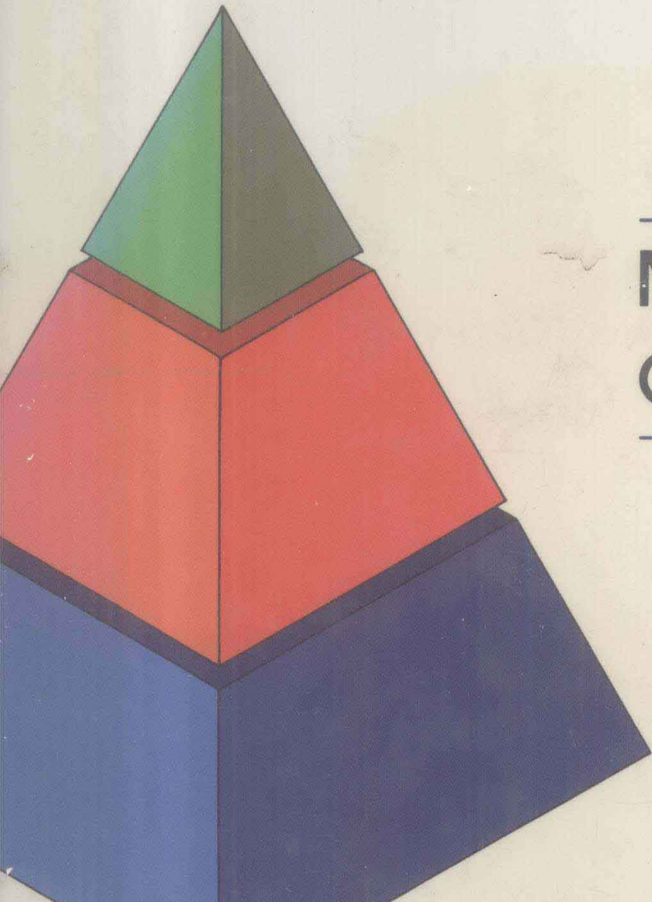
Introductory
ALGEBRA



Michael A. Gallo
Charles F. Kiehl

Introductory Algebra

Michael A. Gallo
Monroe Community College

Charles F. Kiehl
State University of New York, Brockport

WEST PUBLISHING COMPANY
St. Paul New York Los Angeles San Francisco

Production Coordination: Editing, Design & Production, Inc.
Composition: Syntax International

Printed in the United States of America

Library of Congress Cataloging in Publication Data

Gallo, Michael A.
Introductory algebra.

Includes index.
1. Algebra. I. Kiehl, Charles F. II. Title.
QA152.2.G35 1984 512.9 83-21825
ISBN 0-314-78001-7

Introductory Algebra

To my parents, Joe and Rose Gallo,
and to two of the world's finest secondary mathematics teachers,
Jim Sudore and Dick Toole (MAG);
and
To my father, Charles Kiehl,
to the memory of my mother, Thelma,
and to my loyal home supporters:
my wife Margo, and my children,
Amy, Karen, and Christopher (CFK).

Preface

This text is written for college students who are taking a first course in algebra. In writing this text, we have assumed that students using the book have little or no algebraic background.

Specifically, the distinctive pedagogical features and organization of the text include the following:

Introductions: Every topic or concept of each section is first introduced through either a diagram or logic to motivate the student and present rationale for method.

Procedures: A step-by-step algorithm, developed naturally from the introduction and summarizing the necessary steps for the solution to a problem, is provided for nearly every section. All procedures are boxed off from the text and are highlighted in a second color for easy reference.

Examples: A large number of graded examples follow each procedure. The development of the examples are telescopic in nature. The first example, for instance, is completely worked out using the procedure as a directional guide. Successive examples, however, show less detail, encouraging students to gradually build their problem-solving skills.

