# ALGEBRA

# TRIGONOMETRY

TEACHER'S ANNOTATED EDITION

# TEACHER'S ANNOTATED EDITION OF

# *Algebra and Trigonometry*



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*AMERICAN BOOK COMPANY*

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# OVERVIEW FOR *ALGEBRA AND TRIGONOMETRY*

*Algebra and Trigonometry* is the third book in a three-book high school sequence. It may be used as part of this series or used independently for a second course in algebra, a second algebra course with trigonometry, or a fundamental course in trigonometry. In addition to the points made in "A Note to the Student" and in "A Note to the Teacher", the following paragraphs apply to *Algebra and Trigonometry*.

## Algebraic Development

Several chapters deal primarily with a variety of algebraic structures. Since the student has already encountered the system of real numbers and its principal subsystems, these structures are considered first. The concept of an algebraic structure is then applied to polynomials, rational expressions, complex numbers, and by means of a set of enrichment sections, to linear algebra. Substantial attention is directed to the formation and practice of the skills and techniques associated with algebra, the use of algebra in forming models for the solution of problems, and the role of deduction in the development of a mathematical structure.

## Relations and Functions

Perhaps the most recurring, unifying themes to be found in *Algebra and Trigonometry* are the concepts of a relation and a function. Subsets of Cartesian products and their associated graphs are introduced and applied throughout the treatment of the various algebraic systems. Logarithmic functions are considered as the inverses of exponential functions. The entire development of trigonometry is based on the function concept. Only after the trigonometric functions have been defined and thoroughly explored are the triangle and ratio interpretations of this important family of functions introduced. Sequences and series are defined as functions with special domains and probability as a function with a restricted range.

## Deduction

Throughout the book, emphasis is placed upon the role of deduction in mathematics. Many proofs are presented in the developmental material and a large number of upper level exercises require the construction of valid deductive arguments. The preponderance of proofs appearing in *Algebra and Trigonometry* are in paragraph form.

## Other Topics

The enrichment sections found in each chapter afford the student an opportunity to become acquainted with a variety of topics. In addition, Chapters XV and XVI are devoted to sequences and series and to permutations, combinations, and probability. Concepts and techniques from analytic geometry are found throughout the book.

## Subsequent Study

The student who successfully completes a study of *Algebra and Trigonometry* should be well qualified to undertake courses in linear algebra, analytic geometry, and calculus.

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# INDIVIDUAL DIFFERENCES

In order to allow for individual differences the following provisions have been made within these texts:

1) *Self-test paragraphs* are used as "stopper" sections in the exposition. Questions are asked of the student to check his understanding of the material just covered and answers are provided for immediate confirmation. This quasi-programmed feature is easily identified by its color coding and accompanying answers in the margin.

2) *Classroom exercises* are provided to give the student an opportunity to try out his newly acquired concepts and skills under teacher supervision before going on to independent practice.

3) *Graded exercises* are provided and are keyed according to level of difficulty with a color bar. Level one exercises are considered basic. Level two exercises are also basic but are considered to be more "in depth." Level three exercises are provided as an option for the more mathematically mature student.

4) *Proofs as an optional topic*—normally proofs of theorems follow the development of basic concepts and skills within the exposition. It is therefore possible to teach a "minimum" course without involving the marginal student too deeply with proof. Exercises that require an understanding of proofs are generally limited to level three exercises.

# *Algebra and Trigonometry*

THE SERIES

*Structuring Algebra*

*Geometries*

*Algebra and Trigonometry*

# CONTENTS

# A NOTE TO THE TEACHER

The writing of instructional materials is influenced by a psychology of learning and a philosophy of teaching. In the paragraphs that follow, the authors deal with some of the aspects of learning and teaching mathematics that guided the writing of this series.

## Insight through Structure

The authors' aim is that this series should help students to perceive patterns and become aware of relationships between the parts of mathematics and the whole of mathematics. The conceptual aspects of number systems, algebraic systems, and point-set systems are therefore stressed. Computational skills are gradually developed and techniques introduced. By means of such questions as "How do I do this?" and "Why is that true?", the student is led to discover generalizations. The series seeks to develop insight; problems are to be solved on the basis of reasoning rather than through repetition.

