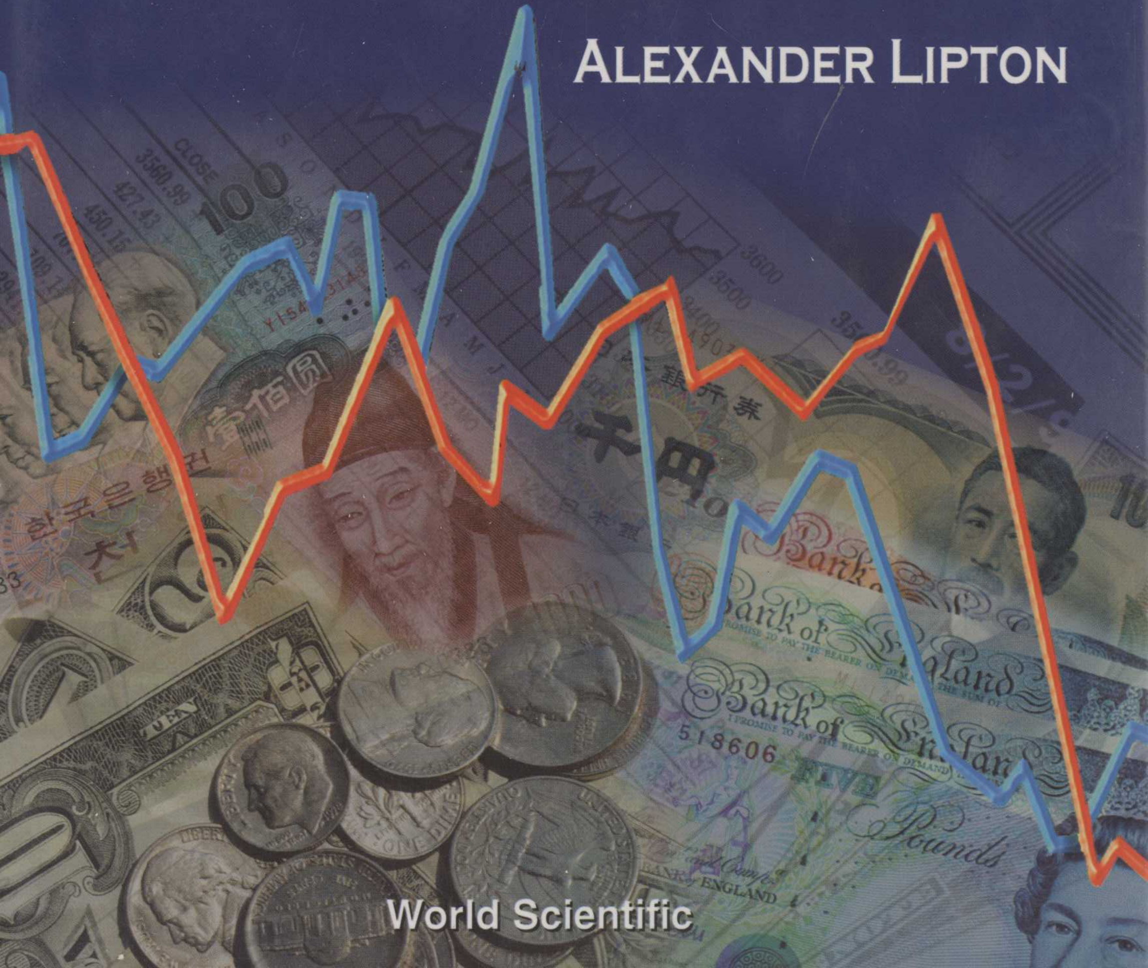
MATHEMATICAL METHODS FOR FOREIGN EXCHANGE

A Financial Engineer's Approach

ALEXANDER LIPTON



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MATHEMATICAL METHODS FOR FOREIGN EXCHANGE

A Financial Engineer's Approach

This Book is Dedicated

To Marsha

The importance of money essentially flows from it being a link between the present and the future.

John Maynard Keynes

Money and currency are very strange things.
They keep on going up and down and no one knows why;
If you want to win, you lose, however hard you try.

