NUMERICAL COMPUTATION OF ELECTRIC & MAGNETIC FIELDS

Numerical Computation of Electric and Magnetic Fields

Second Edition

Charles W. Steele

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PREFACE

Since the first edition of this book was published in 1987, there have been several important changes in the state of numerical field computation, as discussed in the Introduction. These changes have motivated the publication of this second edition.

As with the first edition, the objective of this second edition is to give the newcomer to field computation the information needed to perform practical field computations. Again, clarity of presentation is given greater emphasis than a high degree of sophistication or the state of the art. And again, the basic concepts of field computation are presented as well as the commonly used algorithms.

Several persons have provided much valuable information for this second edition. I wish to thank Professor Giorgio Molinari of the University of Genoa, Italy for advice regarding adaptive mesh generation; Dr. C. R. E. Emson of Vector Fields, Ltd., England and Dr. John Brauer of McNeal-Schwendler Corp. for their advice on transient eddy current computation; and Dr. Zoltan Cendes of Ansoft Corp. for information about their adaptive mesh generator.

Again, I would like to acknowledge the support for this second edition by my wife, Candace. Again, I could not have written this book without her support.

PREFACE TO THE FIRST EDITION

For well over a decade, the numerical approach to field computation has been gaining progressively greater importance. Analytical methods of field computation are, at best, unable to accommodate the very wide variety of configurations in which fields must be computed. On the other hand, numerical methods can accommodate many practical configurations that analytical methods cannot. With the advent of high-speed digital computers, numerical field computations have finally become practical.

However, in order to implement numerical methods of field computation, we need algorithms, numerical methods, and mathematical tools that are largely quite different from those that have been traditionally used with analytical methods. Many of these algorithms have, in fact, been presented in the large number of papers that have been published on this subject in the last two decades. And to some of those who are already experienced in the art of numerical field computations, these papers, in addition to their own original work, are enough to give them the knowledge that they need to perform practical numerical field computations.

But newcomers to the art of numerical field computations need a more orderly presentation of the necessary background information than is available to them in these papers. The objective of this book is precisely to provide this orderly presentation of the necessary information. Clarity of presentation is given greater emphasis than a high degree of sophistication or being up to the state of the art. It is my hope that those who read this book will then be able to read and understand the recent papers and therby achieve the level of sophistication to which they aspire.

Specifically, this book has the objective of presenting information in two different categories:

1. Certain algorithms that have been used successfully for computation of fields.
2. Background information that will help readers to develop their own algorithms.

Putting it another way, the intent of this book is to help readers develop or select the algorithms to be used in these field computations. These algorithms can then be used in computer programs, and these computer programs can be run in high-speed digital computers to compute the fields.

To build these computer programs, the programmer requires knowledge in a number of other areas beyond the algorithms that are discussed in this book. These areas include computer programming, methods for numerical solution of systems of equations, methods for automatic deployment of node points, and construction of finite elements and computer graphics (to display the computed fields). These important topics are discussed in technical journals as well as in other books.

Any numerical field computation is built upon logic that is necessarily an intimate mixture of mathematics, numerical analysis, and electromagnetic theory. This book is just such a mixture. It is assumed that the reader has some prior knowledge of electromagnetic theory, vector analysis, and linear algebra.

This book covers static and quasi-static field problems in media that generally vary in permittivity, permeability, and conductivity. It does not cover dynamic problems, such as antennas, scatterers, radiation in free space, and waveguide modes. These items are topics for books in themselves.

Here we consider only problems in which both the observer of the fields and the medium throughout the entire domain are fixed with respect to a single frame of reference. That is, we do not consider problems in which a portion of the medium moves with respect to the remainder of the medium (and the field is computed in both)—for example, problems in which the field is to be computed in both the moving rotor and stator of an electric motor or generator. Such problems are an important study in themselves.

An objective of this book has been to use standard nomenclature as much as possible. All of the equations related to electric and magnetic fields are based upon the use of the SI system of units. The symbols used are, for the most part, taken from the international recommendations for quantities, units, and their symbols, as published by the United States of America Standards Institute.

I am deeply indebted to a number of people for their contributions to this book. I appreciate my discussion with Mr. Charles Trowbridge, of Rutherford Laboratories, United Kingdom, regarding my approach to Chapter 5. Dr. Alvin Wexler of the University of Manitoba made valuable suggestions related to my presentation of certain of his material in Chapter 7. In addition, Mr. Trowbridge and Dr. Wexler provided thorough technical reviews of my completed manuscript. Dr. Dennis Lindholm of Ampex Corporation reviewed the majority of the manuscript and made many valuable suggestions. Mrs. Bernie Jones of Palo Alto, California, did an excellent job of typing a large portion of the manuscript.

