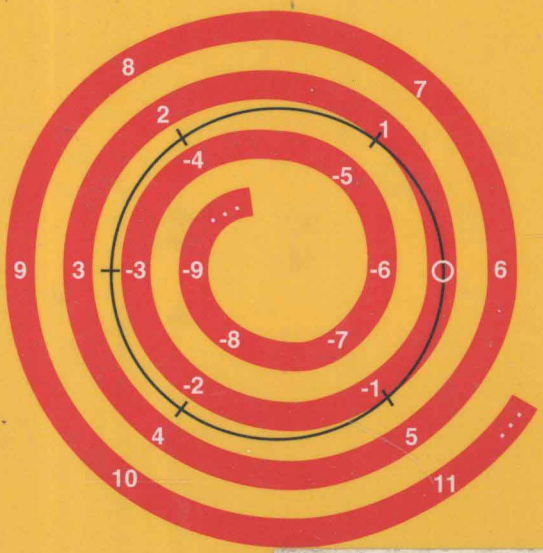
Undergraduate Texts in Mathematics

Lindsay Childs

A Concrete Introduction to Higher Algebra



CHILDS A605
CONCRETE INTRO TO HIGHER ALG
9 780387 903330
8 495400 A605 01 4933600 A605

Springer-Verlag

Lindsay Childs

A Concrete Introduction to Higher Algebra

Springer-Verlag
New York Berlin Heidelberg London Paris
Tokyo Hong Kong Barcelona Budapest

Lindsay Childs
Department of Mathematics
SUNY at Albany
Albany, New York 12222
USA

Editorial Board

J.H. Ewing
Department of Mathematics
Indiana University
Bloomington, IN 47401
USA

F.W. Gehring
Department of Mathematics
University of Michigan
Ann Arbor, MI 48109
USA

P.R. Halmos
Department of Mathematics
Santa Clara University
Santa Clara, CA 95053
USA

To Ashley and Nathan

AMS Subject Classification: 12-01

Library of Congress Cataloging in Publication Data
Childs, Lindsay, N.
A concrete introduction to higher algebra.

(Undergraduate texts in mathematics)
Bibliography: p.
Includes index.
I. Algebra. I. Title.
QA155.C53 512.9 78-21870

Printed on acid-free paper.

With 9 Illustrations.

© 1979 by Springer-Verlag New York, Inc.
All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA) except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.
The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Printed and bound by Edwards Brothers, Ann Arbor, Michigan.
Printed in the United States of America.

9 8 7 6 (Corrected Printing, 1992)

ISBN 0-387-90333-X Springer-Verlag New York Berlin Heidelberg
ISBN 3-540-90333-X Springer-Verlag Berlin Heidelberg New York

Undergraduate Texts in Mathematics

Editors
J. H. Ewing
F. W. Gehring
P. R. Halmos

Undergraduate Texts in Mathematics

Apostol: Introduction to Analytic Number Theory. Second edition.
Armstrong: Groups and Symmetry.
Armstrong: Basic Topology.
Bak/Newman: Complex Analysis.
Banchoff/Wermer: Linear Algebra Through Geometry. Second edition.
Brémaud: An Introduction to Probabilistic Modeling.
Bressoud: Factorization and Primality Testing.
Bressoud: Second Year Calculus.
Readings in Mathematics.
Brickman: Mathematical Introduction to Linear Programming and Game Theory.
Cederberg: A Course in Modern Geometries.
Childs: A Concrete Introduction to Higher Algebra.
Chung: Elementary Probability Theory with Stochastic Processes. Third edition.
Cox/Little/O'Shea: Ideals, Varieties, and Algorithms.
Curtis: Linear Algebra: An Introductory Approach. Fourth edition.
Devlin: The Joy of Sets: Fundamentals of Contemporary Set Theory. Second edition.
Dixmier: General Topology.
Driver: Why Math?
Ebbinghaus/Flum/Thomas: Mathematical Logic.
Edgar: Measure, Topology, and Fractal Geometry.
Fischer: Intermediate Real Analysis.
Flanigan/Kazdan: Calculus Two: Linear and Nonlinear Functions. Second edition.
Fleming: Functions of Several Variables. Second edition.
Foulds: Optimization Techniques: An Introduction.
Foulds: Combinatorial Optimization for Undergraduates.
Franklin: Methods of Mathematical Economics.
Halmos: Finite-Dimensional Vector Spaces. Second edition.
Halmos: Naive Set Theory.
Hämmerlin/Hoffmann: Numerical Mathematics.
Readings in Mathematics.
Iooss/Joseph: Elementary Stability and Bifurcation Theory. Second edition.
James: Topological and Uniform Spaces.
Jänich: Topology.
Klambauer: Aspects of Calculus.
Kinsey: Topology of Surfaces.
Lang: A First Course in Calculus. Fifth edition.
Lang: Calculus of Several Variables. Third edition.
Lang: Introduction to Linear Algebra. Second edition.
Lang: Linear Algebra. Third edition.
Lang: Undergraduate Algebra. Second edition.
Lang: Undergraduate Analysis.
Lax/Burstein/Lax: Calculus with Applications and Computing. Volume 1.
LeCuyer: College Mathematics with APL.

