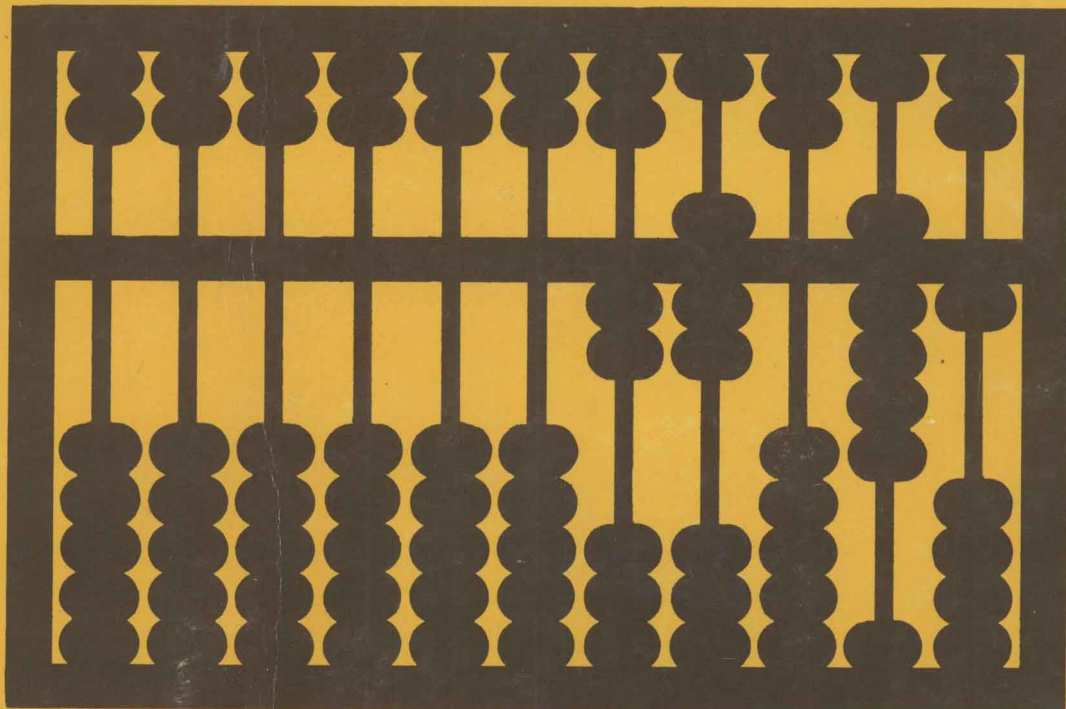


# **INTERNATIONAL CONFERENCE ON EDUCATION, PRACTICE AND PROMOTION OF COMPUTATIONAL METHODS IN ENGINEERING USING SMALL COMPUTERS (EPMESC)**

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**Macau, 5-9 August 1985**



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**VOLUME 2A  
(August, 6th, A. M.)**

V O L U M E   I I

(AUG.   6th)

F R A M E

A N A L Y S I S   I



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## ABSTRACT

Many programs are used for modern engineering analysis. But programs for computer based **engineering** education are not used often. In this paper the unified classical and computerized method in structural analysis and programs are introduced. The programs are parts of a 4-year joint research project on 'Computer Based Education in Structural Engineering' funded by the Croucher Foundation. The programs are used to analyse plane frames and plane trusses by stiffness method. More important, it is used for users to learn computer stiffness method.

## I. INTRODUCTION

Small computer has developed very fast during the last decade. The capacity and speed of operation of small computer have increased very fast by a large reduction in cost. Now, more and more small computers come into use in colleges and in design institutes. The enormous computing power and speed of these machines have opened up new avenues of development in all areas of structural engineering. It is clear that the small computer is already having a important influence on structural engineering education and practice. It is necessary to undertake a fundamental review of curricula and syllabuses in structural engineering to take advantage of the small computer.

The logical approach required by the computer forces the engineer to change his way in engineering design process. The result of this is to transfer the mundane computations to the computer and free the designer to act as a 'higher level' of decision making in the design process. This radical change in approach requires the development of new attitude and expertise.

Therefore we need to develop the new knowledge of computing and computer alongside, and intergrated with, our structural engineering studies. We should have a unified classical method and computerized method. We wish outline some of the features in structural analysis in order that we can **fully use** the **computer** so that we can easily master the science of **the** structural analysis in classical and computerized method.

