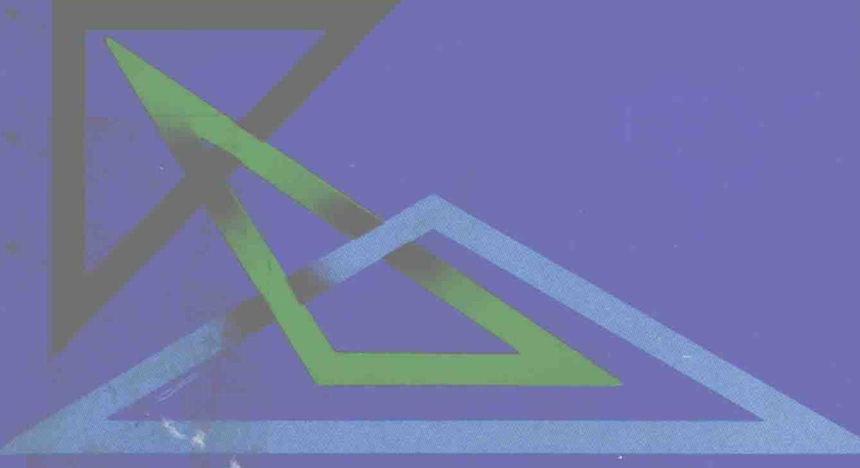
ISIDORE DRESSLER
& BARNETT RICH

Trigonometry



Trigonometry

Isidore Dressler

Barnett Rich

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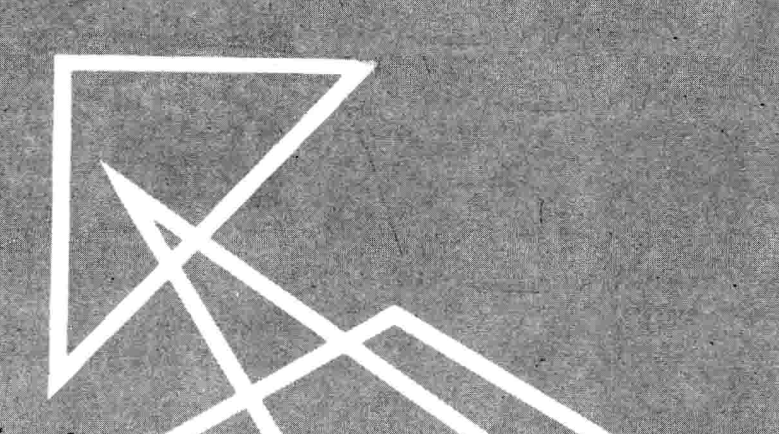


PREFACE

Trigonometry presents a comprehensive course of study that is fully in accord with trigonometry courses throughout the United States.

The book emphasizes an understanding of the structure and processes of mathematics, as well as the acquisition of the necessary manipulative skills. In short, the major objective of the text is to fuse the "why" and the "how" by presenting each topic from a modern point of view. To make mathematics teaching more effective, the book stresses broad, basic, and unifying mathematical concepts such as those of sets and function. For example, function is treated at length in a wide variety of topics such as exponential functions, logarithmic functions, trigonometric functions, inverse trigonometric functions, and circular functions.

Each reader is urged to note the simplicity of language and the informal style. Students should have little or no difficulty reading and understanding the text. Note further the organization and exposition of the materials in a self-teaching order, meaningfully and pedagogically developed.

The first chapter provides for an effective review of elementary trigonometric principles and techniques.

To help students make a smooth transition from working with algebraic expressions and equations to working with trigonometric expressions and equations, Chapter II is devoted entirely to the integration of algebra and trigonometry. To accomplish this, *each practice exercise is presented in two parallel forms, the algebraic form on the left half of the page and the related trigonometric form on the right half.* Each model problem is solved both in an algebraic form as well as in its analogous trigonometric form. If a student has difficulty with the lengthier trigonometric form, the model problems reveal how a problem can be revised into the shorter and simpler algebraic form. This integration technique is continued in later chapters with each new trigonometric unit of work. See, for example, the integrative technique in Chapter VI, which deals with identities and equations.

