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Management— A Quantitative Perspective



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PREFACE

The main purpose of this book is to describe, explain, and illustrate how managers *can* and *should* make decisions for achieving personal as well as organizational goals and objectives. Hence, the book is directed toward two major audiences: (1) men and women who are currently enrolled in colleges and aspire to assume managerial positions in the world of business, politics, government, industry, health services, and social institutions, and (2) practicing managers who wish to keep informed regarding new tools, techniques, and methods of management analysis and decision making.

The book comprises sixteen chapters and five appendixes. A schematic representation of the organization of the book is shown in Exhibit A, which reflects a *conceptual* structure of the book in terms of six parts. The title of a specific part gives an indication of the common thread that runs through the chapters grouped under that part.

It is customary to include in the preface a description of the contents of various chapters. We shall deviate from this custom because we provide, on the page opposite each chapter opening, a list of major concepts and topics discussed in the chapter. However, it is desirable to describe the main purpose of each part.

PART I Conceptual Foundations

The main purpose of Part I is to build a theoretical foundation for the topics, tools, techniques, methods, and models covered in the book. Part I provides a framework for managerial thinking. It describes and explains the quantitative approach to building and solving decision models—and designing managerial strategies.

PART II Decision Theory

The purpose of Part II is to present an integrated view of managerial decision making. This part explains how rational choices can be identified under conditions of certainty, risk, uncertainty, and conflict. In addition, the reader is exposed to decision making under dynamic circumstances in which not one but a set of sequential decisions is required. The overall review of decision theory presented in this part sets the stage for the planning and control models presented in subsequent chapters.

PART III Analysis and Planning

The main purpose of Part III is to describe and explain how managerial planning can be made more efficient and effective by utilizing management science models. Planning and control are related concepts, and in real life they cannot be separated. However, for purposes of classification we have

A

Part I

CONCEPTUAL
FOUNDATIONS

The Integrated Nature of Management

The Quantitative Approach

The Quantitative Approach—
Illustrative Examples

Systems and Models

Part II

DECISION
THEORY

Decision Theory—Basic
Concepts and Applications

Part III

ANALYSIS
AND
PLANNING

Network Models—PERT and CPM

Linear Programming—General Structure
and Graphical Analysis

The Simplex Method

The Dual, Sensitivity Analysis, and
Selected Applications of Linear Programming

Extensions of Linear Programming and
Special Structure Models

Game Theory

Part IV

ANALYSIS
AND
CONTROL

Inventory Control—Deterministic Models

Inventory Control—Probabilistic Models

Simulation

Queuing Models

Part V

IMPLEMENTATION

Implementation of Quantitative
Models

Part VI

APPENDIXES

Probability—Basic Concepts and
Some Applications

Optimization

Basic Concepts of Matrix Algebra

Pivoting

Tables

included *PERT*, *CPM*, *linear programming*, and *game theory* under the umbrella of analysis and planning.

PART IV Analysis and Control

The major focus of this part is on the managerial function of control. Selected models of *inventory control* are described, developed, and illustrated with specific examples. The nature, structure, and potential of *simulation* as well as *queuing* models are examined and explained with reference to actual decision problems.

PART V Implementation

The purpose of Part V is to present some of the important issues and aspects of implementation of quantitative models. The implementation of quantitative models is of the utmost importance because, without actual implementation, quantitative decision models will amount to no more than an interesting academic exercise. This part examines various relationships and interactions involved in the *process of implementation*.

PART VI Appendixes

In Part VI we present a comprehensive coverage of the required probability, statistical, and mathematical concepts. This part also includes a set of tables needed to solve problems given at the end of various chapters.

With rare exceptions, most books dealing with quantitative approaches to management are narrow and mechanistic. Seldom do they provide a “macro” view of the topic under discussion. Nor do they integrate the *behavioral* and *practical* considerations of management with the quantitative decision models. This book seeks to attend to these and other deficiencies and provides the reader with a comprehensive and balanced treatment of management. In each chapter we provide the reader with a “macro” orientation before proceeding to “micro” interests. The perspective and focus are no doubt *quantitative*, but the reader is continuously made aware of the role and importance of *qualitative* and *behavioral* factors in the process of management.

We should mention four additional unique features of this book. First, mathematical content within the chapters is minimal. However, in the appendixes that appear at the end of the book we have provided a comprehensive coverage of the required probability, statistical, and mathematical concepts. Second, throughout the book we have listed, in footnotes, further references and comments to provide additional depth and comprehensiveness. Third, this book is not a collection of unrelated topics. It is, instead, a *unified entity* that integrates different chapters and sections of the book in a meaningful whole. Fourth, and most important, we have prepared a separate volume entitled *Study Guide and Cases* to

reinforce and supplement this book. The major purpose of the *Study Guide and Cases* is to serve as a *linking mechanism* between the main text and the classroom. These features make it possible for the instructor to select the level of mathematical sophistication and the depth of analysis to suit the specific needs of his or her students. They also enable the instructor to present a broadened perspective of each chapter and to utilize the cases as illustrations of the practical applications of management science. It should be emphasized that most of the case studies represent real-life experiences.

This book is specifically written for courses usually taught under such headings as: *Quantitative Management*, *Quantitative Approaches to Management*, *Introduction to Management Science*, *Introduction to Operations Research*, *Introduction to Decision Sciences*. The book affords a vast degree of flexibility in terms of comprehensiveness and level of mathematical sophistication. It can, therefore, be adopted for undergraduate as well as graduate courses.

I am grateful to many of my students, friends, and colleagues for their advice and help. Professors A. O. Holsinger of Baruch College, Donald Moscato of Iona College, J. Donald Phillips of the University of Alabama, Gary Wicklund of the University of Iowa, and George Schneller of Rider College reviewed the entire manuscript and made valuable suggestions. Professors Lou Stern, Georghios Sphicas, John Humes, and Jack Shapiro, who are my colleagues in the Department of Management, Baruch College, gave me their professional as well as personal support in developing this book.

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N. Paul Loomba
Scarsdale, New York

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