Now, many commerical packages are used in engineering design and in engineering analysis. However, these packages are black boxes to students because they can only produce calculation results but they are not able to be used as learning tools. We thank Croucher Foundation to help us to make a program-package for computer based education with the corporation of Hong Kong Polytechnic and Hatfield Polytechnic, England. Due to the limited content required, we present the plane frame programme in detail.

## II. PLANE TRUSS

In classical method we use the method of joints to analyse the internal forces of determinate trusses and the force method and displacement method to analyse the internal forces of indeterminate trusses. In computerized method it is better to use stiffness method which is used to solve problems of determinate trusses as well as indeterminate trusses.

In the method of joint there are  $N+NR$  unknown forces (  $N$  number of members and  $NR=3$  reaction components ) to be determined. At each joint we can set up two equilibrium equations,  $\sum X=0$  and  $\sum Y=0$ . If a truss satisfied the following equation

$$N+NR=2J$$

where  $J$  -- number of total joints.

The  $N+NR$  unknowns can be solved with  $2J$  equations. This is the principle of classical method - method of joint - to solve the problem of determinate truss. Of course we can easily make a program to solve these problems.

Stiffness method is a computerized method which should be emphasized in modern education. This method can solve problems of determinate trusses as well as indeterminate trusses. There are two displacement components ( $\Delta x_i$  and  $\Delta y_i$ ) in each joint. We need  $2J$  equations to solve  $2J$  displacement components in a truss with  $J$  joints. However, we need not care whether the unknown forces is  $N+NR$  or more than that.

Ex.1 Fig.1 shows that four members meet at one joint with external forces  $P_1$  and  $P_2$ . Find the internal forces of the members.

Solution:  $U_1$  and  $U_2$  are the unknowns of displacements components. The  $i$ th bar will be lengthened

$$d_i = U_1 l_i + U_2 m_i$$

where  $l_i$  and  $m_i$  are the direction cosine of the  $i$ th bar.

$$F_i = \frac{EA_i}{L_i}(-d_i) \quad (1)$$

From equilibrium equations of joint 1 we have

$$\sum X=0 \quad \text{and} \quad \sum Y=0$$

that is

$$P_1 = \sum F_i l_i = S_{11}U_1 + S_{12}U_2 \quad (2)$$

$$P_2 = \sum F_i m_i = S_{21}U_1 + S_{22}U_2 \quad (3)$$

where

$$S_{11} = \sum \frac{EA_i}{L_i} l_i^2, \quad S_{12} = S_{21} = \sum \frac{EA_i}{L_i} l_i m_i \quad \text{and} \quad S_{22} = \sum \frac{EA_i}{L_i} m_i^2$$

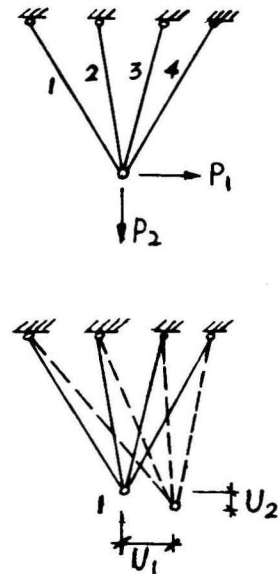


Fig. 1

From Eq.(2) and Eq.(3) we can solve for  $U_1$  and  $U_2$ .

From Eq.(1) we can solve for member force

$$F_i = \frac{EA_i}{l_i} (U_1 l_i + U_2 m_i)$$

Ex.2 (i) Fig.2 shows a truss. If the length of member 3 is not equal to the theoretical length for example  $L_3 + e$ . Find the internal forces in the members.

(ii) If the member 3 is under a temperature change of  $T$  degree. Find the internal forces in the members.

Solution: (i) The effect of the erection of member 3 is equal to that of two force components acting on the joint as shown in Fig.2. The load vector is

$$\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} \frac{EA_3 e}{L_3} l_3 \\ \frac{EA_3 e}{L_3} m_3 \end{pmatrix}$$

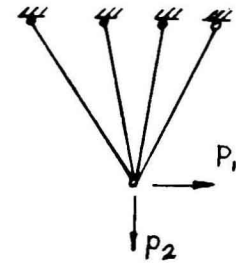
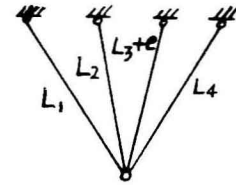


Fig.2

Then we can solve for the internal forces as Ex.1.