Included in the text are the following features:

1. The comprehensive treatment of the function concept as a unifying theme.
2. The presentation of coordinate geometry as a most effective means of understanding complex numbers as well as trigonometric functions.

3. The presentation and development of logarithmic and exponential properties and their applications.
4. The extensive treatment of forces and vectors.
5. The use of circular functions as another approach to trigonometry.
6. The thorough treatment of the numerical aspects of trigonometry in both the non-logarithmic and logarithmic solutions of right and oblique triangles.
7. The provision of numerous applications in such fields as mensuration, physics, and navigation.
8. The extension of the study of numbers to include the system of complex numbers and its use in the understanding of vectors and the sets of real numbers.
9. The last chapter is devoted to the proofs, derivations, and interrelationships of the important trigonometric formulas.
10. The inclusion of challenging honor units for the superior student: (1) vectors, (2) complex numbers, and (3) circular functions.

An important feature of the text is the character of its organization:

1. Each chapter is divided into related and sequential learning units which, with proper application, a student can readily master.
2. The basic concepts and principles of each unit are carefully developed using precise, yet simple, language and symbolism.
3. New terms are clearly defined.
4. Explanations and problems suitable for teaching purposes lead to the statement of the general principles and procedures involved in a unit.
5. Model problems, whose solutions are accompanied by detailed step-by-step explanations, are most helpful in showing students how to apply the related principles and how to follow the necessary procedures.
6. A set of expertly selected and carefully graded exercises covering types of problems appropriate for a course in trigonometry enables a student to understand thoroughly the basic concepts and related procedures of each unit. The exercises enable him to test the extent of his mastery of the unit.
7. Each chapter includes exercises of more-than-average difficulty, as well as enrichment materials to challenge the superior student.

In summary, *Trigonometry* makes available to both students and teachers an abundance of teaching and practice materials for the mastering of a course in trigonometry.

The authors are deeply grateful to Mr. Harry Schor, Mr. Melvin Klein, and Dr. Robert E. Dressler for their many valuable suggestions and criticisms.

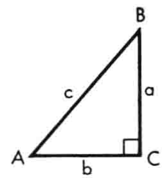
Isidore Dressler
Barnett Rich

CONTENTS

CHAPTER I

TRIGONOMETRY OF THE RIGHT TRIANGLE

1. Defining the Six Trigonometric Functions



In the figure, triangle ABC is a right triangle in which C is the right angle. $\overline{AB}$, the side opposite $\angle C$, is the ***hypotenuse*** of the right triangle; the length of $\overline{AB}$ is represented by c. The other two sides of $\triangle ABC$, $\overline{BC}$ and $\overline{AC}$, are called the ***legs*** of the triangle. We call $\overline{BC}$ the leg opposite $\angle A$; the length of $\overline{BC}$ is represented by a. We call $\overline{AC}$ the leg opposite $\angle B$; the length of $\overline{AC}$ is represented by b. We may also call $\overline{AC}$ the leg adjacent to (next to) $\angle A$. We may also call $\overline{BC}$ the leg adjacent to (next to) $\angle B$.

Now we will define six ratios, each of which involves two sides of the right triangle. These ratios are called ***trigonometric ratios.***

For either acute angle in a right triangle:

$$\text{sine (sin) of the angle} = \frac{\text{length of leg opposite the angle}}{\text{length of hypotenuse}}$$

$$\text{cosine (cos) of the angle} = \frac{\text{length of leg adjacent to the angle}}{\text{length of hypotenuse}}$$

$$\text{tangent (tan) of the angle} = \frac{\text{length of leg opposite the angle}}{\text{length of leg adjacent to the angle}}$$

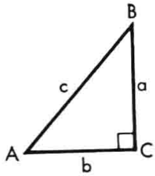
$$\text{cotangent (cot) of the angle} = \frac{\text{length of leg adjacent to the angle}}{\text{length of leg opposite the angle}}$$

$$\text{secant (sec) of the angle} = \frac{\text{length of hypotenuse}}{\text{length of leg adjacent to the angle}}$$

$$\text{cosecant (csc) of the angle} = \frac{\text{length of hypotenuse}}{\text{length of leg opposite the angle}}$$

Note. Next to the name of each ratio, we find in parentheses the abbreviation for that name.