(continued after index)

此为试读,需要完整PDF请访问: www.ertongbook.com

Preface

This book is written as an introduction to higher algebra for students with a background of a year of calculus. The book developed out of a set of notes for a sophomore–junior level course at the State University of New York at Albany entitled Classical Algebra.

In the 1950s and before, it was customary for the first course in algebra to be a course in the theory of equations, consisting of a study of polynomials over the complex, real, and rational numbers, and, to a lesser extent, linear algebra from the point of view of systems of equations. Abstract algebra, that is, the study of groups, rings, and fields, usually followed such a course.

In recent years the theory of equations course has disappeared. Without it, students entering abstract algebra courses tend to lack the experience in the algebraic theory of the basic classical examples of the integers and polynomials necessary for understanding, and more importantly, for appreciating the formalism. To meet this problem, several texts have recently appeared introducing algebra through number theory.

This book combines the newer number-theoretic approach with the old theory of equations. In fact, the book contains enough of each of elementary number theory and the theory of equations that a course in either could be taught from it (see below). But the algebraic similarities of the two subjects are such that both subjects can be developed in parallel, and ideas customarily associated with one can be transferred to the other. Thus the ideas of congruence and congruence classes, normally arising in elementary number theory, can also be used with polynomials. Doing so permits passage from the study of polynomials to the study of simple field extensions, and in particular, leads to an exposition of finite fields.

There are, I feel, several advantages in beginning the study of higher algebra by studying number theory and polynomial theory.

First, the algebra is built on the student's entire mathematical experience. The study of numbers and polynomial equations dominates the precollege mathematical training. By building on this background a course in algebra will be building on the strongest possible intuitive base. And given such a base, the potential for reaching results of significance is high. I hope that this potential is realized by this book's theoretical development, numerous applications, and exercises.

Second, the dominating algebraic idea in the development of the book is that of congruence classes. The concept of quotient structure is perhaps the most difficult of the concepts of abstract algebra. The experience of seeing it in a variety of concrete contexts, and seeing worthwhile consequences of its use, should greatly aid the student when subsequently it is seen in abstract presentations. This particular feature of our approach is one which was missing from traditional theory of equations courses, and also is missing in courses in linear algebra used as background for abstract algebra.

Third, the subject matter of the book is intrinsically worth studying. Both number theory and the theory of equations have attracted the attention of the very greatest mathematicians. In particular, two of Gauss's greatest achievements, the fundamental theorem of algebra and the law of quadratic reciprocity, are important results in this book. One of the important lines of research in modern algebraic geometry stems from A. Weil's 1949 paper on solutions of equations in finite fields, a topic which is beyond the level, but very much in the tradition, of the material in this book (see Ireland and Rosen (1982) for an exposition). But even at the level of this book the subjects have attracted the notice of combinatorial analysts and computer scientists in recent decades. A surprising amount of the material in this book dates from since 1940. As I discovered only late in the writing of this book, there is considerable overlap between it and the mathematics in Chapter 4, "Arithmetic," of D. E. Knuth's fundamental treatise, *The Art of Computer Programming* (Knuth, 1969). Thus the mathematics in this book is worth learning for its own sake, apart from any value it has in preparing for more advanced mathematics.

The explicit prerequisites of the book consist for the most part only of high school algebra (in the *de facto* sense, not in the sense of Abhyankar (1976)—in his sense this *is* predominantly a high school algebra text). In various places we assume some acquaintance with calculus; however, the subject of differentiating polynomials is developed from the beginning (Chapter II-6), and the one place where integration occurs explicitly (Section II-5C) may be omitted.[1] Two-variable calculus is mentioned only in the proof of the fundamental theorem of algebra (II-3), but either the facts needed there can be taken on faith or the proof can be omitted. The

[1] Chapter 6 of Part II is referred to as Chapter II-6 or simply as II-6; if the reference occurs within Part II, simply as Chapter 6. II-5C refers to Section C of Chapter II-5.

use of infinite series is more substantial, however, particularly in connection with decimal expansions of fractions.

