



CALCULUS

WITH APPLICATIONS

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The Gallup Organization

McGRAW-HILL PUBLISHING COMPANY

New York St. Louis San Francisco Auckland Bogotá Caracas
Hamburg Lisbon London Madrid Mexico Milan Montreal
New Delhi Oklahoma City Paris San Juan São Paulo
Singapore Sydney Tokyo Toronto

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1 2 3 4 5 6 7 8 9 0 VNH VNH 8 9 4 3 2 1 0 9

ISBN 0-07-557045-9

This book was set in Times Roman.
The editors were Robert A. Weinstein and John M. Morriss;
the designer was DeNee Skipper;
the production supervisor was Michael Weinstein.
Von Hoffmann Press, Inc., was printer and binder.

Library of Congress Cataloging-in-Publication Data

Burgmeier, James W.

Calculus with applications/James W. Burgmeier, Monte B. Boisen, Jr.
Max D. Larsen.

p. cm.

Bibliography: p.

Includes index.

ISBN 0-07-557045-9

1. Calculus. I. Boisen, Monte B., Jr. II. Larsen, Max D.

III. Title

QA303.B926 1990

515—dc 19

88-18295

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WITH APPLICATIONS

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WITH APPLICATIONS

This text is dedicated to our families, for their patience, support, and love.

Pat, Glenn, and Robin

Helen, John, and Jennifer

Lillie, Mike, Paul, and Charlie

PREFACE

The purpose of this text is to develop the student's understanding of the basic concepts of calculus through realistic applications from the managerial, social, and life sciences. Numerous examples motivate the theoretical concepts and show how calculus can be applied in these areas. Realistic applications show that the mathematics presented in this course will have an impact on the student's future work. This approach ensures that the student learns the concepts, power, and usefulness of calculus.

The text presents the fundamentals of calculus intuitively and clearly without sacrificing accuracy. The balance of prose explanations and algebraic details found in the solutions to examples is the result of years of classroom experience. Instructors and students alike will find the text clear, concise, accurate, and readable.

Distinguishing Features

- ♦ ***Emphasis on examples:*** The many carefully explained examples illustrate procedures, show applications of the material, and elaborate on mathematical content. In many cases, the examples are somewhat idealized: we have adjusted the numerical constants to avoid tedious and distracting arithmetic. Students can thus focus attention on the calculus concept involved and grasp the mathematics more easily and quickly.
- ♦ ***Emphasis on applications:*** We have included a wealth of applications to the managerial, social, and life sciences adapted from the recent literature of these sciences. Each application has been chosen to represent faithfully the discussion from which it came as well as to provide a straightforward illustration of the calculus topic under study. Examples from each of the managerial, social, and life sciences are presented for most of the mathematical topics in this text, so instructors can emphasize one discipline or another according to the interests of the students. These examples require no specialized knowledge. The range of applications provides evidence that the mathematics learned in this course has relevancy and usefulness beyond the classroom.
- ♦ ***Exercises:*** Nearly 3400 exercises are included to give students ample opportunity to test their understanding and apply their

skills to real-world problems. Our experience shows that the student's understanding of calculus is linked directly to the number of problem-types solved successfully; we believe that the quality and variety of our exercises will foster this success. The answers to the odd-numbered exercises are given at the back of the book, and many of these include intermediate results so students can check their work. Answers to the even-numbered exercises are given in the Instructor's Manual.

- ♦ **Figures and photographs:** Wherever possible, we have used figures to motivate new concepts and as an aid in problem-solving. These figures are clearly drawn and carefully captioned and labeled to enhance student comprehension. In addition, the abundance of photographs throughout the text attests to the range of applied problems in this book and the diverse uses of mathematics. Every photo is carefully keyed to an applied example or topic in the text, illustrating the usefulness of calculus in the real world.
- ♦ **Bibliography:** The annotated bibliography lists sources for the applied problems and other works in which calculus is used at the level presented in this text. Readers who would like more information on the applications will find numerous source materials listed here.

Special Pedagogical Issues

- ♦ **Limits and continuity:** While our discussion of limits is intuitive, it does not present the material in a dishonest fashion. We have made a special effort to use applications to set the stage in Section 2.1 for the need for limits. Section 2.2 then discusses limits and how to calculate the limits of various functions. We have deliberately postponed continuity until Section 2.7 so that students can go immediately to the derivative in Section 2.3. Instructors who wish to omit Section 2.7 may do so without disruption. This organization should hold the student's attention and clarify why these concepts are important.
- ♦ **Derivatives:** Section 2.3 uses the informal treatment of limits given in Sections 2.1 and 2.2 to define the derivative and cover the simple algebraic rules for finding derivatives. The product, quotient, and chain rules are postponed until Section 3.1 in order to present some of the uses of the derivative immediately. Our experience has shown that this approach has more appeal for students.

Sections 2.4 to 2.6 discuss several ways to interpret the derivative. Section 2.4 covers the slope of the tangent line and how it is related to the derivative. Section 2.5 discusses increasing and decreasing functions so students see a direct implementation of the tangent line concept. Section 2.6 uses applications to explore the derivative as a rate of change. These sections discuss various interpretations of the derivative and several of its applications.