(ii) The changed value of the length of member 3 is  $e = \alpha TL$  ( $\alpha$  is the coefficient of linear thermal expansion). The procedure of solution is the same as that of (i).

Ex.3 Fig.3 shows a truss with elastic supports. Find the internal forces of the bars and reactions under loads. The stiffness of the supports are  $S_8, S_9$  and  $S_{10}$ .

Solution: We can use three substitute bars for the three vertical springs. The length of the substitute bar may be of arbitrary value but areas of the three bars should be as follows

$$A_8 = \frac{L_8}{E} S_8, A_9 = \frac{L_9}{E} S_9 \text{ and } A_{10} = \frac{L_{10}}{E} S_{10}$$

Then by stiffness method the internal forces of members and the elastic reactions can be solved easily.

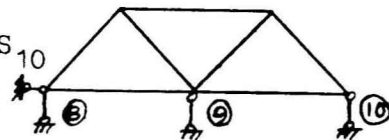
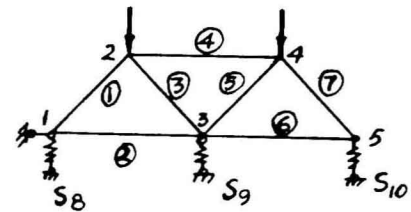


Fig.3

Ex.4 If the truss shown in Fig.3 with different support settlements. Find the internal forces and the support reactions.

Solution: Using equation

$$\begin{pmatrix} A_D \\ A_R \end{pmatrix} = \begin{pmatrix} S & S_{DR} \\ S_{RD} & S_{RR} \end{pmatrix} \begin{pmatrix} D \\ D_R \end{pmatrix} \quad (4)$$



where  $A_D, A_R$  are the vectors of loads on free joints and unknown reactions on supports respectively.

$D, D_R$  are the vectors of unknown displacements of free joints and the known settlements of supports.

From Eq.4 we can find the unknown displacements of the free joints

$$D = S^{-1}(A_D - S_{DR}D_R)$$

and we can also find the unknown reactions of the supports

$$A_R = S_{RD}D + S_{RR}D_R$$

### III. PLANE FRAME

The classical methods used in frame analysis are force method and displacement method. And the axial deformations of the bars are ignored. For example, if we want to analyse the frame shown in Fig.1, we must determine the sideways displacement relationship. This concept is not easily grasped by undergraduate students.

We use the stiffness method in the frame analysis with axial deformations considered. This is like that the stiffness method to be used in truss analysis, but in rigid frame there are three displacement components in **each** joint instead of two in each joint in truss analysis.

**Ex.1** A rigid frame shown in Fig.1. Find the internal forces of each member.

Solution:  $U_1, U_2, U_3, U_4, U_5$  and  $U_6$  are the unknown of displacement components. From the Fig. we can obtain

$$\left. \begin{aligned} d_{11} &= U_1 l_1 + U_2 m_1 \\ d_{12} &= -U_1 m_1 + U_2 l_1 \\ d_{13} &= U_3 \\ d_{2i} &= U_i \quad (i=1,2,\dots,6) \\ d_{34} &= U_4 l_3 + U_5 m_3 \\ d_{35} &= -U_4 m_3 + U_5 l_3 \\ d_{36} &= U_6 \end{aligned} \right\} \quad (1)$$

