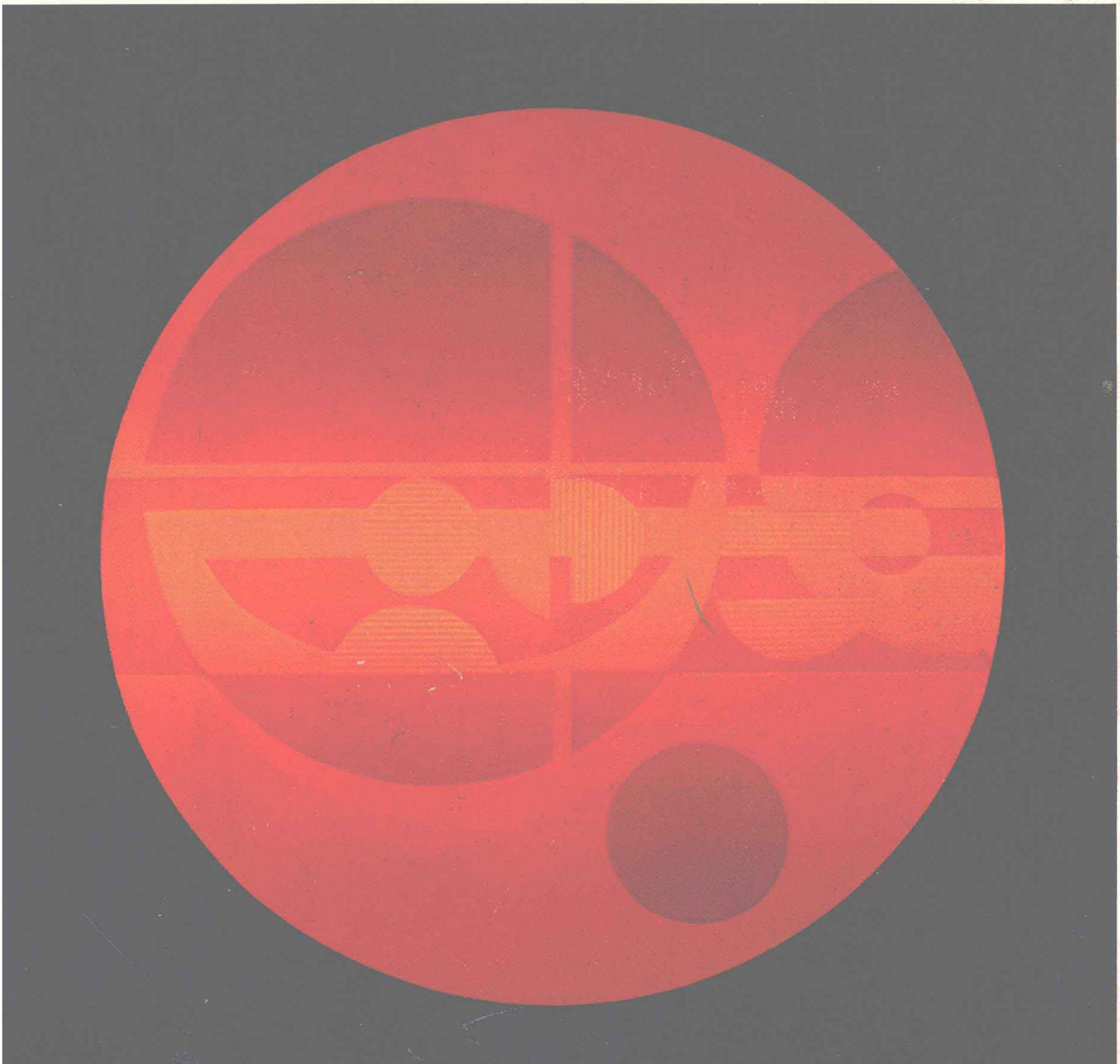
Fundamentals of Algebra and Trigonometry

FOURTH EDITION



EARL W. SWOKOWSKI

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FOURTH EDITION

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Preface

The fourth edition of *Fundamentals of Algebra and Trigonometry* reflects the continuing change in the needs and abilities of students who enroll in precalculus mathematics courses. The goal of this new edition is to maintain the mathematical soundness of earlier editions, but to make the tone of the book less formal by means of rewriting, placing more emphasis on graphing, and adding many new exercises, applied problems, examples, and figures. Optional exercises for students who use hand-held calculators are included in appropriate sections. The use of a second color to highlight figures and important statements will further enhance the appeal of the text to students.

