

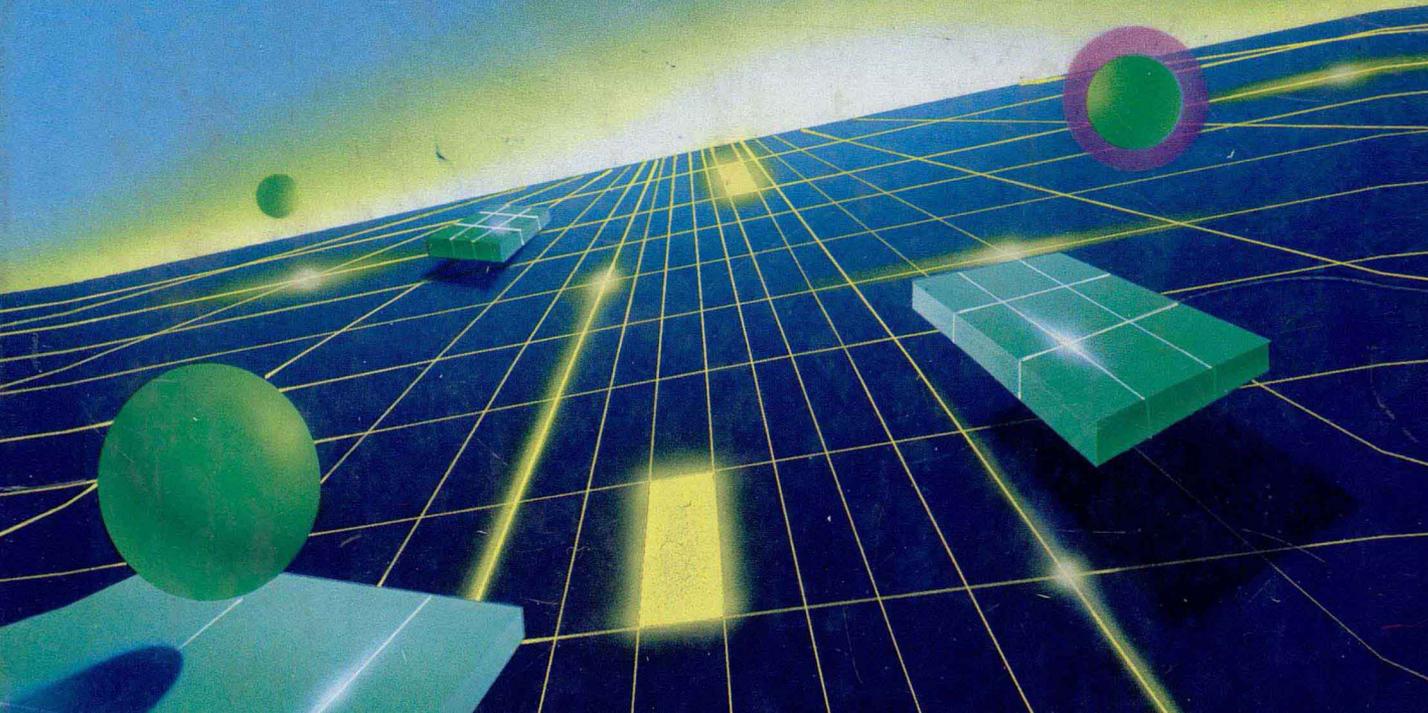
FINITE

FOURTH EDITION

MATHEMATICS

AND ITS APPLICATIONS

GOLDSTEIN / SCHNEIDER / SIEGEL





Fourth Edition

FINITE MATHEMATICS AND ITS APPLICATIONS

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Finite Mathematics and Its Applications, fourth edition

Larry J. Goldstein/David I. Schneider/Martha J. Siegel



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Mathematics and Its Applications

This volume is one of a collection of texts for freshman and sophomore college mathematics courses. Included in this collection are the following.

Calculus and Its Applications, fifth edition by L. Goldstein, D. Lay, and D. Schneider. A text designed for a two-semester course in calculus for students of business and the social and life sciences. Emphasizes an intuitive approach and integrates applications into the development.

Brief Calculus and Its Applications, fifth edition by L. Goldstein, D. Lay, and D. Schneider. Consists of the first eight chapters of the above book. Suitable for shorter courses with a slight reordering of some topics

Finite Mathematics and Its Applications, fourth edition by L. Goldstein, D. Schnieder, and M. Siegel. A traditional finite mathematics text for students of business and the social and life sciences. Allows courses to begin with either linear mathematics (linear programming, matrices) or probability and statistics.

Mathematics for the Management, Life, and Social Sciences, second edition by L. Goldstein, D. Lay, and D. Schneider. A text for a two-semester course covering finite mathematics, precalculus, and calculus.

