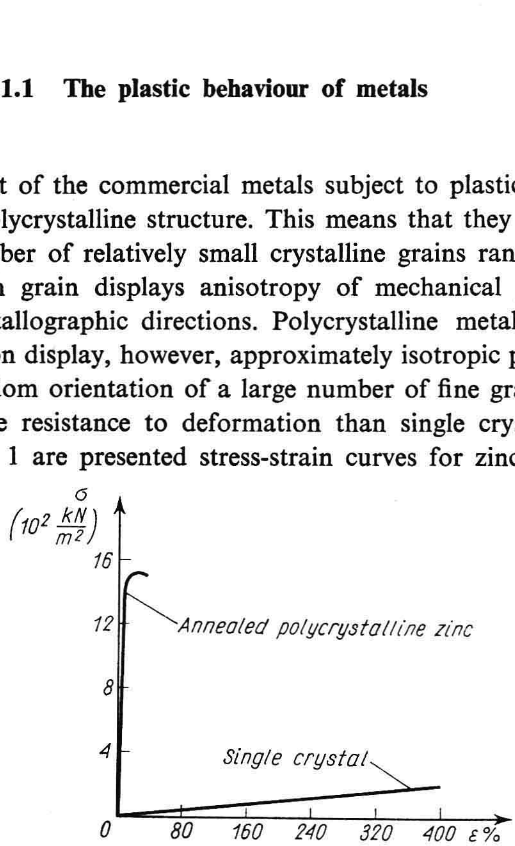


Fig. 1. Stress-strain curves for zinc (after C. F. Elam, see [4])

particularly evident (see [4]). Many factors are held responsible for the different formability of monocrystals and polycrystalline metals, the main role being attributed to the complex effects connected with grain boundaries in an polycrystalline aggregate.

Applying uniaxial tension to standard specimens is the most frequently used method for testing the mechanical behaviour of metals. Also compression of relatively short solid cylinders and torsion of thin-walled tubular specimens is used in routine practice. From the tension test one obtains the diagram of loading force $P$ versus elongation $\Delta l$ from which the stress-strain curve can be constructed. Two measures of strain are usually employed. If the length of a tensile specimen is increased from $l_0$ to $l$, the amount of deformation may be measured either as the conventional strain $e = (l-l_0)/l_0$ or as the logarithmic (natural) strain, obtained by adding increments of strain referred to the instantaneous length of the specimen $\varepsilon = \int_{l_0}^{l} dl/l = \ln(l/l_0)$. There exists the simple relationship $\varepsilon = \ln(1+e)$ between these two measures. As a measure of stress one usually takes the loading force $P$ divided by the area of the initial cross-section of the specimen, i.e. $\sigma_0 = P/A_0$. Such a measure of stress does not correspond to the actual stress since the area of the cross-section decreases gradually during the test. The decreasing of the cross-section area is particularly strong at the final stage of the test, when a local necking occurs at a certain segment of the specimen. Necking is accompanied by a characteristic drop of the conventional stress $\sigma_0 = P/A_0$. However, if we take the quotient $\sigma_r = P/A_{\min}$ of the loading force $P$ by the area $A_{\min}$ of the instantaneous narrowest cross-section area as a measure of stress, then the stress-strain curve increases continuously up to the occurrence of fracture (Fig. 2).

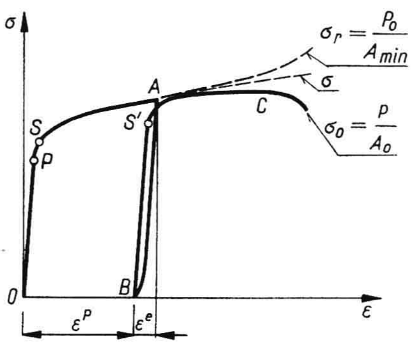


Fig. 2. Stress-strain curve for metals

Frequently, the slope of the $\sigma_r$-curve increases after the neck is formed, which is connected with the triaxial state of stress created in this region. A detailed analysis of this phenomenon is given in specialist monographs (see e.g. [92]) on material testing, and it will not be presented here because of its small significance for the metal forming theory. If the triaxiality of the stress state in the neck is accounted for, one obtains the corrected curve $\sigma$ which represents the properties of deformed material.