This edition has greatly benefited from suggestions and comments of the following reviewers and survey respondents:

A. N. Aheart *(West Virginia State College),* C. C. Alexander *(University of Mississippi),* J. Baker *(Western Carolina University),* P. Bauer *(University of Wisconsin, Marshfield),* J. Bennett *(University of Evansville),* B. P. Bockstege *(Broward Community College),* R. Dana *(Lake City Community College),* D. Deckard *(University of Michigan),* B. C. Detwiler *(Western Kentucky University),* J. Dewar *(Loyola Marymount University),* F. Dodd *(University of South Alabama),* W. Duncan *(McLennan Community College),* C. V. Duplissey *(University of Arkansas–Little Rock),* L. Estergard *(Brevard Community College),* H. Fox *(University of Wisconsin, Waukesha),* L. E. Fuller *(Kansas State University),* R. Geo
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In addition, I wish to single out as especially helpful the detailed reviews of various stages of the revised manuscript by Donald L. Dykes *(Kent State University),* Mark P. Hale, Jr. *(University of Florida),* Douglas W. Hall *(Michigan State University),* A. J. Hulin *(University of New Orleans),* Burnett Meyer *(University of Colorado),* and Russell J. Rowlett *(University of Tennessee).*

I am also grateful to the staff of Prindle, Weber & Schmidt, Inc., for their cooperation and valuable assistance. In particular, Elizabeth Thomson was very helpful in her role as production editor, and Executive Editor John Martindale was a constant source of advice and encouragement.

Special thanks are due to my wife Shirley and the members of our family: Mary, Mark, John, Steve, Paul, Tom, Bob, Nancy, and Judy. All have had an influence on the book — either directly, through working exercises, proofreading, or typing, or indirectly, through continued interest and moral support.

To all of the people named here and to the many unnamed students and teachers who have helped shape my views about precalculus mathematics, I express my sincere appreciation.

Earl W. Swokowski

Table of Contents

Fundamental Concepts of Algebra

The material in this chapter is basic to the study of algebra. We begin by discussing properties of real numbers. Next we turn our attention to exponents and radicals, and how they may be used to simplify complicated algebraic expressions.

1 ALGEBRA: A POWERFUL LANGUAGE AND TOOL

A good foundation in algebra is essential for advanced courses in mathematics, the natural sciences, and engineering. It is also required for problems which arise in business, industry, statistics, and many other fields of endeavor. Indeed, every situation which makes use of numerical processes is a candidate for algebraic methods.

Algebra evolved from the operations and rules of arithmetic. The study of arithmetic begins with addition, multiplication, subtraction, and division of numbers, such as

$$4 + 7, \quad (37)(681), \quad 79 - 22 \quad \text{and} \quad 40 \div 8.$$

In *algebra* we introduce symbols or letters a, b, c, d, x, y, etc., to denote *arbitrary* numbers and, instead of special cases, we often consider *general* statements such as

$$a + b, \quad cd, \quad x - y \quad \text{and} \quad x \div a.$$

This *language of algebra* serves a two-fold purpose. First, it may be used as a shorthand, to abbreviate and simplify long or complicated statements. Second, it is a convenient means of generalizing many specific statements. To illustrate, at an early age, children learn that

$$2 + 3 = 3 + 2, \quad 4 + 7 = 7 + 4, \quad 5 + 9 = 9 + 5, \quad 1 + 8 = 8 + 1$$

and so on. In words, this property may be phrased "if two numbers are added, then the order of addition is immaterial; that is, the same result is obtained whether the second number is added to the first, or the first number is added to the second." This lengthy description can be shortened, and at the same time made easier to understand, by means of the algebraic statement

$$a + b = b + a$$

where a and b denote arbitrary numbers.