## Individual Proficiency in Problem Formulation and Solution

The authors believe that students must become proficient in solving problems. Hence, the series contains numerous exercise sets designed to develop techniques and to maintain skills already acquired. The self-test paragraphs (colored type between colored line segments) enable the student to check for understanding as he proceeds. Classroom exercises for oral discussion and supervised study are a feature of each section. Exercises are graded throughout, levels of difficulty being indicated by graduated colored bars. This color scheme allows assignments to be adjusted to individual needs. The three end-of-chapter tests are intended to guide the student in his review, to test techniques acquired, and to maintain skills. Test items are keyed to the appropriate section number in each cases. The enrichment topics will be found suitable, usually, for all class members; the average and below-average student as well as the bright student will profit from reading these topics.

## Rationalizing Deductively

The authors believe that all students should be given the opportunity to study correct mathematics. Throughout the series emphasis is placed upon correct use of language. The role of deduction and axiomatics in building sound mathematical systems is stressed particularly.

## Applications and Models

The authors believe that, for the most part, people are motivated to learn mathematics so that they can solve problems which may confront them. Therefore, the series attempts to show how mathematics may be used to solve problems from a variety of areas and affords practice in the techniques of problem formulation and solution. On the other hand, the authors believe that human curiosity will always find the study of mathematics worthwhile for its own sake.

## Teacher's Guide and Annotation

A teacher's guide and overprints in the Teacher's Annotated Edition for the series answer all questions raised and give suggestions for the effective use of textual material. Objectives are stated so that teachers will know what the authors expect of the student on completion of a particular chapter or section.

We welcome any suggestions you may wish to send us regarding this text.

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# REVIEWING THE REAL NUMBER SYSTEM

After completing this chapter, the student should be able to

1. classify numbers according to membership in
   - **a.** the set of natural (counting) numbers, $N$
   - **b.** the set of whole numbers, $W$
   - **c.** the set of integers, $I$
   - **d.** the set of rational numbers, $Q$
   - **e.** the set of real numbers, $R$;
2. give examples of real numbers that are not rational numbers;
3. identify the system of real numbers, $\{R,+,\cdot\}$, as the system used in coordinatizing a line in geometry so that there is a one-to-one correspondence between the set of points on the line and the set of numbers in the number system;
4. use the structural aspects of $\{R,+,\cdot\}$ and the equivalence relation 'is equal to' to make logical deductions;
5. associate addition and multiplication with their respective inverse operations—subtraction and division—and inverse operations with inverse elements.

This chapter provides a review of mathematical ideas from the student's preceding years of study and an outline for organizing the mass of details about numbers which the student has accumulated. Real numbers are viewed as members of a system such that the numbers behave in prescribed ways. The operational and equivalence properties dictate the behavior of real numbers and form the structure of the real number system for this chapter.

Some postulates for the real number system are discussed in detail, with the exception of the Completeness Postulate which is properly stressed in a development of the limit concept necessary for the calculus. Establishing the relationship between the real number system and a line in Euclidean geometry suffices for our purposes.

The operational postulates display the fact that addition and multiplication are of prime importance. The secondary operations of subtraction and division are derived from these basic operations.

A minimal treatment of this chapter would include stressing the different number sets, with a cursory treatment of the sections on structure. The student, however, should be able to illustrate (by means of specific examples) the postulates and basic theorems for the real number system. A more stringent treatment requires weaning the student from following teacher-constructed proofs to developing student-constructed proofs. An alternate plan involves giving an initial minimal treatment to this chapter followed, near the end of the academic year, by a second and deeper look at the material.

## Section 0-1

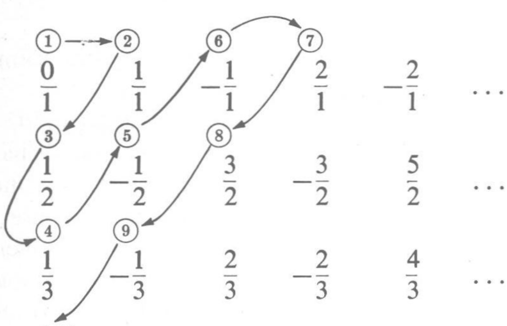
**OBJECTIVES:** After studying the section, **Different Sets of Numbers,** the student should be able to

1. identify a set by listing its members and by using a phrase to characterize its elements;
2. answer questions such as 'Is $A$ a subset of $B$?', 'What are the elements of $A \cup B$?', and 'What are the elements of $A \cap B$?' when given specific sets $A$ and $B$ like those found in the exercises;
3. indicate (in written form) the members which comprise each of the sets $N$, $W$, $I$, and $Q$, and classify a given real number as belonging to, or not belonging to, each of these sets;
4. name any member of $Q$ with either a fraction or a decimal numeral;
5. select the set of real numbers from a list of the number sets discussed in this section as the only set which can be matched one-to-one with the set of points on a line in Euclidean geometry.