Let us now consider the main properties of the tension curve in the $\sigma, \varepsilon$ coordinate system. The initial portion $OP$ of the diagram is linear. The end point $P$ of that portion represents the stress which is called the *proportional limit.* Deformations within this range are elastic, and they vanish when the load is removed. The elastic type of deformation slightly overuns the sector $OP$, thus the stress at which plastic (permanent) deformations begin to appear is greater than the proportional limit. This stress corresponds to the point $S$ on the tension curve, whose ordinate determines the elastic limit. The exact measurement of the elastic limit is rather difficult and depends mainly on the accuracy of the measuring device. Hence, for the sake of clarity the notion of conventional elastic limit has been introduced. The *conventional elastic limit* represents the stress level accompanied by a certain very small permanent deformation, most commonly equal to 0.01 or 0.02 percent. Thus the conventional elastic limit is denoted as $\sigma_{0.01}$ or $\sigma_{0.02}$, respectively. To give rise to further plastic deformation the load must be increased. This very important effect is referred to as the *strain-hardening* of the material. The slope of that portion of the stress-strain diagram $c_1 = d\sigma/d\varepsilon$ is called the *hardening modulus.* The physical meaning of that term is clear if the unloading and consecutive reloading of the specimen, previously deformed until a certain point $A$, is considered. During unloading from $A$ to $B$ a certain small part of deformation vanishes; it represents the elastic part of strain $\varepsilon^e$. The remainder constitutes the plastic part of strain $\varepsilon^p$. If the specimen is loaded again, deformation up to a certain point $S'$ is elastic. Since the stress level corresponding to $S'$ is higher than that corresponding to $S$, the plastically deformed material increases its elastic limit. Under the still increasing loading force the material begins again to deform plastically. It is important to note that after a short transitory sector of strong curvature the diagram becomes an extension of the initial part of the stress-strain curve obtained before unloading. If the loading had been continued uninterruptedly after the point $A$ is reached, the two lines practically would coincide.

Some metals as for example mild steel, certain aluminium alloys, polycrystalline molybdenum and cadmium display certain characteristic features of the stress-strain diagram (Fig. 3). The stress after reaching the point $G$ sudenly drops. The stress $\sigma_u$ is called *upper yield point.* Then the material undergoes plastic deformation at an almost constant value of stress $\sigma_l$, called *lower yield point.* The remaining part of the diagram is similar to that on Fig. 2. The magnitude of the upper yield point depends mainly on the conditions under which the tension test is run, whereas the lower yield point has a fixed value.

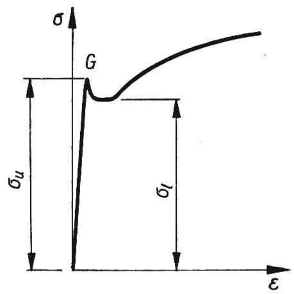


Fig. 3. Stress-strain curve for mild steel, certain aluminium alloys, polycrystalline molybdenum and cadmium

Certain difficulties arise concerning determination of the yield point for materials whose tension diagram is of the type schematically shown in Fig. 2. In engineering practice the conventional yield point $\sigma_{0.2}$ is used, equal to stress accompanied by 0.2 percent of permanent deformation.

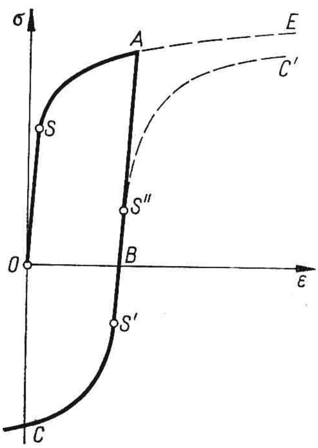


Fig. 4. Schematic diagram of the Bauschinger effect. Curve $BC$ is shifted to the position $BC'$