where  $l_1 = \cos r_1, m_1 = \sin r_1$

$l_3 = \cos r_3, m = \sin r_3$

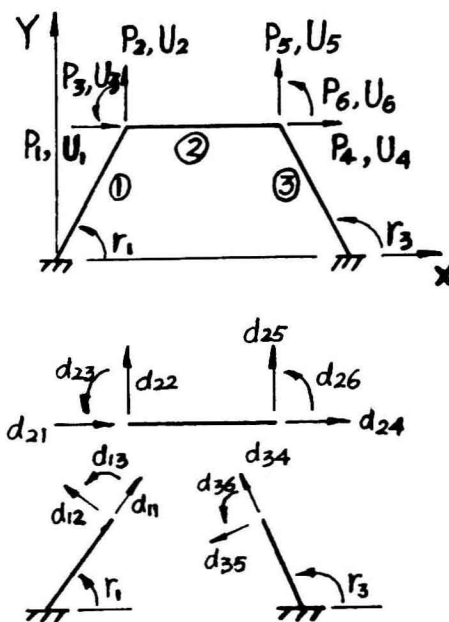


Fig. 1

And we can obtain

$$\begin{aligned}
 F_{11} &= \frac{EA_1}{L_1} d_{11} \\
 F_{12} &= \frac{12EI_1}{L_1^3} d_{12} - \frac{6EI_1}{L_1^2} d_{13} \\
 F_{13} &= -\frac{6EI_1}{L_1^2} d_{12} + \frac{4EI_1}{L_1} d_{13} \\
 F_{21} &= \frac{EA_2}{L_2} d_{21} - \frac{EA_2}{L_2} d_{24} \\
 F_{22} &= \frac{12EI_2}{L_2^3} d_{22} + \frac{6EI_2}{L_2^2} d_{23} - \frac{12EI_2}{L_2^3} d_{25} + \frac{6EI_2}{L_2^2} d_{26} \\
 F_{23} &= \frac{6EI_2}{L_2^2} d_{22} + \frac{4EI_2}{L_2} d_{23} - \frac{6EI_2}{L_2^2} d_{25} + \frac{2EI_2}{L_2} d_{26} \\
 F_{24} &= -\frac{EA_2}{L_2} d_{21} + \frac{EA_2}{L_2} d_{24} \\
 F_{25} &= -\frac{12EI_2}{L_2^3} d_{22} - \frac{6EI_2}{L_2^2} d_{23} + \frac{12EI_2}{L_2^3} d_{25} - \frac{6EI_2}{L_2^2} d_{26} \\
 F_{26} &= \frac{6EI_2}{L_2^2} d_{22} + \frac{2EI_2}{L_2} d_{23} - \frac{6EI_2}{L_2^2} d_{25} + \frac{4EI_2}{L_2} d_{26} \\
 F_{34} &= \frac{EA_3}{L_3} d_{34}, \quad F_{35} = \frac{12EI_3}{L_3^3} d_{35} - \frac{6EI_3}{L_3^2} d_{36} \\
 F_{36} &= -\frac{6EI_3}{L_3^2} d_{35} + \frac{4EI_3}{L_3} d_{36}
 \end{aligned} \tag{2}$$

From equilibrium equations of each joint

$$\sum X=0, \sum Y=0 \quad \text{and} \quad \sum M_z=0$$



We obtain

$$\begin{aligned} P_1 &= F_{11}l_1 - F_{12}m_1 + F_{21} \\ &= S_{11}U_1 + S_{12}U_2 + S_{13}U_3 + S_{14}U_4 + S_{15}U_5 + S_{16}U_6 \end{aligned}$$

where

$$S_{11} = \frac{EA_1}{L_1}l_1^2 + \frac{12EI_1}{L_1^3}m_1^2 + \frac{EA_2}{L_2}, \quad S_{12} = \left(-\frac{EA_1}{L_1} - \frac{12EI_1}{L_1^3}\right)l_1m_1$$

$$S_{13} = \frac{6EI_1}{L_1^2}m_1, \quad S_{14} = -\frac{EA_2}{L_2}, \quad S_{15} = S_{16} = 0$$

Do as above, we can obtain the following equation

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{pmatrix} \quad (3)$$

Solving Eq.(3), we can obtain the displacement components, and we can obtain the internal forces in each member by Eq.(1) and Eq.(2).

If there are loads act on members or there are some temperature changes on members, we can first fix the member and calculate the equivalent joint loads. Then solve the problem as done in Ex.1. The final internal forces can be obtained by fixed end forces + forces due to displacements.

If there are support settlements, we can solve it as done in II. Ex.4.

#### IV. A PROGRAM OF COMPUTER BASED EDUCATION FOR PLANE FRAMES

This program is one of a 4-year joint research project on 'Computer Based Education in Civil Engineering' funded by the Croucher Foundation. It is used to analyse plane frames by stiffness method. More important, it is used to provide students a mean of learning stiffness method of analysis step by step.

This program-package is divided into three parts. In the first part the students can understand the terminology, data preparation and steps of the computer stiffness method of the plane frame analysis with graphics. In the second part a detail example is illustrated so that the students

can study other problems more easily. In the last part some problems and discussions are introduced according to the teaching experience of the authors, so that the students can understand the analysis of plane frame thoroughly. There is also a solution part in the package for students to analyse other problems by themselves.