**VOCABULARY:** *empty set* ($\varnothing$), *finite set, infinite set, integer, intersection* ($\cap$), *irrational number, is an element of* ($\in$), *is not an element of* ($\notin$), *natural number, one-to-one correspondence, proper subset, rational number, real number, real number line, subset, whole number, union* ($\cup$)

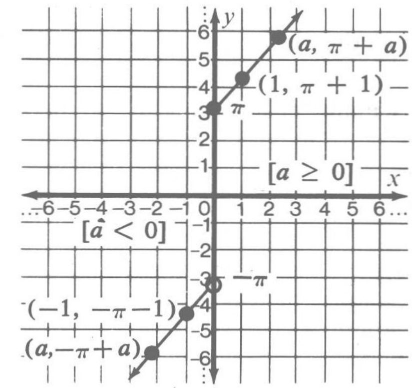
PROCEDURES: Because this section is the first one and we need vehicles for expressing mathematical ideas, a liberal amount of set notation is used. Usually, the student will have encountered this standard set symbolism in previous courses; however, he may have used '$\subseteq$' for 'is a subset of' and a vertical bar for the colon in set-builder notation. Be sure everyone understands that '$\subset$' denotes 'is a subset of' and ':' is read "such that" in set-builder notation. For example, $\{x: x \in Q \text{ and } x < 2\}$ is read, "The set of all $x$ such that $x$ is a rational number and $x$ is less than 2." The self-test paragraph at the top of page 5 tests for student understanding of the fact that $A \subset A$ and $\varnothing \subset A$, for any set $A$.

The definition of an infinite set may be a new one for the student. The matchings below illustrate this definition for the sets of rational and real numbers.



For any real number $a$, $\begin{cases} a \leftrightarrow \pi + a, \text{ where } a \geq 0 \\ a \leftrightarrow -\pi + a, \text{ where } a < 0 \end{cases}$

In the first matching, the one-to-one correspondence is between $Q$ and its proper subset $N$. In the second matching, the one-to-one correspondence is between $R$ and the subset of $R$ formed by deleting all real numbers less than $\pi$ but greater than or equal to $-\pi$. See the diagram below for this correspondence.




The set of natural numbers, $N$; the set of whole numbers, $W$; the set of integers, $I$; the set of rational numbers, $Q$; and the set of real numbers, $R$, are the important collections discussed in this section. These abbreviations for sets should be memorized since they are used throughout the book.

## Section 0-2

**OBJECTIVES:** After studying the section, **Postulates for the Real Number System,** the student should be able to

1. complete phrases such as the following for the real number system, $\{R,+,\cdot,\}$:
   $(5, \pi) \xrightarrow{+} \square$ $\quad (5, \pi) \xrightarrow{\cdot} \square$
   $(-3, 2\sqrt{3}) \xrightarrow{+} \square$ $\quad (-3, 2\sqrt{3}) \xrightarrow{\cdot} \square$
   $(x, y) \xrightarrow{+} \square$ $\quad (x, y) \xrightarrow{\cdot} \square$
2. make an operational table for a finite number set, given the general instruction for assigning the number to the ordered pair;
3. write the complete statement of the operational postulate, given its name; illustrate the postulate with specific numbers, given the complete statement of the postulate; and identify the postulate by name given specific cases. (See the Classroom Exercises.)

**VOCABULARY:** *additive identity, additive inverse, associative, binary operation, closure, commutative, completeness postulate, distributive, field, multiplicative identity, multiplicative inverse, ordered pair, postulate, real number system* ($\{R,+,\cdot\}$)

PROCEDURES: Simple finite systems having one operation (Exercise 11–15, pages 11 and 12) can be employed in developing the concept of a binary operation. Emphasize that a binary operation on a set of numbers involves assigning at most one number of that set to each ordered pair having first and second coordinates from that set of numbers. Stress that addition and multiplication are the basic binary operations of the real number system.