- **Curve sketching:** In Section 4.2 we present a checklist for graphing a function. Sign charts and carefully labeled preliminary sketches accompany the examples; the final graphs are labeled with the important features of the functions. A detailed summary of the data is given under the caption of each figure. The examples and exercises in this section cover various types of functions to give students plenty of practice in graphing functions.
- **Numerical methods:** Computing technology has evolved enormously in the past several years and an increasing number of disciplines use numerical methods to approximate solutions to mathematical problems and to analyze data. Therefore, we introduce a variety of numerical methods throughout the text to acquaint students with the power and scope of these techniques. In Chapter 12 we discuss Newton's root-finding method, Euler's method for solving differential equations, and function evaluation via Taylor and Maclaurin series. Furthermore, we discuss the total differential of several variables (Section 8.5), numerical integration (Sections 6.5 and 7.4), and the method of least squares (Section 8.6).

Features for the Student

- **Algebra review:** We have assumed that students taking this course will have had two years of high-school algebra or the equivalent. However, Appendix A includes a review of basic algebra. The numerous examples and exercises enable students to refresh their knowledge of algebra and to pinpoint those areas in which a deficiency may exist. They also give the instructor an option to cover as much review as needed.
- **Algebraic steps:** We know that students get discouraged if too many algebraic details are omitted. Therefore, in the exposition and in the examples, algebraic steps are clearly marked and explained. Many answers in the back of the book also include steps to the solution so students can check intermediate results. By providing intermediate steps we encourage students to persist in their problem-solving efforts.
- **Table format:** Except for some of the examples, the material in the text has been classroom-tested over a period of several years. Many pedagogical aspects of the text stem from this testing, including our special table format for many procedures. This format takes advantage of the fact that a compact array of information is more easily assimilated than a long list. These table formats have proved to be successful for learning and implementing many procedures, including integration by parts, the trapezoidal rule, Simpson's rule, the derivative tests for one and two independent variables, the method of least squares, and the Newton root-finding method.
- **Plausibility arguments:** The primary pedagogical tool we use is

examples. Therefore, we have included only those proofs and theoretical discussions that contribute to an understanding of calculus and its applications. Even in these cases we often use informal arguments since we believe students will find them more meaningful than formal mathematical proofs.

For Each Chapter

- **Chapter introduction:** Most of the chapter introductions discuss applications of the material; these applications and many others are then explored and developed in the examples and exercises of that chapter.
- **Summary:** Each chapter concludes with a concise summary highlighting the chapter's important concepts, definitions, formulas, and procedures. Students will find this summary an excellent study and review tool.
- **Review exercises:** In addition, at the end of each chapter, there is an extensive set of review exercises to test student comprehension and problem-solving skills. In keeping with our emphasis on applications, almost every review exercise set includes applied problems as well as numerous exercises to test skills and understanding.

Accuracy

Every effort has been made to verify that this text is free from error. The exercises were solved independently by James W. Burgmeier and Wendy Fenwick. The examples have been solved and checked by several people, including Garret Etgen, Linda Holden, and Vincent P. Schielack. We are grateful for their keen eyes and attention to detail. If, however, you do find an error, please advise our publisher immediately so that it can be corrected in subsequent printings.