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In the analysis there are many functions in our program:

- 1) Beside flexural deformations, the axial deformations and shear deformations can be considered.
- 2) We can ignore shear deformations or axial deformations.
- 3) We can ignore both shear and axial deformations.

In the application there are many functions in our program too:

- 1) It can be used to analyse rigid frames with or without hinges. The effects of temperature changes and effects due to support settlements are considered.
- 2) It can be used to analyse continuous beams and trusses.
- 3) Multiple loading cases are considered.

This program is written in BASIC language in APPLE II, in FORTRAN 77 in CYB system and in IBM PC. So it can be used to teach students how to realise mathematical model in program languages. All the steps in analysis including problem solving are accompanied with detail graphics to help users to understand them clearly.

## V. PROGRAM SPECIFICATION

This program-package consists of three parts and a plane frame program. Users are suggested to follow the order of the parts. However, users may take any one of the parts and any part may be repeated individually if so required. The contents of the parts are given in the following sections.

### 1. TERMINOLOGY

This part is a glossary of terminology including terms used in the stiffness method introduced in this program. They are briefly explained on the screen. They are:

- 1) Computer stiffness method
- 2) Local axes
- 3) Global axes
- 4) Coordinates transformation
- 5) Element stiffness matrix
- 6) Structural stiffness matrix

Also in this part, the stiffness method based on matrix formation is explained on the screen. It is illustrated step by step to help users to understand this method. The steps are as follows

- 1) To explain the structure identifications.
- 2) To choose a convenient set of coordinates axes and calculate the length and cosine directions of members.
- 3) To formulate element stiffness matrix using direct-stiffness method. And to set up the structural stiffness matrix and boundary conditions.
- 4) To solve the simultaneous equations for displacements and evaluate the member forces.

## 2. DATA PREPARATION

In this part, how to prepare data for stiffness method especially for our program is introduced. They are divided into several sets as follows

- 1) Number of total joints, number of total members, number of loading cases, elastic modulus and shear modulus.
- 2) Are there any hinges in the frame?  
Consider axial deformations or not?  
Consider shear deformations or not?
- 3) Joint coordinates.
- 4) Member properties. They are number of 1st joint and number of 2nd joint of the member, moment of inertia of the member, area of cross section of the member (if axial or shear deformations are considered) and area shear factor of the member (if shear deformations are considered).
- 5) Support condition and support settlements.
- 6) Loading condition and condition of temperature changes.

## 3. EXAMPLE

In this part, a detail example is illustrated. It is analysed step by step with graphics, and different conditions are considered, such as whether we consider all the flexural, axial and shear deformations or ignore axial deformations, etc.

## 4. PROBLEMS AND DISCUSSIONS

In this part, there are 18 problems displayed on the screen, such as

- 1) What are the effects on the internal forces due to flexural, axial and shear deformations? The advantages and disadvantages of consideration of axial or shear deformations.

2) The influence of rearrange the member of frame.

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3) How to treat the frame with members of variable moment of inertia?

4) How to choose a better form of structure in design work?  
And so on.

After considering these problem by themselves, the users can get some discussions on the screen.

## VI. CONCLUSIONS

The present paper has shown a program for computer based education. It has been used by some students and teachers and shown that it is a very interesting package. It is said that this package is a very good tool in study structural analysis. We thank for Croucher Foundation very much. Other researches on 'Computer Based Education in Civil Engineering' studied in South China Institute of Technology, Hong Kong Polytechnic and Hatfield Polytechnic will be issued soon.

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**COMPUTER AIDED INSTRUCTIONS  
FOR  
UNDERGRADUATE STRUCTURAL ANALYSIS**

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## **INTRODUCTION**

Undergraduate Civil Engineering students are required to take at least one course of structural analysis. Educators have been challenged to preserve the fundamental nature of instruction in the classical methods, while adequately preparing students for the industry where computer based methodology is commonplace.

Many classical methods are introduced to give students a firm basis in which there is almost no time left for the discussion of the computer method. Therefore a computer aided instruction structure analysis package has been developed with these students in mind. This has been experimented at the University of Miami. Instructor spent about two hours on the program, after that the students were assigned problems to run with the program on their own time.

