

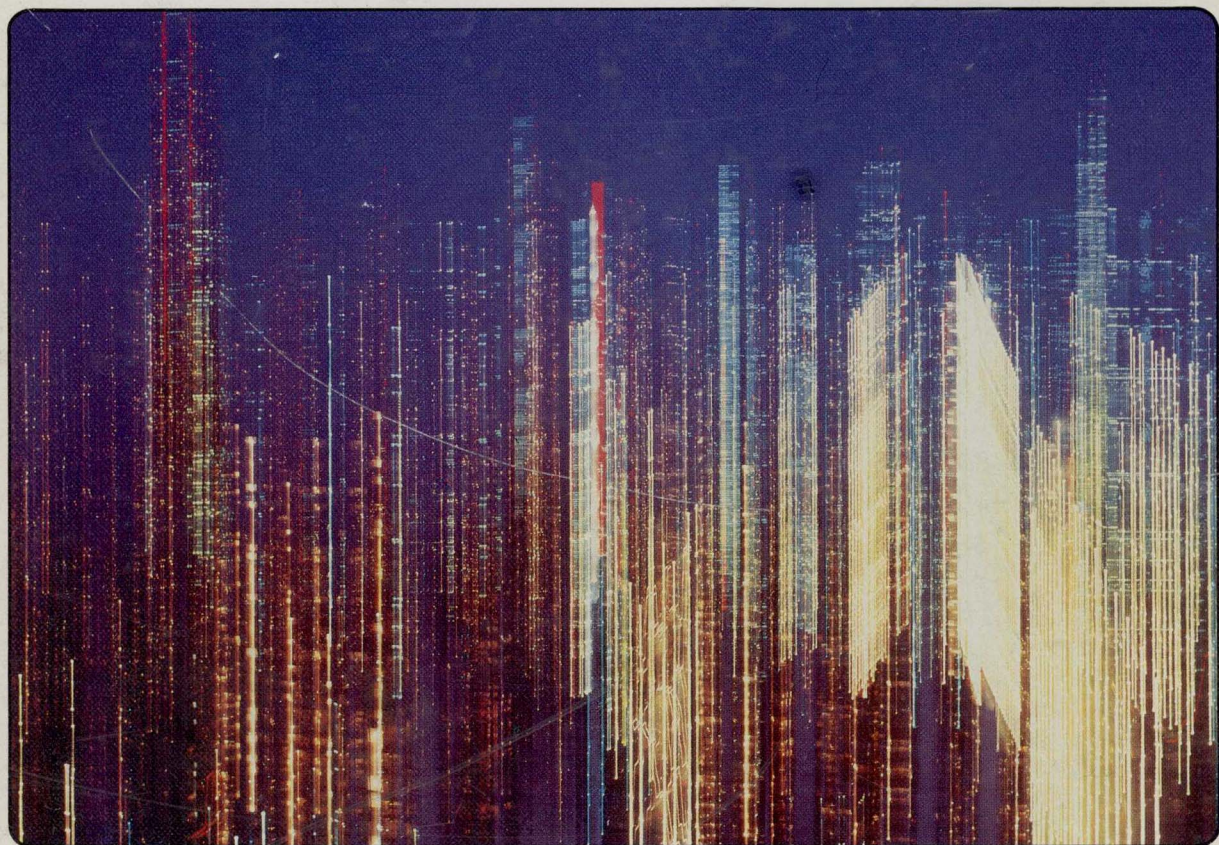
# Mathematics

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with applications to

**Business, Economics & Social Sciences**

**Bouldin**





# Mathematics

with Applications  
to Business, Economics,  
and Social Sciences

**Richard Bouldin**

University of Georgia, Athens



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# Preface for Instructor

The purpose of this book is to present the material frequently taught in a two-quarter or two-semester mathematics course offered to business students and others. The goal of the book is to be clear and persuasive. I believe that the ideas are offered in a way that would convince anyone that this material is natural, accessible and powerful. The book takes the points of view of the small-business person, investor and consumer. Most of the students taking such courses are not destined to be corporate managers but rather sales people, clerks, assistant managers, and small-business managers. The book also considers macroeconomics from the point of view of a business person and voting citizen.

**Exercises** There are more exercises than an instructor will ordinarily assign, and the number assigned will probably vary from one class to another. One instructor might assign every fourth exercise to a well-prepared class with good aptitude, while another instructor might assign every other exercise to a class that needs a lot of practice. Since the odd-numbered exercises have answers given in the back of the text and the even-numbered exercises do not, an instructor can assign only problems with answers given, or only problems without answers given, or any combination desired.