Practice Exercises: At strategic points in the discussion practice exercises consisting of problems dealing with a specific concept are provided. These practice exercises afford students the opportunity to test their understanding of a given topic before doing the section exercises. Typically, practice exercises immediately follow the examples of a section, with the problems keyed directly to the previously completed examples. Answers to the practice exercises are provided in the right-hand margin for immediate reinforcement.

Section Exercises: A substantial number of graded exercises are provided at the end of each section. In addition to providing the necessary practice needed to master the material of each section, certain exercises also offer some thought-provoking problems designed to further enhance specific concepts. Answers to all odd-numbered problems for section exercises are provided in the back of the text.

Chapter Review Exercises: A separate set of chapter review exercises that provides a mix of problems discussed in the chapter can be found at the end of each chapter. Exercises in color represent typical problems that might be included on a chapter test. Answers to all chapter review exercises are provided in the back of the text.

Notes: Special notes in boxes provide added depth to a particular concept. In most cases the notes reinforce a previously learned idea. In other instances students are alerted to typical errors to be avoided.

Verbal Problems: Chapter 5 is devoted entirely to the development of problem-solving techniques for solving verbal problems. The problems presented in this chapter are of the traditional type (number, coin, mixture, investment, and motion), and are carefully developed with highly detailed examples. This technique is further applied in subsequent chapters (e.g., Chapter 8—work problems; Chapter 11—using systems of equations to solve verbal problems). Each type of problem is treated separately, with matching exercise sets. The chapter review exercises do, however, provide a mix of problem types.

Graphics: In addition to enhancing the overall physical attraction of the text, the use of a second color and shading have been thoughtfully and carefully incorporated within the examples to aid in visualizing the solution of a problem. For the most part, shaded items represent a quantity that is being manipulated in a particular step of the process, and a second color is used to represent the result or answer of this manipulation. Such use of graphics is both effective and complements the learning process.

Basic Arithmetic Review: Chapter 1 provides a complete and thorough review of basic arithmetic.

Sets: A separate discussion of sets is provided in the Appendix.

INSTRUCTIONAL RESOURCES

In addition to the main text, an instructor's manual, student study guide, and computer-assisted instruction disks are available.

The instructor's manual contains six different but parallel chapter tests for each chapter in the textbook, two comprehensive final examinations, answers to the chapter tests included in the manual, answers to the final examinations included in the manual, and answers to all exercises found in the text.

The student study guide briefly reviews key concepts and all procedures for students. It also contains examples taken from the odd-numbered problems in the text to illustrate procedures. Additional drill exercises with answers and additional chapter tests with answers are included in the study guide.

The computer-assisted instruction disks provide students with additional drill, practice, and review in areas of difficulty.

ACKNOWLEDGMENTS

As it is with any work of this magnitude, there are many behind-the-scene individuals who either have assisted us or have provided us with guidance, direction, and encouragement. It is here that we acknowledge their tangible, and, in some cases, intangible effect.

The following reviewers read all or part of the manuscript during its various stages. We thank you for sharing your suggestions and insights with us.

Benjamin Bockstege, Jr. (Broward Community College)
Ray Boersema (Community College of Denver)

Tommy Caldcleuth (North Harris County College)
Ellen Casey (Massachusetts Bay Community College)
Robert Christie (Miami-Dade Community College, South)
Eunice Everett (Broward Community College)
K. Elayn Gay (University of New Orleans, Lakefront)
Alleyne Hartnett (Corning Community College)
Paul Hutchens (St. Louis Community College at Florissant Valley)
Corinne Irwin (Austin Community College)
Richard Linder (Southwestern College)
Rose Novey (Saginaw Valley State College)
Jack Porter (Cuyoga Community College)
Susan Wagner (Northern Virginia Community College, Annandale)
Edward B. Wright (Oregon College of Education)

Typists Barbara Kays, Taeko Heiser, Linda McIntyre, LuAnn Merz, and Dorothy Reed were responsible for typing various parts of the manuscript. We appreciate your time and skill and are grateful for the quick turn-around time that you provided us.