The program has been developed for the IBM personal computer family, due to the availability of such machine throughout the campus. The stiffness method is utilized. Emphasis is made on the graphical displays which is the best way to comprehend the behaviors of structure under load. User's manual is built into the package. The limitations of the program, sign conventions used, and concept of the stiffness method are explained in this user's manual.

## **PROGRAM DESCRIPTION**

The program is divided into four major modules; the user's manual, the input, the analysis, and the output modules. All modules reside in the same diskette which is called the program diskette. User needs to provide one data diskette for each problem solved.

User's manual module is divided into three major portions. The first portion is the limitation of that program version. The sign convention used in the program is explained in the second portion. The last portion gives a brief explanation of the stiffness method including the flowchart of the method of analysis. Figure 1 shows the sign convention used for the element load. Flowchart of the analysis is illustrated in Figure 2.

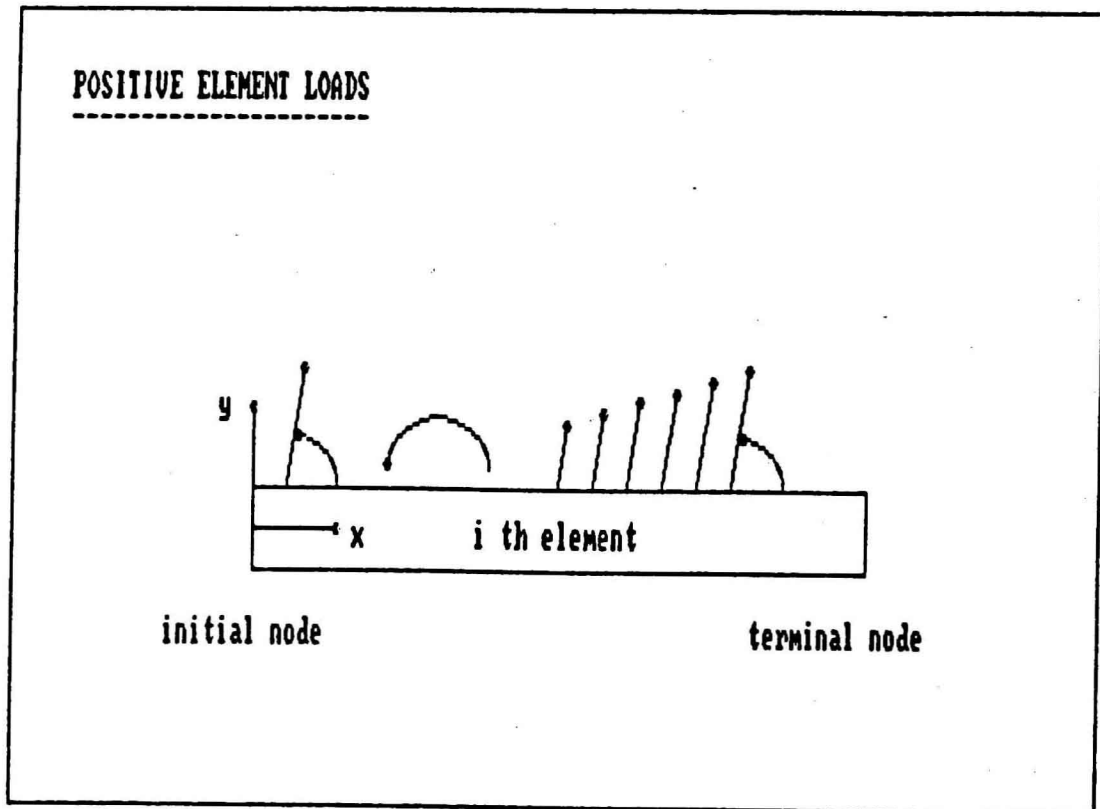


Figure 1. Positive directions of element load as explained in user's manual module.

In the input module, user has options to create a new data files for new problem, modify the existing data files, or display the undeformed structure geometry. The module is written such that user can interactively communicate with the computer. Mistakes can be checked and changed with ease. All the input data is stored in input data files on the data diskette. There are total of eight input data files, namely,

- PBID.DAT - problem identification input data file,
- CONO.DAT - control number input data file,
- NOCO.DAT - nodal coordinate input data file,
- ELCN.DAT - element connectivity input data file,
- ELPR.DAT - element property input data file,
- BCON.DAT - boundary condition input data file,
- NOFO.DAT - nodal force input data file, and
- ELLD.DAT - element load input data file.