Remind the student that postulates are statements which he accepts as properties of the mathematical system and that definitions are agreements about words, symbols, and phrases. Of course, the operational postulates for $\{R,+,\cdot\}$ have been carefully selected by mathematicians so that they form a foundation for the structure of the system. When discussing these postulates, use descriptive titles such as 'the Identity Postulate for Addition', rather than 'Postulate 4'. Note that the identity postulate for an operation must precede the corresponding inverse postulate since the latter uses the identity element. Relative to the Completeness Postulate, emphasize that the points for rational numbers are not adequate for covering a Euclidean line ($\sqrt{2}$ and $\pi$ are irrational examples) and that $R$ is adequate for the desired one-to-one correspondence.

## Section 0-3

**OBJECTIVES:** After studying the section, **Elementary Theorems in $\{R,+,\cdot\}$**, the student should be able to

1. write the complete statements of the reflective, symmetric, and transitive properties for the equals relation in $\{R,+,\cdot\}$;
2. state which of the three properties for an equivalence relation are possessed by familiar relations on sets (see Exercises 1–6, pages 15 and 16) and furnish counterexamples for showing that a property does not hold;
3. match given illustrations (see Classroom Exercises, page 15) with the theorems of this section;
4. write the converse of a given implication;
5. write the two implications which comprise a given biconditional ('if and only if' statement).

**VOCABULARY:** *antecedent, consequent, converse, counterexample, equals relation, equivalence relation, if and only if (iff), implication, reflexive, symmetric, transitive, theorem*

PROCEDURES: The set of real numbers together with the basic operations, addition and multiplication, and their postulated properties forms the real number system which is identified by the notation $\{R,+,\cdot\}$. In the context of the real number system, stress that the equals relation '=' for numbers means 'is the same real number as'. The equals relation is no trivial concept and should be treated carefully. For example, $\frac{22}{7}$ and 3.1416 are both approximations for the real number $\pi$, but are not equal to $\pi$. Displays such as

$$\xcancel{\pi = \tfrac{22}{7}} \quad \text{and} \quad \xcancel{\pi = 3.1416}$$

illustrate corrected misusages of the equals relation. Carefully check a student's written work for correct usage of the symbol for 'is equal to' throughout the entire course.

Review the fact that theorems are statements that can be deduced from previously accepted postulates, definitions, and previously proved theorems. Theorems 1 and 2 are trivial results from the fact that addition and multiplication are *operations*. Thus, supporting statements in a proof such as 'if equals are added to equals, the sums are equal' and 'if equals are multiplied by equals, the products are equal' can easily be replaced by 'addition is an operation' and 'multiplication is an operation'. The authors prefer that these replacements be made.

The use of the 'if ... then' sentential connective is reviewed in this section. Review the terminology, *antecedent* and *consequent*. Emphasize that the term *converse* means the converse of a particular implication. Include a discussion of deriving the converse of the converse of a given implication. Also, since the conjunction of an implication and its converse is so commonly denoted by the phrase 'if and only if' (iff), make sure that the student can identify the components of the basic biconditional statement. For example

$$x + 2 > 5 \quad \text{iff} \quad x > 3$$

is a concise form of the statement

$$\text{If } x > 3 \text{ then } x + 2 > 5, \text{ and if } x + 2 > 5 \text{ then } x > 3.$$

## Section 0-4

**OBJECTIVES:** After studying the section, **Subtraction and Division,** the student should be able to

1. employ the basic operations to see if $a - b$ is equal to $c$ and $d \div e$ is equal to $f$ for specific numbers $a$, $b$, $c$, $d$, $e$, and $f$;
2. apply Theorems 1 and 2, page 17, in converting specific cases of $a - b$ and $a \div b$ to the sums and products of two numbers, respectively;
3. rename specific cases of $a \div b$ by $\frac{a}{b}$ notational form, and conversely;
4. state which part of the definition of division is not satisfied when considering each of the cases $0 \div 0$ and $2 \div 0$.