We also thank Marta Fahrenz, Production Editor, West Publishing Company, and Mark Walter, Project Editor, Editing, Design & Production, Inc., for their harmonious efforts in producing this text, and to Kristen McCarthy, Marketing Coordinator. To Peter Marshall, editor and friend, we extend a very warm thank you for having faith in this project and helping us through our "dark period." Also, special acknowledgments are deserving to Jack Pritchard, Larry Gilligan (Mattatuck Community College), Jane Edgar (Brevard Community College), and Nancy Keymel (Steno Services, Monroe Community College), who all helped in his or her own way.

Finally, we thank our wives, Mary Ellen and Margo, for their patience, love, and understanding during our periods of writing and frustration.

MAG
CFK

Contents

CHAPTER 1

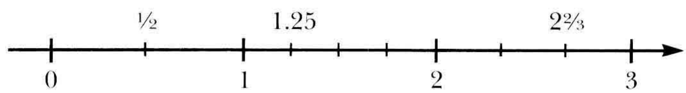
The Structure of Arithmetic

1-1 THE NUMBER LINE AND THE NUMBERS OF ARITHMETIC

In studying arithmetic, we operate with many different types of numbers—counting numbers, whole numbers, decimal numbers, fractional numbers, and so on. In this section, we will review these *numbers of arithmetic*.

A. The Number Line

Before we begin our review of the numbers of arithmetic, we introduce the *number line*. A number line is simply a line whose points are named by numbers. By way of an example, a number line with some of the numbers of arithmetic is pictured here.



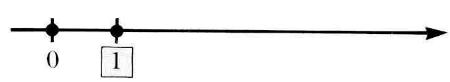
To construct a number line:

1. Draw a straight line.

2. Place an arrowhead at the right end of the line. (This means the line may be extended without ever ending.) The number line also extends to the left, but this will not be discussed now.

3. Pick a point on the line and label it "0."

4. To the right of 0, pick another point and label it "1."



5. Mark off more points to the right of 1, using the length between 0 and 1 as a unit of measure. Label these points 2, 3, 4, and so on.

We now use the number line as an aid in our review of the numbers of arithmetic.

B. Counting Numbers or Natural Numbers

Whenever we count anything we begin with a first number [which we call *one* (1)], followed by a second number [which we call *two* (2)], followed by a third number [which we call *three* (3)], and so on. Each of these numbers is one more than the number it follows (e.g., 2 is *one more* than 1; 3 is *one more* than 2; etc.).

These *counting numbers* are also consecutive in nature. That is, each number follows one after the other, in order, without any interruptions. As a result, the method of counting has no end; that is, there is no last counting number. (The counting numbers are also commonly referred to as the *natural numbers*.)

The counting numbers are easily pictured on a number line.

The counting numbers may also be represented as

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, . . .

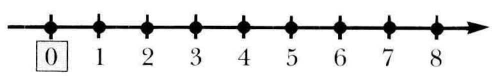
NOTE:

> The three dots after the 10 have the same meaning as the arrowhead on the number line; these numbers continue in the exact same pattern without ever ending.

C. Whole Numbers

As previously mentioned, whenever we count something we start counting with the number 1, followed by 2, 3, and so on. We never begin counting with zero (0). Thus, zero is not a counting number. However, if we include zero with the counting numbers, we have a new collection of numbers which we call the *whole numbers*.

The whole numbers are easily pictured on a number line.



The whole numbers may also be represented as

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, . . .

D. Decimal Numbers

Decimal numbers (or just *decimals*) represent a subdivision of whole numbers. This subdivision is regarded as a special subdivision because decimal values are always expressed in powers of ten.