Generally, metals under compression have the same stress-strain curve as under tension, provided the signs of stress and strain are changed. However, materials behave in a different manner if previous tensile plastic deformation is followed by compression. Let tensile loading until the point $A$ (Fig. 4) and unloading until $B$ be followed by compression causing a shortening of the specimen. If now the segment $BC$ is shifted to take up the position $BC'$, then it turns out that it will run below the tension curve $OAE$. Even more remarkable is the difference between ordinates of the point $S''$ and of the corresponding point on the tension-following-previous-tension curve (Fig. 2). This phenomenon is known as the *Bauschinger effect*, and is observed also in the reversible torsion test and other more complex loadings of plastically prestrained metals. The Bauschinger effect is of great significance in the formulation of the strain-hardening hypotheses, which will be discussed in more detail in Chapter 3.

The stress-strain curve strongly depends on the test temperature. By rising the temperature sufficiently one obtains considerable lowering of the yield point and of the curve itself. This phenomenon is widely exploited in the practice of plastic forming of metals. In Fig. 5 are presented tension

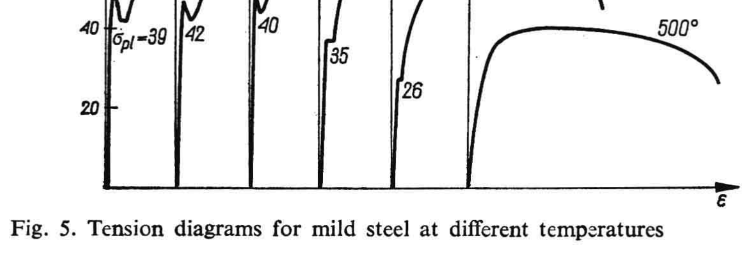


Fig. 5. Tension diagrams for mild steel at different temperatures

diagrams for mild steel at different temperatures. At temperatures of 100° and 200°C a slight rise of the yield point is observed, but beginning from 300°C the yield point decreases remarkably. As shown in Fig. 6, copper for any temperature increment displays lowering of stresses required to deform it plastically.

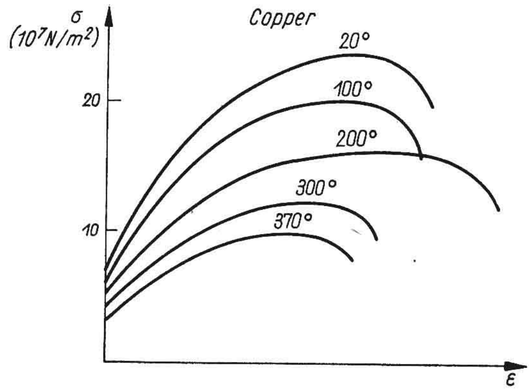


Fig. 6. Tension diagrams for copper at different temperatures

The tension test is not particularly advantageous for determining the plastic properties of metals, which need to be known in the theoretical analysis of various plastic forming processes. The main disadvantage of this test consists in the loss of stability connected with necking occurring at the relatively small plastic deformations. The triaxiality of the stress state in the neck adds to the intricacy of interpretating the test results. Although the end part of the stress-strain diagram can be corrected by means of respective calculations [21, 12] and so the effect of triaxiality of the stress state can be eliminated, the fracture of the specimen at relatively small deformations remains the main weakness of the tension test. A substantial amount of total elongation is concentrated in the necking region, and the so-called *uniform deformation* reached, before necking begins to form, does not usually exceed the magnitude of several percent. In plastic forming processes deformations are in most cases, particularly when compressive stresses predominate, remarkably large. For that reason, of great practical significance are methods of material testing which permit to obtain the stress-strain diagram for as large as possible strains.

Being confined in space, we are not able to describe here all the various methods of testing, which may be found in specialist works. Two methods, however, will be shortly mentioned below for their ingenious simplicity. In [35] the specimens having the form of a flat ring shown in the bottom part of Fig. 7, are twisted between two punches. Moreover, the specimen is compressed between the two punches by means of an axial force acting along the axis of the testing device. An internal mandrel and