PREFACE

This work is the fourth edition of our text for the traditional finite mathematics course taught to first- and second-year college students, especially those majoring in business and the social and biological sciences. Finite mathematics courses exhibit tremendous diversity with respect to both content and approach. Therefore, in revising this book, we have incorporated a wide range of topics from which an instructor may design a curriculum, as well as a high degree of flexibility in the order in which the topics may be presented. In the case of the mathematics of finance, we have even allowed for flexibility in the approach of the presentation.

In this edition, we have attempted to maintain our popular student-oriented approach. This approach manifests itself throughout and, in particular, in the following features:

- Applications** We provide realistic applications that illustrate the uses of finite mathematics in other disciplines. The reader may survey the variety of applications by turning to the Index of Applications on page xv. Wherever possible, we have attempted to use applications to motivate the mathematics. For example, the idea of linear programming is introduced in Chapter 3 via a discussion of production options for a factory with a labor limitation.
- Examples** We have included many more worked examples than is customary (549). Furthermore, we have included computational details to enhance readability by students whose basic skills are weak.
- Exercises** The more than 1900 exercises comprise about one-quarter of the text—the most important part of the text in our opinion. The exercises at the ends of the sections are usually arranged in the order in which the text proceeds, so that the homework assignments may easily be made after only part of a section is discussed. Interesting applications and more challenging problems tend to be located near the ends of the exercise sets. Supplementary exercises at the end of each chapter expand the

other exercise sets and provide cumulative exercises that require skills from earlier chapters.

Practice Problems The practice problems have proved to be a popular and useful feature of the book. The practice problems are carefully selected exercises located at the end of each section, just before the exercise set. Complete solutions are given following the exercise set. The practice problems often focus on points that are potentially confusing or are likely to be overlooked. We recommend that the reader seriously attempt the practice problems and study their solutions before moving on to the exercises. In effect, the practice problems constitute a built-in workbook.

Minimal Prerequisites Because of great variation in student preparation, we have kept the formal prerequisites to a minimum. We assume only a first year of high school algebra. Furthermore, we review, as needed, those topics which are typically weak spots for students.

NEW IN THIS EDITION

Among the changes in this edition, the following are the most significant.

1. *Additional Exercises.* The stock of exercises from the previous edition has been significantly expanded. Among the new exercises are some that test understanding and others that challenge the exceptional students.

2. *New Chapters on Logic and Graphs.* The recent recognition of the value of discrete mathematics in the curriculum has produced a shift in content in many finite mathematics courses. These chapters have been added to serve courses being modified in that direction. With the inclusion of this material, the book now covers nearly all the topics recommended by the CUPM Subcommittee on Discrete Mathematics. (See “Discrete Mathematics in the First Two Years,” *MAA Notes* 15, 1989.)

(a) The chapter on logic emphasizes the elements of propositional calculus, implication, and valid argument. The section on the predicate calculus pays particular attention to the precision of mathematical language and relates to sets and computing.

(b) The chapter on graph theory and its applications includes basic definitions, elementary theorems, and applications to business, social sciences, and computing. The introduction to PERT in the digraph section is enhanced by optional material on stochastic scheduling methods, thus linking this material to the chapters on probability and statistics. Matrix representation of graphs and digraphs provides a further application of matrix algebra.

This edition has more material than can be covered in most one-semester courses. Therefore, the instructor can structure the course to the students’ needs and inter-

ests. The book divides naturally into four parts. The first part consists of linear mathematics: linear equations, matrices, and linear programming (Chapters 1–4); the second part is devoted to probability and statistics (Chapters 5–7); the third part covers topics utilizing the ideas of the other parts (Chapters 8–10); and the fourth part explores key topics from discrete mathematics that are sometimes included in the modern finite mathematics curriculum (Chapters 11–13). We prefer to begin with linear mathematics since it makes for a smooth transition from high school mathematics and leads quickly to interesting applications, especially linear programming. Our preference notwithstanding, the instructor may begin this book with Chapter 5 (Sets and Counting) and then do either the linear mathematics or the probability and statistics.

Answers to the odd-numbered exercises are included at the back of the book.

If you have comments or suggestions, we would like to hear from you. We hope that you enjoy using this book as much as we have enjoyed writing it.

ACKNOWLEDGMENTS

While writing this book, we have received assistance from many persons. And our heartfelt thanks goes out to them all. Especially, we should like to thank the following reviewers, who took the time and energy to share their ideas, preferences, and often their enthusiasm, with us.

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Reviewers of the third edition: Robin G. Symonds, Indiana University at Kokomo; Elizabeth Teles, Montgomery College; Charles J. Miller, Foothill Community College.

Reviewers of the fourth edition: Phil Steitz, Beloit College. Barry Cipra.

The authors thank the many people at Prentice Hall who contributed to the success of our books. We appreciate the efforts of the production, art, manufacturing, marketing, and sales departments. An extra special thanks to Steve Comny, acquisitions editor at Prentice Hall, for his help in planning and executing this new edition. His partnership and friendship have added a warm personal dimension to the writing process.

*Larry J. Goldstein
David I. Schneider
Martha J. Siegel*

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INTRODUCTION