Using these definitions, we can represent the trigonometric ratios involving acute angles A and B in right triangle ABC as follows:



$$\sin A = \frac{a}{c} \qquad \sin B = \frac{b}{c}$$

$$\cos A = \frac{b}{c} \qquad \cos B = \frac{a}{c}$$

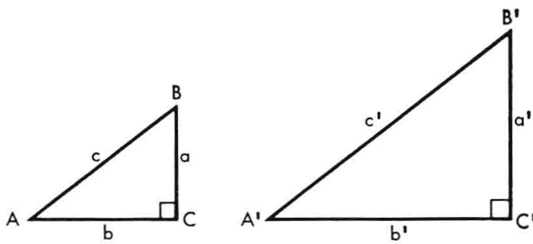
$$\tan A = \frac{a}{b} \qquad \tan B = \frac{b}{a}$$

$$\cot A = \frac{b}{a} \qquad \cot B = \frac{a}{b}$$

$$\sec A = \frac{c}{b} \qquad \sec B = \frac{c}{a}$$

$$\csc A = \frac{c}{a} \qquad \csc B = \frac{c}{b}$$

It might appear that each of these trigonometric ratios, for example sin A, depends upon the size of the right triangle that contains $\angle A$. However, this is not the case. In the following figure, consider the two right triangles ABC and $A'B'C'$.



The lengths of the corresponding sides are different; however, $\angle A \cong \angle A'$. It follows that right triangle ABC is similar to right triangle $A'B'C'$ because they agree in two angles. Therefore, the lengths of the corresponding sides of these triangles are in proportion, giving $\frac{a}{c} = \frac{a'}{c'}$ or $\sin A = \sin A'$.

This proves that the number which is the value of sin A does not depend on the size of the right triangle which contains $\angle A$; it depends only on the measure of $\angle A$. The same reasoning is true for the five other trigonometric ratios.

Thus, with each acute angle there is associated one and only one number called the *sine*. Therefore, we have here an example of a function in which the first coordinate of every ordered pair is the measure of an acute angle and the second coordinate is the sine of that acute angle; that is, the set of ordered pairs $(A, \sin A)$ is a function. If the measure of $\angle A$ in degrees is represented by A, the domain of the function is the set of numbers between 0 and 90, $0 < A < 90$, and the range is the set of positive real numbers less than 1, $0 < \sin A < 1$. ($\sin A = \frac{a}{c}$ must be less than 1 because in a right triangle the hypotenuse is always greater than either leg.) Similarly, the remaining five trigonometric ratios are functions having the same domain $0 < A < 90$. These six functions—sine, cosine, tangent, cotangent, secant, and cosecant—are called ***trigonometric functions.***

In future statements, the symbols $m\angle A = 50$ and $A = 50°$ will both mean "the measure of $\angle A$ is 50°."

MODEL PROBLEMS

1. In right triangle ABC, $a = 4$, $b = 3$, $c = 5$. Find all trigonometric functions of angle A and angle B.

Solution:

$$\sin A = \frac{a}{c} = \frac{4}{5} \qquad \sin B = \frac{b}{c} = \frac{3}{5}$$

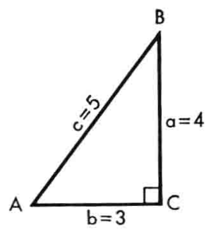
$$\cos A = \frac{b}{c} = \frac{3}{5} \qquad \cos B = \frac{a}{c} = \frac{4}{5}$$