Decimal numbers contain three parts: whole number(s), decimal point, and decimal digit(s). For example, in the decimal 12.06:

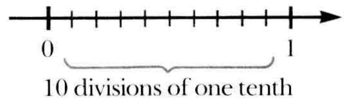
The whole number part is 12.	12 .06
The decimal digits are 0 and 6.	12.0 6
The decimal point separates the whole number part and decimal number part.	12 . 06

The term *decimal places* means the number of decimal digits (or the number of places to the right of the decimal point). Thus,

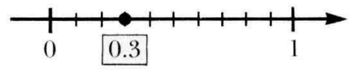
12.06 contains two decimal places — 12. 06 (2 PLACES)

Decimals may also be represented on a number line. For example, to represent the decimal three tenths (0.3):

Divide the interval between the whole numbers into ten equal parts—each part represents one tenth (0.1) of the whole.



The decimal, 0.3, is the point that represents three tenths of the whole.



If we want to represent the point three hundredths (0.03) or three thousandths (0.003), we would divide the interval between the whole numbers into one hundred parts (where each part represents one hundredth [0.01] of the whole) or into one thousand parts (where each part represents one thousandth [0.001] of the whole).

E. Fractional Numbers

Fractional numbers (or just *fractions*) also represent a part of a whole. Fractions are written in the form of a/b where a is a whole number (0, 1, 2, 3, . . .) and b is a natural number (1, 2, 3, . . .).

NOTE:

> The top number, a, is called the *numerator*, and the bottom number, b, is called the *denominator*. The line separating the top and bottom numbers is called a *fraction bar* and represents a division.
>
> Fraction bar ↗ $\frac{a}{b}$ ← numerator (a), ← denominator (b)

Some examples of fractions are

$$\frac{0}{5}, \frac{1}{3}, \frac{2}{8}, \frac{3}{7}, \frac{5}{9}, \frac{12}{16}, \frac{99}{100}, \frac{8}{5}, \text{ etc.}$$

In arithmetic, we learned that there are many different classifications of fractions. We briefly review them here.

- *Proper fractions* are fractions in which the numerator is smaller in value than the denominator.

  $\frac{1}{5}, \frac{2}{3}, \frac{7}{10}, \frac{99}{100}$, etc. proper fractions

- *Improper fractions* are fractions in which the numerator is equal to or greater in value than the denominator.

  $\frac{4}{4}, \frac{5}{3}, \frac{7}{2}, \frac{99}{80}$, etc. improper fractions

- *Like fractions* are fractions that contain the same denominator.

  $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$, etc. like fractions

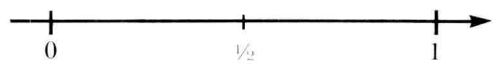
- *Unlike fractions* are fractions that contain different denominators.

  $\frac{1}{7}, \frac{3}{6}, \frac{5}{4}, \frac{7}{8}, \frac{9}{10}$, etc. unlike fractions

Fractions may easily be illustrated by using a number line. We simply divide the interval between the whole numbers into the number of equal parts indicated by the denominator.

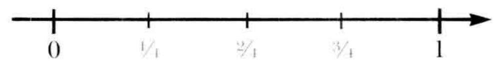
For example, we can divide the interval into

- *Halves*:



- *Thirds*:

- *Fourths*:



- etc.

F. Mixed Numbers

A mixed number is considered a fractional number. However, it is a special fractional number. Mixed numbers represent fractional parts greater than one.

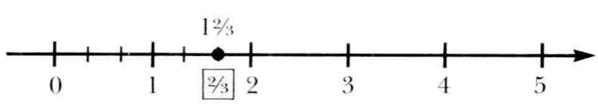
Mixed numbers are usually written in the form of: whole number + fraction . Some examples follow.

$$1\frac{2}{3},\ 2\frac{1}{5},\ 3\frac{4}{8},\ 5\frac{6}{7},\ \text{etc.}$$

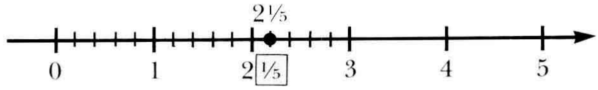
NOTE:

> Mixed numbers represent the *sum* of a whole number and a proper fraction. Thus, $1\frac{2}{3}$ means $1 + \frac{2}{3}$.