Flexibility

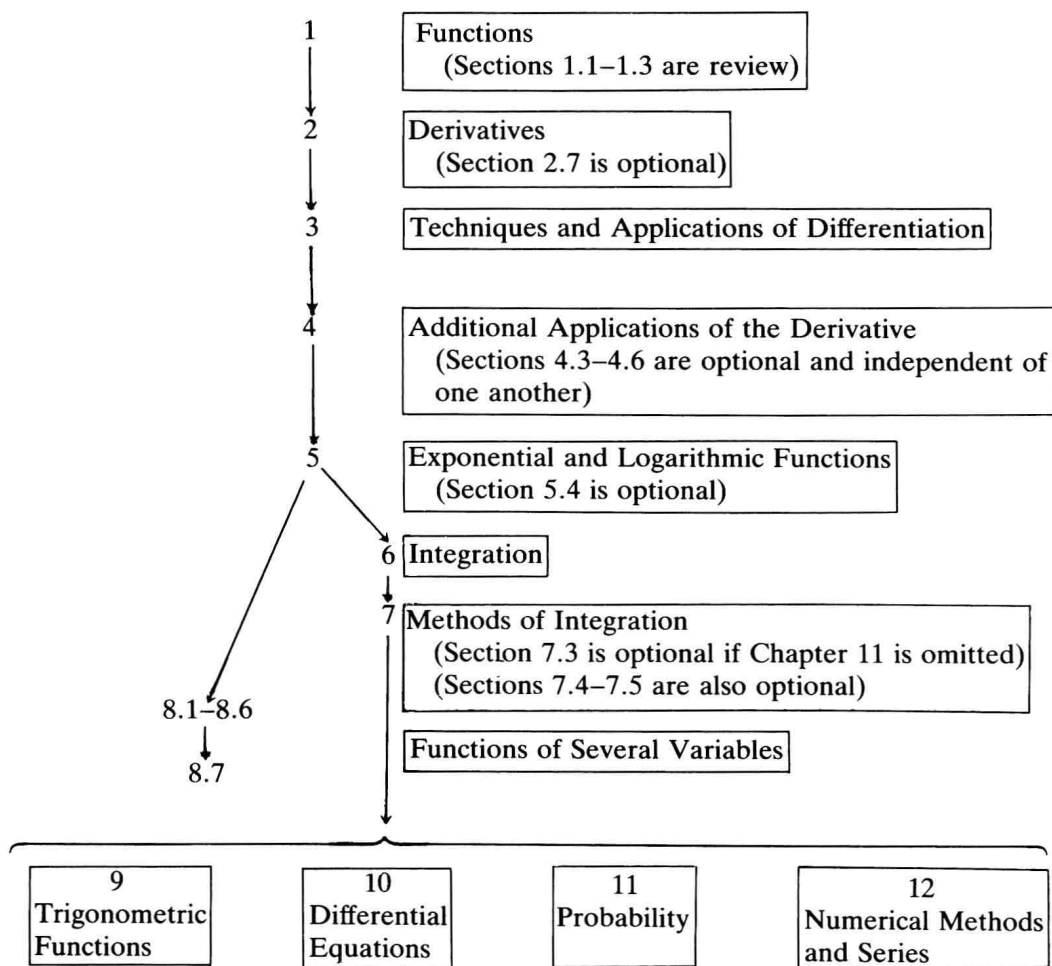
Chapters 1 through 6 can be covered in one semester and the entire text in two semesters. Since several chapters are independent of one another, there are various possible sequences of chapters, as the schema on the next page illustrates. For a one-semester course for business or managerial students, Sections 8.1 through 8.6 may be substituted for Chapter 6.

Students who have a thorough precalculus background could skip Sections 1.1 through 1.3 and briefly study the material in the remainder of Chapter 1. The other optional sections in Chapters 1 through 7 are 2.7; 4.3 through 4.6; 5.4; 6.5; 7.3 (if Chapter 11 is omitted); 7.4 and 7.5. Chapter 8 is also optional. Chapters 9 through 12 are independent from each other except for an occasional example and some exercises. These examples and exercises can be ignored without disruption. Section 12.1 on Newton's root-finding method can be covered any time after Chapter 5. An edition of this text consisting of Chapters 1 through 8 is available for schools with a one-semester or two-quarter course.

Section Length

Generally, each section presents the amount of material that is appropriate for one fifty-minute lecture. Exceptions to this rule, for example, Sections 4.2, 5.5, and 6.5, contain many examples and applications, rather than new mathematical material. Time devoted to these sections should depend on the disciplines represented in a particular class.

Chart of Dependent Chapters for *Calculus with Applications*



Supplements

This text is accompanied by a well-rounded supplements package, including Instructor's Manual, a Student Solutions Manual, Test Bank, Computerized Test Bank and software.

♦ **Student Solutions Manual:** The Student Solutions Manual was prepared by James W. Burgmeier and Wendy Fenwick and contains detailed solutions to selected odd-numbered exercises.

♦ **Instructor's Manual:** The Instructor's Manual was prepared by James W. Burgmeier and Wendy Fenwick, who prepared answers to

even-numbered exercises, and Jan E. H. Johansson, who prepared sample tests. It includes three forms of a chapter test for each chapter, three forms for two mid-term exams, and three forms of two final exams to insure a plentiful supply of questions for instructors.

• **Test Bank:** The questions found in the sample tests, and additional test questions, are also available in a test bank form which corresponds to a computerized version for IBM-PC and compatibles.

The supplements package also includes student software. For more details, please contact your local McGraw-Hill Sales Representative.

Reviewers

We would like to thank the many reviewers who have read all or part of the manuscript. Their encouragement, comments, and suggestions have been most helpful.

Ann O'Connell	Providence College
Bruce H. Edwards	University of Florida
Garret Etgen	University of Houston
Brian D. Hassard	S.U.N.Y.-Buffalo
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Vincent P. Schielack	Texas A&M
Daniel Shea	University of Wisconsin
Clifford W. Sloyer	University of Delaware
Robert E. White	North Carolina State University

Acknowledgments

We would like to express our appreciation to Wendy Fenwick at the University of Vermont for her work on the Student Solutions Manual and the Instructor's Manual; to Jan E. H. Johansson for preparing the Test Bank; to W. David Klemperer for his helpful suggestions on the forestry applications; and to Pat Burgmeier for typing the original manuscript.

Finally, we want to thank the editorial and production staff at McGraw-Hill and Random House: John Martindale, Senior Editor; Alexa Barnes, Development Editor; Margaret Pinette, Project Manager; her assistant, Julia Kerr; Michael Weinstein, Production Manager; his assistant, Susan Brown; and John Morriss, Editing Manager. Their help and support on this project have been invaluable.

*James W. Burgmeier
Monte B. Boisen, Jr.
Max D. Larsen*

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