$$\tan A = \frac{a}{b} = \frac{4}{3} \qquad \tan B = \frac{b}{a} = \frac{3}{4}$$

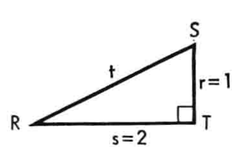
$$\cot A = \frac{b}{a} = \frac{3}{4} \qquad \cot B = \frac{a}{b} = \frac{4}{3}$$

$$\sec A = \frac{c}{b} = \frac{5}{3} \qquad \sec B = \frac{c}{a} = \frac{5}{4}$$

$$\csc A = \frac{c}{a} = \frac{5}{4} \qquad \csc B = \frac{c}{b} = \frac{5}{3}$$



2. In right triangle RST, $\angle T = 90°$, $r = 1$, and $s = 2$. Find all trigonometric functions of $\angle R$.



Solution: In order to find all the functions of $\angle R$, it is first necessary to find the length of the hypotenuse, t.

$t^2 = r^2 + s^2$

$t^2 = 1 + 4$

$t^2 = 5$

$t = \sqrt{5}$

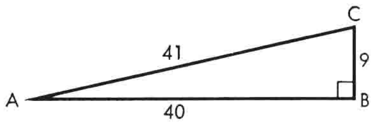
$\sin R = \frac{r}{t} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\qquad$ $\cot R = \frac{s}{r} = \frac{2}{1} = 2$

$\cos R = \frac{s}{t} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\qquad$ $\sec R = \frac{t}{s} = \frac{\sqrt{5}}{2}$

$\tan R = \frac{r}{s} = \frac{1}{2}$ $\qquad$ $\csc R = \frac{t}{r} = \frac{\sqrt{5}}{1} = \sqrt{5}$

Exercises

In 1–6, find all trigonometric functions of the acute angles in the right triangles:



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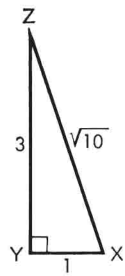

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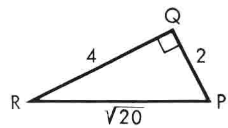

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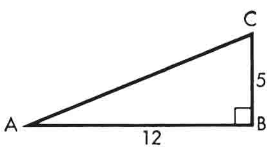

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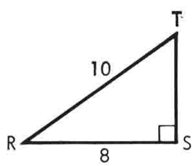

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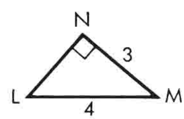


6.

7. In right triangle RST, $m\angle T = 90$, $RS = 50$, and $ST = 30$. Find all the trigonometric functions of $\angle R$.

8. The three sides of a right triangle are 8, 15, 17. Find the trigonometric functions of the smaller acute angle.

9. In a right triangle, the hypotenuse is 4 and the shorter leg is 2. Find the trigonometric functions of the larger acute angle.

10. In right triangle RST, $m\angle T = 90$, $\tan S = \frac{5}{12}$, and $s = 10$. Find r.

11. In right triangle DEF, $m\angle F = 90$, $\cos D = \frac{15}{17}$, and $f = 68$. Find e.

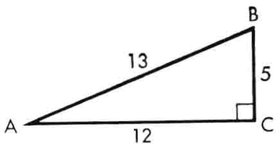
12. In triangle ABC, $m\angle C = 90$. If $\sec A = \frac{6}{5}$ and $c = 30$, find b.

13. In triangle ABC, $m\angle C = 90$, $c = 51$, and $\sin B = \frac{8}{17}$. Find b.

2. Understanding Reciprocal Relations Among the Trigonometric Functions

We know that if the product of two numbers is 1, one of the numbers is the reciprocal of the other number. For example, since $\left(\frac{a}{c}\right)\left(\frac{c}{a}\right) = 1$, then $\frac{a}{c}$ is the reciprocal of $\frac{c}{a}$, and $\frac{c}{a}$ is the reciprocal of $\frac{a}{c}$.

The trigonometric functions of A in right triangle ABC are arranged so that those functions which are reciprocals of each other are on the same line.