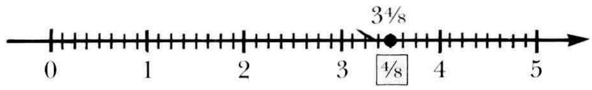
Mixed numbers are also easily illustrated by using a number line. For example, $1\frac{2}{3}$ means one (1) whole interval plus an additional $\frac{2}{3}$ of an interval.



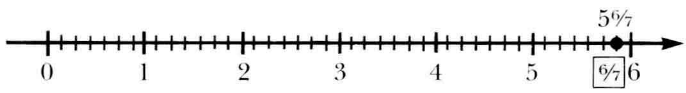
$2\frac{1}{5}$ means two (2) whole intervals plus an additional $\frac{1}{5}$ of an interval.



$3\frac{4}{8}$ means three (3) whole intervals plus an additional $\frac{4}{8}$ of an interval.



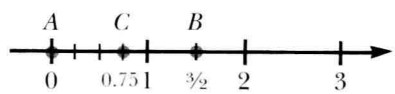
$5\frac{6}{7}$ means five (5) whole intervals plus an additional $\frac{6}{7}$ of an interval.



etc.

We offer the following examples involving the number line and the numbers of arithmetic.

EXAMPLE 1: State the number that represents the points *A*, *B*, and *C* on the number line shown here.



SOLUTIONS:

- Point *A* is represented by 0.
- Point *B* is represented by $\frac{3}{2}$.
- Point *C* is represented by 0.75.

NOTE:

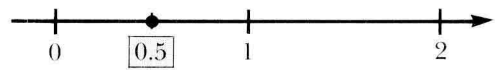
> Whenever a number represents a point on the number line the number is commonly referred to as the *coordinate* of that point. Thus, in example 1:
>
> - The *coordinate* of point *A* is 0.
> - The *coordinate* of point *B* is $\frac{3}{2}$.
> - The *coordinate* of point *C* is 0.75.

EXAMPLE 2: *Graph* the following numbers on the number line: 0.5; $1\frac{3}{4}$; $\frac{12}{12}$.

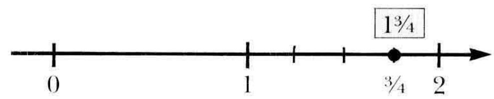
To graph a number on the number line means to locate the point that can be represented by that number.

SOLUTIONS:

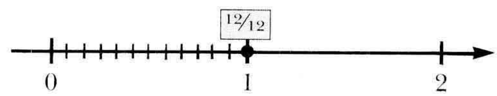
- 0.5 is located halfway between 0 and 1.



- $1\frac{3}{4}$ is located three fourths of the interval between 1 and 2.

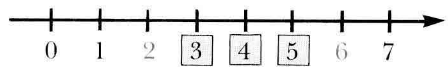


- $\frac{12}{12}$ is simply the number 1.



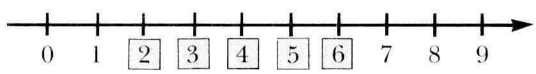
EXAMPLE 3: Using a number line, determine how many *whole numbers* there are between 2 and 6.

There are three whole numbers between 2 and 6. They are 3, 4, and 5.



NOTE:

If we want to *include* 2 and 6, we would use the word inclusive. For example, "How many whole numbers are there between 2 and 6, *inclusive*?" The answer is now five: 2, 3, 4, 5, and 6.



Do the following practice set. Check your answers with the answers in the right-hand margin.

PRACTICE SET 1-1

Using the number line given, state the coordinates of each of the following points (see example 1).