Figure 3 gives a sample of input screen, while the editing mode or changing mode is shown in Figure 4. User may use graphical display of the undeformed structure geometry option to check the input data for nodal coordinates and element connectivities as shown in Figure 5.



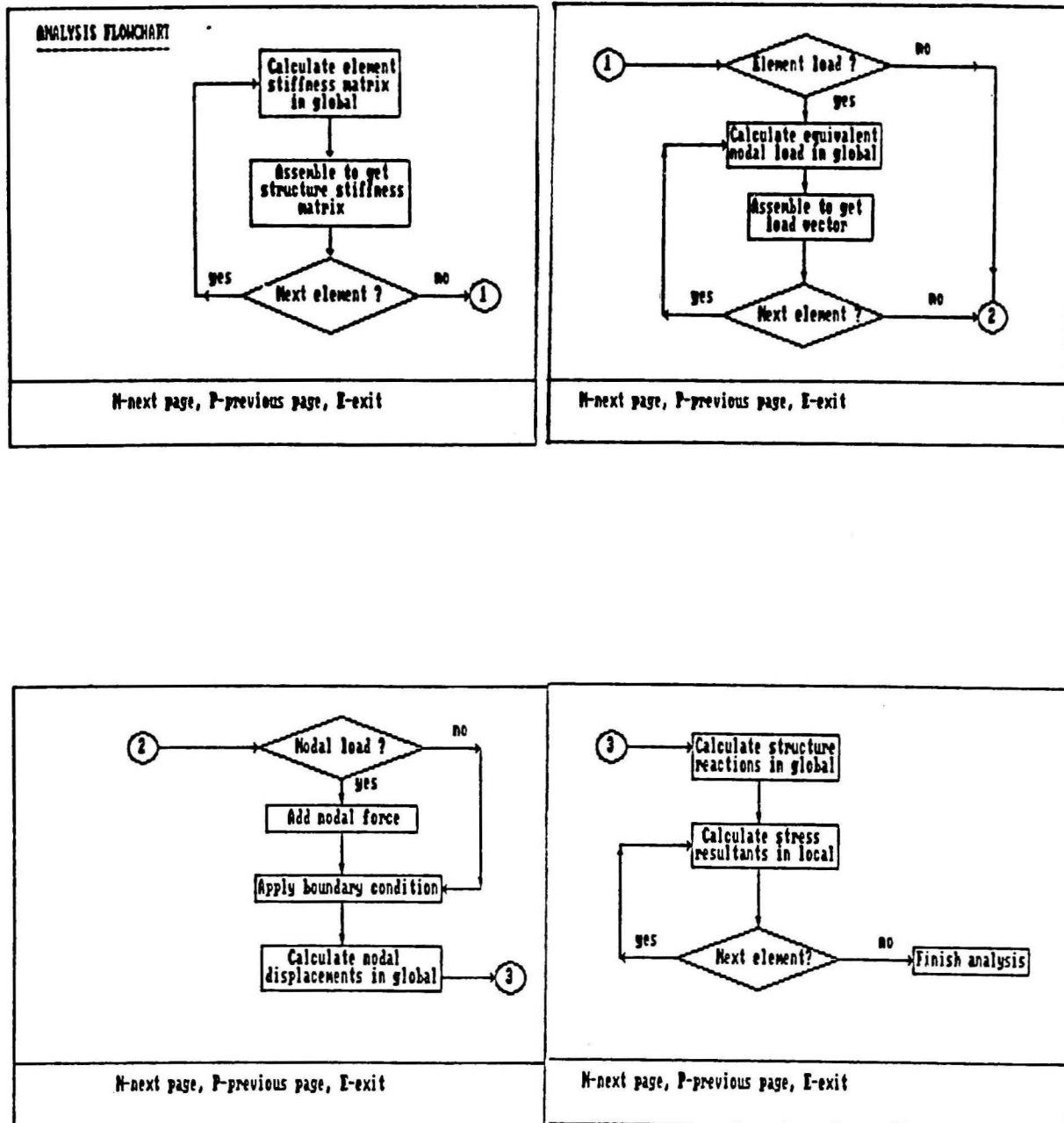


Figure 2. Flowchart of the analysis method as displayed on the screen.