Finally, and most important, I wish to acknowledge my wife, Candace. She has gently tolerated my work on this book over four annual vacations and

countless weekends. Without her unfailing support and encouragement I could not have written it.

If the reader wishes to communicate with the author regarding this book, he can send the communication to the publisher, who will then forward it to the author.

CHARLES W. STEELE

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1
INTRODUCTION

For the last 30 years, the computer has been revolutionizing the computation of electric and magnetic fields, to the point that most practical computations of fields are now done on a computer. This is because many, if not most, practical problems that arise in engineering and science can be solved numerically but cannot be solved analytically, and the computer is practically necessary for numerical solutions.

As a result, a new science has emerged, the science of numerically computing electric and magnetic fields. This science is necessarily a mixture of electromagnetic theory, mathematics, and numerical analysis. In fact, each *problem* of numerical computation is an intimate mixture of these three disciplines. By now, this science has progressed substantially beyond its status when the first edition of this book was published.

This book has several objectives. The first of these objectives is to present information that the reader needs to solve certain field problems numerically. This presentation is then a mixture of electromagnetic theory, numerical analysis, and mathematics.

The second objective of this book is to make it unnecessary for any reader to read the whole book (unless, of course, he or she *wants* to.) It is for this reason that a glossary of the most important and commonly used symbols is included. Furthermore, at points where latter chapters require support from a former chapter, the appropriate references to the former material is made.

The third objective of this book results from the progress that this science has made since the first edition was published. This objective is to present several new and important algorithms that have come into use, as discussed below.

Any field computation necessarily starts with consideration of the properties of the fields themselves. Accordingly, Chapter 2 reviews electromagentic theory to the extent necessary to provide the basis in physics of the field problems considered in this book.

Chapter 3 provides a unified treatment for the definition of field problems. This chapter is intended to underscore the importance of having a clear definition of the problem at the outset.

In most of our numerical field computations, our solution can best be thought of as an element of a finite-dimensional linear space that we construct ourselves. Chapter 4 discusses, in some detail, the construction of these linear spaces.

Since most of the algorithms that we use in field computations are projection methods. Chapter 5 provides a unified treatment of the projection methods that we use.

Chapters 2, 3, 4, and 5 provide support for the material presented in the subsequent chapters.

Chapters 6 and 7 discuss the most commonly used method of numerical field computation, the finite element method.

Chapter 8 discusses mesh generation. This topic has always been an aspect of finite element field computation on which accuracy of computation depends critically and which has usually required much tedious attention. The new developments in automatic and adaptive mesh generation discussed represent significant steps in making finite element field computation practical.

Chapter 9 discusses the integral equation method of field computation.

Chapter 10 discusses the numerical solution of static potential problems.

Chapter 11 presents and discusses specific algorithms that have been used in numerical solution of eddy current problems. This includes a discussion, new to this second edition, of time-domain or transient eddy current computations.

2
FIELD PROPERTIES

2.1. INTRODUCTION

This chapter develops and presents the equations in electromagnetic theory that provide the basis for the computational methods and algorithms presented in later chapters.

Nonuniformities and discontinuities in the media (the permeability, permittivity, and conductivity) are a major problem in practical field computations. This is true whether the computations are made analytically or numerically. For numerical computations, these nonuniformities can tax the skill, patience and endurance of the scientist, and the capacities of his computer as well (in terms of high-speed memory capacity and computational speed and time). To cope with this problem, a variety of methods for dealing with nonuniformities have been presented in the published papers on numerical field computations.

This chapter takes the approach of the aggregate of these papers. To deal with the problem of these nonuniformities, the chapter presents a variety of approaches and theoretical formulations. Emphasis is placed on developing formulations that form the basis for practical numerical algorithms. Emphasis is also placed on developing ways for the scientist to develop a physical "feel" for the behavior of the fields, since a good feel for field behavior is an essential ingredient for a successful field computation.

Emphasis is placed on a careful presentation of the differences between static, quasi-static, and dynamic field behaviors. These differences are extremely important in practical numerical field computations. Static field computations are simpler than quasi-static field computations, and quasi-static field computations are simpler than dynamic field computations. And for field computations, it is *always* best to use the simplest acceptable approach.

2.2. MAXWELL'S EQUATIONS IN THE DYNAMIC, QUASI-STATIC, AND STATIC CASES

2.2.1. Dynamic Case

The methods or algorithms discussed in this book are based largely upon Maxwell's equations. These equations, for the fully dynamic case, are the following:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{2.2.1-1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.2.1-2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.2.1-3}$$

$$\nabla \cdot \mathbf{D} = \rho \tag{2.2.1-4}$$

In these equations **H** and **E** are the magnetic and electric fields, **J** is the conduction current density, and ρ is the electric charge density. Furthermore, **B** is the magnetic flux density and **D** is the displacement, or electric flux, density.