Gilles li Muisis

Preface

This book is devoted to some mathematical problems encountered by the author in his capacity as a mathematician turned financial engineer at Bankers Trust and Deutsche Bank. The exposition is restricted mainly to problems occurring in the foreign exchange (forex) context not only due to the fact that it is the author's current area of responsibility but also because mathematical methods of financial engineering can be described more vividly when the exposition is centered on a single topic. Studying forex is interesting and important because it is the grease on the wheels of the world economy. Besides, while the meaning of some financial instruments is difficult to comprehend without prior experience, everyone who has ever travelled abroad has had to exchange currencies thus acquiring direct experience of such concepts as spot forex rate, bid-ask spread, transaction costs (in the form of commissions), etc. At the same time, the reader who acquires working knowledge of the material presented in this book should be able to handle efficiently most of the problems occurring in equity markets and some of the problems relevant for fixed income markets.

If one were to choose just one word in order to characterize financial markets, that word would be *uncertainty* since it is their dominant feature. Some investors consider uncertainty a blessing, while others think it a curse, yet both groups participate in the intricate inner workings of the markets. The fact that foreign exchange rates (relative prices of different currencies), as well as prices of bonds (government or corporate obligations to repay debts) and stocks (claims on future cash flows generated by companies) are random and financial investments are risky was realized long ago and has been a source of fascination for economists, mathematicians, speculators, philosophers, and moralists, not to mention laymen.

Due to the random nature of financial markets, trying to predict future prices of individual financial instruments makes little sense. However, one can introduce so-called derivative instruments (the name indicates that their prices are derived from the prices of some underlying financial instruments, namely, currencies, bonds, stocks, credits, etc.), which can be used in order to cope with financial risks and uncertainties. Alternatively, one can develop optimal investment strategies in the presence of uncertainty which are based on diversification and creation of portfolios of different instruments which are less risky than individual instruments. In the present book we show how to value derivatives and construct optimal portfolios in the forex context by using modern mathematical methods.

This book is devoted to various problems which financial engineers face in the market place and gives a detailed account of mathematical methods necessary for their solution. Even though the exposition is presented from a financial engineer's prospective, the author tried his best to expose all the necessary details. At the same time, mathematical rigor as such was not high on the author's priority list. In particular, most of the results are not formulated as theorems and lemmas since this traditional format is not adequate for the purposes of the present exposition. We start with a brief survey of relevant mathematical concepts. After that we present an in-depth discussion of discrete-time models of forex. We distinguish between single-period and multi-period models. In both cases the corresponding models are too stylized to be of practical importance but they do allow the reader to understand some of the issues which occur in more complicated situations. For this reason, and because of their aesthetic appeal, these models deserve a careful study. We analyze conditions which guarantee that a particular model is financially reasonable and show how to price derivatives and solve the optimal investment problem for such models. Once discrete-times models are mastered, we switch our attention to more practically useful continuous-time models. We describe in detail a variety of models, starting with the standard Black-Scholes model and ending with rather involved stochastic volatility models with a special emphasis on practical aspects. We then show how to price derivatives and solve the optimal investment problem in the continuous setting.

Recently, several very good (and some not so good) books dealing with various aspects of financial engineering were published. The author hopes that the present book can complement the existing literature on the subject and will be useful to the reader in more than one way. In fact, when deciding whether to write this book, he followed the advice of Franz Kafka who once said "Such books as make us happy, we could, if need be, write ourselves".

In the process of writing this book the author enjoyed help, advice, and support of various individuals. First and foremost, he is grateful to his wife Marsha, father Yefim, mother Eugenia, and daughter Rachel. Next, he is also deeply grateful and much indebted to his fellow quants, especially to Christo-

pher Berry, Stewart Inglis, and William McGhee, as well as to Peter Carr, Brian Davidson, Vladimir Finkelstein, Ken Garbade, Arvind Hariharan, Tom Hyer, Andrew Jacobs, Bin Li, Dmitry Pugachevsky, Eric Reiner, and Paul Romanelli. Last but not least, he greatly benefited from the interactions with a group of outstanding managers and traders including Hal Herron, Dan Almeida, Jim Turley, Kevin Rodgers, Matt Desselberger, Perry Parker, and Andrew Baxter.

Reasonable efforts were made to publish reliable information. However, in a book like this one typing and other errors are unavoidable. The author and the publisher do not assume any responsibility or liability for the validity of the information presented in this book and for the consequences of its use or misuse. The book represents only the personal views of the author and does not necessarily reflect the views of Deutsche Bank, its subsidiaries or affiliates.

Finally, a few words about the epigraphs. J. M. Keynes needs no introduction. The Abbot Gilles li Muisis of Tournai lived in the fourteenth century. His wonderful verse is quoted by P. S. Lewis in "*Later Medieval France*" and by B. W. Tuchman in "*A Distant Mirror*".

Alexander Lipton
New York and London
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