Several of the applications and a few of the theoretical sections use matrices and ideas from linear algebra. Chapter I-9 is an overly concise review of the necessary ideas. Sections C–F of I-9 should be used only for reference. If linear algebra is lacking in the student's background, the chapters particularly to avoid are II-12 and the last half of III-9. The remaining uses of linear algebra mainly involve simple matrix manipulations as described in Section I-9A, and these are quickly learned.

Exercises are scattered throughout the text as well as collected at the ends of sections. They range from routine examples to ingenious problems to extensions of theory. The most nontrivial ones are starred; comments on them are collected in the back of the book. Exercises which are mentioned subsequently either in the text or in exercises are marked with a dagger, and are indexed together with the subsequent references in the back of the book.

There is more material in this book than would be appropriate for a one semester course. For a year course it could be supplemented with a not-too-geometric introduction to linear algebra (such as Zelinsky (1973)). For a one semester course there are a variety of routes through the book.

The main development in Parts I and II is contained in

I—: 1–3, 4A, 5A, B, D, 6–8, 11, 14A;
II—: 1, 2, 3A, 4, 6, 8–10, 11A.

To get to the classification of finite fields (III-14) most efficiently, follow the main development with

III—: 1, 4, 7, 8, 10, 11, (12), 13, 14.

To get to algebraic numbers (III–18) most efficiently, follow the main development with

III—: 1, 4, 7, 8, 10, 16A, B, 18–21.

To concentrate on elementary number theory, see

I—: all except 9C–F and 4B, 4C, 5C, 9B, 10, 13, 14B, 15 as desired;
II—: 1, 2A;
III—: 1–5, 16A, B, 17; then
II—: 3A, 8–10, 11A;
III—: 7, 8, 10, 18–21.

To concentrate on theory of equations, see:

I—: 1–3, 4A, 6–8, 11, 14A;
II—: all, omitting 3C, 5, 7, 11B, 12 as desired;
III—: 1, 4, 6–8, 10, 11, 13–16.

It is probably unwise to spend too much time in Chapter I-2. Also, part of the uniqueness of this book lies in the chapters on applications, so it is hoped that any route through the book will be chosen to allow time for visits to some of the scenic wayside areas.

Finally, I wish to acknowledge with appreciation the contributions of people who in various ways influenced the book: Ed Davis, for developing and selling the idea for the course for which the book was written, and for a number of useful comments on an early version of the notes; Bill Hammond, for teaching from the notes and offering a number of improvements; Violet Larney, for teaching from the notes graciously even though her book (for a competing course) had just appeared; Morris Orzech, Paulo Ribenboim, Tony Geramita, Ted Turner, and Ivan Niven, for a variety of mathematical insights and ideas; and, especially, Malcolm Smiley, for reading through and teaching from the manuscript in its late stages and offering many substantial suggestions for improving the exposition, and David Drasin, for reading through and making many helpful comments on the nearly completed manuscript. Also I wish to thank: Michele Palleschi for typing most of the manuscript even though she didn't have to, and Mrs. Betty Turner and her staff for typing most of the manuscript even though they weren't supposed to; the Universities of Illinois (Urbana) and Oregon for their hospitality during part of the time the book was written; and Springer-Verlag, particularly Walter Kaufmann-Buehler and Joe Gannon, for their professional treatment of the manuscript. Most of all, my greatest thanks go to my wife Rhonda, for putting up with my working on the manuscript at inconvenient hours at inconvenient locations.

Fall, 1978 L. CHILDS

My thanks to those who pointed out misprints and errors in the first printing, including Louis Brickman, Linda Deneen, William Hammond, Irving Kaplansky, Keith Kendig, Richard Patterson, Alan Sprague, Mel Thornton, Ted Turner, S. Wang, and especially Ernst S. Selmer.

May, 1984 L. CHILDS

Many thanks to Frank Gerrish of Surrey, England, for finding and communicating to me over 300 misprints and other comments on the third printing.

April, 1992 L. CHILDS

Contents

此为试读,需要完整PDF请访问: www.ertongbook.com

I. INTEGERS

This part of the book is about the natural numbers and integers. Among the highlights of this part, we show that every natural number factors uniquely into a product of primes, define congruence mod m, and invent new sets called congruence classes mod m, which for each $m \geqslant 2$ add and multiply to form a new algebraic system called $\mathbb{Z}_m$. Various related results about numbers, and applications, fill out this part.

此为试读,需要完整PDF请访问: www.ertongbook.com