Integral forms can derived from the four Maxwell's equations. Using Stokes' Theorem, we derive from Equations (2.2.1-1) and (2.2.1-2) that

$$\oint_C \mathbf{H} \cdot \mathbf{dl} = I + \frac{\partial \Psi}{\partial t} \tag{2.2.1-5}$$

$$\oint_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial \Phi}{\partial t} \tag{2.2.1-6}$$

The integrals in these equations are line integrals around the closed contour C. In other words, they give the *circulation* of **H** and **E** around the path C. The symbols Ψ and Φ represent the *electric flux* and the *magnetic flux* that thread path C, and I is the conduction current that flows through path C. Using the divergence theorem, we derive from Equations (2.2.1-3) and (2.2.1-4), that

$$\oint_S \mathbf{B} \cdot \mathbf{dS} = 0 \tag{2.2.1-7}$$

$$\oint_S \mathbf{D} \cdot \mathbf{dS} = Q \tag{2.2.1-8}$$

where Q is the electric charge inside the surface S. The integrals in Equations (2.2.1-7) and (2.2.1-8) give the *outflow* of vectors **B** and **D** over the closed surface S.

These equations, in this fully dynamic form, are needed for computations of radiated fields, that is, fields radiated and received by antennas, fields radiated down waveguides, and fields in electronic devices such as klystrons and magnetrons. For many other applications, simpler approximate versions of these equations are adequate.

2.2.2. Quasi-Static Case

The quasi-static case differs from the fully dynamic case only by its neglect of the displacement current.* That is, in the quasi-static case, we say that

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{2.2.2-1}$$

and that

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I \tag{2.2.2-2}$$

and that Equations (2.2.1-2), (2.2.1-3), (2.2.1-4), (2.2.1-6), (2.2.1-7), and (2.2.1-8) still hold. Since the divergence of the curl of a vector is zero, we see from (2.2.2-1) that in the quasi-static case,

$$\nabla \cdot \mathbf{J} = 0 \tag{2.2.2-3}$$

The quasi-static approximation is used for time-varying fields in many conducting media. This is because, for good conductors, the conduction current greatly exceeds the displacement current for frequencies that usually concern us (right up to X-ray frequencies). A good example of this is the calculation of time-varying magnetic fields in iron cores, the so-called "eddy current" problem. This has practical applications in electric motors, generators, magnetic recording heads, and solenoid actuators.

2.2.3. Static Case

In the static case, both the electric displacement current *and* the time-varying magnetic flux density are neglected. In this case, then, Equation (2.2.1-2)

* The displacement current can be neglected when

$$\sigma \gg \omega e$$

where σ, ω, and e are the conductivity, radian frequency, and permittivity.

becomes

$$\nabla \times \mathbf{E} = 0 \qquad (2.2.3\text{-}1)$$

and Equation (2.2.1-6) becomes

$$\oint_C \mathbf{E} \cdot \mathbf{dl} = 0 \qquad (2.2.3\text{-}2)$$

In addition to these equations, Equations (2.2.1-3), (2.2.1-4), (2.2.1-7), (2.2.1-8), (2.2.2-1), and (2.2.2-2) still hold.

Static field calculations are made wherever the dictates of physical reality permit, because, as we will see below, they are of relative simplicity compared to dynamic and quasi-static calculations. As an example, many calculations in the magnetic fields of magnets are static calculations.

2.3. POLARIZATION AND MAGNETIZATION

2.3.1. Polarization

Figure 2-1 shows an electric dipole which consists of two point charges, $-Q$ and $+Q$ (equal in magnitude and opposite in sign), that are separated by a distance d. There is the vector $\mathbf{d}$, of magnitude d, and pointing in the direction toward the positive charge and away from the negative charge. This dipole has an *electric dipole moment* $\mathbf{p}$, given by

$$\mathbf{p} = Q\mathbf{d} \qquad (2.3.1\text{-}1)$$

Suppose that there is a system of charges (say, a molecule) that is neutral (i.e., has a total charge of zero). Since the sum of all positive charges in the system equals in magnitude the sum of all negative charges, then that molecule

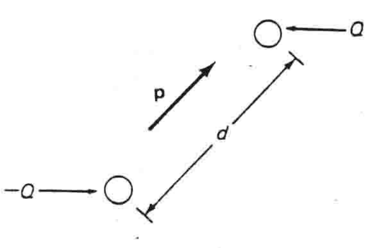

Fig. 2-1. Electric dipole.