**VOCABULARY:** *corollary, division, inverse operation, subtraction*

PROCEDURES: The subtraction and division operations for the real number system are inverse operations of addition and multiplication, respectively, which were built into the postulational structure. The basic operations are used in giving birth to their inverse operations. Emphasize that the meaning of subtraction depends upon the already present addition operation; similarly, stress that division is a by-product of multiplication. For example, the argument that 4 is the unique real number such that $4 + 3 = 7$ suffices to prove that $7 - 3$ is the number 4. Similarly, the assertion that 5 is the unique real number such that $5 \times 2 = 10$ verifies that $10 \div 2 = 5$.

Subtraction and division can each be handled in an alternate manner in $\{R,+,\cdot\}$ because each number has an additive inverse and each nonzero number has a multiplicative inverse. For example, $2 - (-5)$ is the sum

$$2 + [-(-5)] = 2 + 5$$

and $\frac{2}{3} \div \frac{4}{5}$ is the product

$$\frac{2}{3} \cdot \left(\frac{4}{5}\right)^{-1} = \frac{2}{3} \cdot \frac{5}{4}.$$

However, we justify the alternate technique (Theorems 1 and 2, page 17) on the basis of the definitions of subtraction and division.

The definition of division rules out "division by zero". There are two types of difficulties encountered. First, *every* real number $r$ satisfies $r \cdot 0 = 0$; hence, no *unique* number can be denoted by $0 \div 0$ Second, if $a \neq 0$, then no real number $t$ exists such that $0 \cdot t = a$; thus, $a \div 0$ also cannot denote a real number.

## Section 0-5

**OBJECTIVES:** After studying the section, **Theorems in $\{R,+,\cdot\}$ Relating to the Additive Inverse Postulate,** the student should be able to

1. classify $-a$ as a positive or a negative real number, when $a$ is a number such as 5, $-2$, $\frac{-3}{4}$, and $\frac{-1}{-4}$;
2. apply the theorems of this section in finding a simple notation to identify numbers such as those in the Classroom Exercises and in Exercise 1.

PROCEDURES: Emphasize that $-a$ is the additive inverse of $a$ and might not be a negative real number. Hence, avoid the term 'negative $a$' when reading '$-a$'; 'the additive inverse of $a$' is preferable.

Theorems 1–4, pages 20 and 21, are proved by testing a number which is proposed as the additive inverse of a given number. There is only *one* number which, when added to the given number, yields zero. Hence, the proof is accomplished by showing that the sum of the given number and its proposed additive inverse is, indeed, zero. If the sum is zero, the test has been successful and the proof is completed. Theorem 4 and Theorem 1 can then be employed in proving Theorem 5.

## Section 0-6

**OBJECTIVES:** After studying the section, **Theorems in $\{R,+,\cdot\}$ Relating to the Multiplicative Inverse Postulate,** the student should be able to

1. state the postulate, theorem, or definition in $\{R,+,\cdot\}$ which is used at each step in the proofs of the theorems of this section;
2. apply Theorems 1–11 to rational number arithmetic using the specifications that answers are to have the form $\frac{a}{b}$, where $a$ and $b$ are integers with no common prime factors and $b$ is positive, and that for $b = 1$, $\frac{a}{1}$ is replaced by $a$.

**VOCABULARY:** *algorithm, lemma*

PROCEDURES: A non-negative rational number can be denoted by $\frac{m}{n}$ where $m$ and $n$ are whole numbers and $n \neq 0$. If $m$ and $n$ are integers, then $\frac{m}{n}$ is a rational number. Rational number arithmetic is a basic part of the student's previous mathematics education. He knows that in adding with fractions, "first change to a common denominator, then add numerators while keeping the common denominator"; also, to multiply, "multiply the numerators and multiply the denominators".

The results in Section 0–6 are generalizations of these basic procedures. In this section, we have the fractional form $\frac{a}{b}$, but $a$ and $b\,(b \neq 0)$ refer to *real* numbers, and not necessarily to integers. The key to the $\frac{a}{b}$ fractional notation lies in the equalities

$$\frac{a}{b} = a \div b = a \cdot b^{-1}$$

where $a$ and $b$ are real numbers and $b \neq 0$. Recall that the equality

$$\frac{a}{b} = a \div b$$

is just a notational convention; that is, $\frac{a}{b}$ means the same thing as $a \div b$. The equality

$$a \div b = a \cdot b^{-1}$$

holds since the number $(a \cdot b^{-1})$ satisfies the definition of division. Armed with these various forms, the theorems of this section are not difficult to understand.