The exercises increase in algebraic complication as the numbers increase. The beginning exercises require very few steps, and the answers are frequently integers. Thus, the instructor can assign only the easier early exercises if desired, or the instructor can construct homework assignments using predominantly the later more difficult exercises. Word problems are provided at the end of almost every section. Exercises not assigned can be used for classroom examples, quizzes, and review work.

The section at the end of each chapter entitled *Review Problems* is intended as a comprehensive test on the problem-solving methods of the preceding chapter. Since students will ordinarily use this section as a study device, it is substantially longer than an in-class exam on the same material. The Review Problems give the student the opportunity to practice at associating a technique of solution with a problem that is not identified according to section. The answers to all Review Problems are given in the back of the text.

**Applications** Applications to business, economics, and personal finance are integrated into the body of the text. Most of these applications are introduced through examples given shortly after the relevant mathematics is presented. If new terms are required, they are covered immediately prior to the examples. The personal-finance applications should interest all students. Understanding concepts like “rate of return,” “profit margin,” “inflation,” and “mortgage” is important for any educated consumer.

The section at the end of each chapter entitled *Social Science Applications* provides applications which can be used to motivate the material for social science students not involved with business or economics. Since each application is clearly labeled by discipline, the instructor can choose those applications most appropriate to a particular class. Occasionally some non-mathematical material will be presented in the first paragraphs of the section to facilitate the subsequent applications.

**Sequence of Chapters** The chapters of this text have been constructed in a way that permits flexibility in the order of presentation. Chapter 13, Sets and Counting, is independent of all other chapters, and all earlier chapters are independent of it. Chapter 14, Probability, depends only on Chapter 13, except for the optional Section 14.5, which also depends on Chapter 3, Systems of Equations and Matrices. In a program that emphasizes calculus the chapters might be covered in the order that they appear in the book. In another program with less emphasis on calculus, Chapters 13 and 14 might be taught immediately after Chapter 6, Mathematics of Money, or immediately after Chapter 5, Linear Programming, if Chapter 6 is not covered.

In any program that leaves the teaching of linear programming to the business school, Chapters 13 and 14 might be inserted in place of Chapter 5. It is even possible to teach Chapter 13, Sets and Counting, before Chapter 1, Elementary Algebra; in this case the instructor can use the language of sets while teaching subsequent chapters. Below we symbolically indicate some of the possible sequences for teaching the chapters.

**A calculus-oriented  
two-semester sequence**

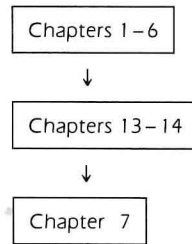
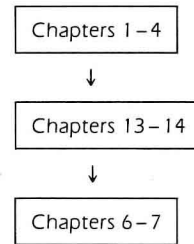
Chapters 1–12

**A calculus-oriented  
two-quarter sequence**

Chapters 1–4



Chapters 6–11

**A two-semester sequence that only introduces calculus****A two-quarter sequence with no linear programming that only introduces calculus**

**Career Profiles** Following each chapter a Career Profile, accompanied by a photograph, gives a brief insight into an occupation. The Career Profiles are not intended to be definitive, or even thorough; they are quick glimpses intended to stimulate the student's imagination. Staffs specializing in career counselling can provide the interested student with more extensive sources such as *The Occupational Outlook Handbook* issued by the U.S. Department of Labor.

**Characteristics of Quantitative Education** This text does not ordinarily present ideas merely for the sake of exposing the student to them. Only material that is needed is presented, and the topics introduced are developed and applied. This approach and the informal style aimed at developing the student's intuition permit the text to concentrate on basic skills and central concepts. In contrast, the survey technique is frequently used in the humanities and is sometimes employed in texts such as this one. Some breadth is sacrificed for the sake of developing reliable skills and a sound grasp of central concepts.

**Calculator Use** A student does not need a calculator in order to use this book. However, the text does try to exploit the fact that many students do have a calculator. An inexpensive calculator with keys to evaluate exponential and logarithm functions can be used to make "function," "limit," and other concepts more concrete. Almost every chapter contains Calculator Examples, which are problems that would be unreasonable without the use of a calculator. Many sections have Calculator Problems, which require the use of a calculator, at the end of the regular exercises. Many of the regular exercises can be solved more quickly and easily with a calculator, but they can also be solved by the customary use of a pencil and paper.

**Acknowledgments** I am indebted to my following colleagues at the University of Georgia who provided me with helpful reactions, suggestions and advice: C.H. Edwards, Jr., Thomas C. Gard, D. Kannan, Frank G. Lether, Carol W. Penney, and David E. Penney.

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Richard Bouldin

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