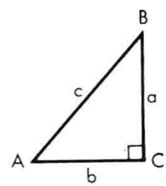


$\sin A = \frac{5}{13}$ $\quad$ $\csc A = \frac{13}{5}$

$\cos A = \frac{12}{13}$ $\quad$ $\sec A = \frac{13}{12}$

$\tan A = \frac{5}{12}$ $\quad$ $\cot A = \frac{12}{5}$

In general, the six trigonometric functions of acute angle A are related reciprocally:



Since $\sin A = \frac{a}{c}$ and $\csc A = \frac{c}{a}$, then $\sin A$ and $\csc A$ are reciprocals of each other.

Since $\cos A = \frac{b}{c}$ and $\sec A = \frac{c}{b}$, $\cos A$ and $\sec A$ are reciprocals of each other.

Since $\tan A = \frac{a}{b}$ and $\cot A = \frac{b}{a}$, $\tan A$ and $\cot A$ are reciprocals of each other.

These reciprocal relationships may be expressed as follows:

$$\sin A = \frac{1}{\csc A} \qquad \csc A = \frac{1}{\sin A} \qquad \sin A \cdot \csc A = 1$$

$$\cos A = \frac{1}{\sec A} \qquad \sec A = \frac{1}{\cos A} \qquad \cos A \cdot \sec A = 1$$

$$\tan A = \frac{1}{\cot A} \qquad \cot A = \frac{1}{\tan A} \qquad \tan A \cdot \cot A = 1$$

Similar relationships may be expressed for the reciprocal functions of acute angle B.

MODEL PROBLEMS

1. If $\tan A = \frac{12}{5}$, find $\cot A$.

Solution: $\cot A$ is the reciprocal of $\tan A$.

$$\cot A = \frac{1}{\tan A} = \frac{1}{\frac{12}{5}} \text{ or}$$

$$\cot A = \frac{5}{12} \quad \textit{Ans.}$$

2. If $\sin B = \frac{4}{7}$, name the trigonometric function of angle B which is equal to $\frac{7}{4}$.

Solution: Since $\frac{7}{4}$ is the reciprocal of $\frac{4}{7}$, the required function of angle B must be the reciprocal of $\sin B$, or $\csc B$. *Ans.*

Exercises

In 1–6, find the value of the indicated function.

1. $\sin A = \frac{3}{5}$; $\csc A = ?$

2. $\tan B = \frac{2}{3}$; $\cot B = ?$

3. $\cos A = \frac{1}{4}$; $\sec A = ?$

4. $\sec A = \frac{13}{5}$; $\cos A = ?$

5. $\tan B = .3$; $\cot B = ?$

6. $\sec C = 1.6$; $\cos C = ?$

In 7–9, state the proper acute angle, or the proper trigonometric function, that can replace the question mark and make the resulting statement true.

7. $\sin 30° = \frac{1}{\csc\ (?)}$

8. $\cos 47° = \frac{1}{(?)\ 47°}$

9. $\tan 85° = \frac{1}{(?)\ 85°}$

In 10–15, without the use of tables, find the product.

10. $\sin 30° \cdot \csc 30°$

11. $\cos 45° \cdot \sec 45°$

12. $\tan 60° \cdot \cot 60°$

13. $\frac{1}{2} \sec 50° \cdot \cos 50°$

14. $10 \cot 22° \cdot \tan 22°$

15. $m \cdot \cos 74° \cdot \sec 74°$

In 16 and 17, state a trigonometric function of an acute angle which can replace the question mark and make the resulting statement true.

16. $\sin 45° \cdot (?) = 1$

17. $\tan 52° \cdot (?) = 1$

18. Express $\sec 45°$ in terms of $\cos 45°$.

19. Express $\csc 60°$ in terms of $\sin 60°$.

20. Express $\tan 35°$ in terms of $\cot 35°$.

21. If $\cos A \cdot \sec A = 1$, solve for $\sec A$ in terms of $\cos A$.

22. If $\tan A \cdot \cot A = 1$, express $\cot A$ in terms of $\tan A$.