When assessing behavior relative to Objective 1, include both those proofs displayed in the text and teacher-constructed proofs. A number of the students should be able to construct their own proofs.

The student can apply the results of this section by reviewing computational algorithms from basic arithmetic such as

$$\frac{5}{-6} + \frac{-2}{-15} = \frac{-25}{30} + \frac{4}{30} = \frac{-21}{30} = \frac{-7}{10}$$

and verifying each step of the algorithm with a property of $\{R,+,\cdot\}$.

## Guiding Your Review

The GUIDING YOUR REVIEW section for this chapter and the remaining chapters of the text provide an outline of study for the topics in each chapter. The basic general concepts for each section are highlighted by the review questions.

If the student can easily answer the review questions for a section, then the corresponding TESTING

YOURSELF exercises serve as a verification that the topics have, indeed, been mastered. However, if the student is unsure of his answers to the review questions, an actual review of the associated section is in order before responding to other end-of-chapter items for the same section.

Some teachers will want to devote a substantial amount of class time to reviewing material. In this case, GUIDING YOUR REVIEW provides a basis for instructional planning. Other teachers may wish to stress individual review, with class time used to clear up a few of the student's inadequately answered questions. In using the second approach, the end-of-chapter questions provide material from which assignment selection can be made for directing the student in his self-study review work.

## Testing Yourself

The exercises in the TESTING YOURSELF section are written to test for specific applications of the general concepts outlined in the GUIDING YOUR REVIEW section. A student who has mastered the general concepts is able to apply the concepts to specific cases. Not being able to make the applications indicates the need for more concentrated study on the topics. Hence, TESTING YOURSELF items serve to indicate to the student that he seems to have mastered the topics or that he needs more review. Further review may take the form of working similar exercises in CHECKING AGAIN and, if need be, studying the corresponding section in the text.

## Checking Again

The items in the CHECKING AGAIN section are primarily for the student who has not exhibited mastery of the topics upon answering the TESTING YOURSELF items. The identification of the exercises with the corresponding sections of the text makes it easy for the student to concentrate on those topics which cause him difficulty. The student who performs well on the TESTING YOURSELF section may want to use the items in this last test section to serve as a double check.

The three end-of-chapter tests are similar for each chapter in the book. Hence, the above comments apply to the remainder of the course and will not be repeated in the guide for the other chapters. The end-of-chapter sections with the reference system used for the different items are quite useful in providing for individualized reviewing for each student. A student's weaknesses can be identified, and pertinent sections and exercises may readily be prescribed for correcting the difficulties.

The teacher may also consider the items of the TESTING YOURSELF and CHECKING AGAIN sections as models for representative test items for instructor-constructed examinations.

## Enrichment Section

**OBJECTIVES:** After exploring this section about an algebra of sets, the student should be able to

1. construct operational tables for set union and set intersection for the set of all subsets of a set containing at most three elements;
2. illustrate the distributive properties for set union with set intersection for specific sets;
3. identify the elements of the complement of a given set relative to a specified universal set;
4. construct Venn diagrams corresponding to sets formed by applying combinations of set union, set intersection, and set complementation.

**VOCABULARY:** *absorption property, algebra of sets, complement of a set, De Morgan's Laws, distributive property of set intersection over set union, distributive property of set union over set intersection, idempotent property, universal set, Venn diagram*

PROCEDURES: The language of sets has become a common vehicle for expressing the content of arithmetic, algebra, geometry, and other branches of mathematics studied by the student. An acquaintance with the binary operations of set union and set intersection as well as the unary operation of set complementation is desirable for every student. A student should be able to identify the elements of the union and intersection of two given sets, and the members of the complement of a specific set relative to a chosen universal set. This enrichment section can be used as another means for reviewing these elementary set concepts.

The thrust of the section, however, is to introduce the student to a structure for these operations on a set of sets. The material presented for an algebra of sets is only an introduction. The properties of the algebra are not proved; however, the technique of drawing Venn diagrams to illustrate these properties is an acceptable procedure at this level for convincing the student that these statements are at least reasonable.

If this topic catches the interest of a student, there are many references that he might pursue. The abstract structure for an algebra of sets is known as a Boolean algebra (George Boole, 1815–1864).

## Bibliography

The bibliography section appearing at the end of each chapter provides references for additional independent study in each of the following categories: the mathematician featured at the beginning of the chapter, the textual material, and the enrichment topic.