Many illustrations of the generality of algebra may be found in formulas used in science and industry. For example, if an airplane flies at a constant rate of 300 mph (miles per hour) for two hours, then the distance it travels is given by

$$(300)(2), \quad \text{or } 600 \text{ miles.}$$

If the rate is 250 mph and the elapsed time is 3 hours, then the distance traveled is

$$(250)(3), \quad \text{or } 750 \text{ miles.}$$

If we introduce symbols, and let r denote the constant rate, t the elapsed time, and d the distance traveled, then the two illustrations we have given are special cases of the general algebraic formula

$$d = rt.$$

When specific numerical values for r and t are given, the distance d may be found readily by an appropriate substitution in the formula. Moreover, the formula may also be used to solve related problems. For example, suppose the distance between two cities is 645 miles, and we wish to find the constant rate which would enable an airplane to cover that distance in 2 hours and 30 minutes. Thus we are given

$$d = 645 \text{ miles}, \qquad t = 2.5 \text{ hours}$$

and the problem is to find r. Since $d = rt$ it follows that

$$r = \frac{d}{t}$$

and hence for our special case,

$$r = \frac{645}{2.5} = 258 \text{ mph.}$$

That is, if an airplane flies at a constant rate of 258 mph, then it will travel 645 miles in 2 hours and 30 minutes. In like manner, given r, the time t required to travel a distance d may be found by means of the formula

$$t = \frac{d}{r}.$$

The preceding example indicates how the introduction of a general algebraic formula not only allows us to solve special problems conveniently, but also to enlarge the scope of our knowledge by suggesting new problems that can be considered.

We have given only two elementary illustrations of the value of algebraic methods. There are an unlimited number of situations where a symbolic approach may lead to insights and solutions that would be impossible to obtain using only numerical processes. As you proceed through this text and go on to either more advanced courses in mathematics or fields which employ mathematics, you will become further aware of the importance and the power of algebraic techniques.

2 REAL NUMBERS

Real numbers are used considerably in all phases of mathematics and you are undoubtedly well acquainted with symbols which are used to represent them, such as

$$1, \quad 73, \quad -5, \quad \frac{49}{12}, \quad \sqrt{2}, \quad 0, \quad \sqrt[3]{-85}, \quad 0.33333\ldots, \quad 596.25$$

and so on. The real numbers are said to be **closed** relative to operations of addition (denoted by $+$) and multiplication (denoted by $\cdot$). This means that to every pair a, b of real numbers there corresponds a unique real number $a + b$ called the **sum** of a and b and a unique real number $a \cdot b$ (also written ab) called the **product** of a and b. These operations have the following properties, where all lower-case letters denote arbitrary real numbers, and where 0 and 1 are special real numbers referred to as **zero** and **one**, respectively.

(1.1) **Commutative Properties**

$$a + b = b + a, \qquad ab = ba$$

(1.2) **Associative Properties**

$$a + (b + c) = (a + b) + c, \qquad a(bc) = (ab)c$$

(1.3) **Identities**

$$a + 0 = a = 0 + a, \qquad a \cdot 1 = a = 1 \cdot a$$

(1.4) **Inverses**
For every real number a, there is a real number denoted by $-a$ such that

$$a + (-a) = 0 = (-a) + a$$

For every real number $a \neq 0$, there is a real number denoted by $1/a$ such that

$$a\left(\frac{1}{a}\right) = 1 = \left(\frac{1}{a}\right)a$$

(1.5) **Distributive Properties**

$$a(b + c) = ab + ac, \qquad (a + b)c = ac + bc$$

The equals sign, =, used in properties (1.1)–(1.5) means, of course, that the expressions immediately to the right and left of the sign represent the same real number. The real numbers 0 and 1 are sometimes referred to as the **additive identity** and **multiplicative identity**, respectively. We call $-a$ the **additive inverse** of a (or the **negative** of a). If $a \neq 0$, then $1/a$ is called the **multiplicative inverse** of a (or the **reciprocal** of a). The symbol a^{-1} is often used in place of $1/a$. Thus, by definition,

$$a^{-1} = \frac{1}{a}$$

Example 1 Verify the following special cases of properties (1.2) and (1.5).

(a) $2 + (3 + 4) = (2 + 3) + 4$

(b) $2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$

(c) $2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$

(d) $(2 + 3) \cdot 4 = 2 \cdot 4 + 3 \cdot 4$

Solutions To verify each of parts (a)–(d) we perform the operations indicated on opposite sides of the equals sign and observe that the resulting numbers are identical. Thus

(a) $$2 + (3 + 4) = 2 + 7 = 9$$
$$(2 + 3) + 4 = 5 + 4 = 9$$

(b) $$2 \cdot (3 \cdot 4) = 2 \cdot 12 = 24$$
$$(2 \cdot 3) \cdot 4 = 6 \cdot 4 = 24$$

(c) $$2 \cdot (3 + 4) = 2 \cdot 7 = 14$$
$$2 \cdot 3 + 2 \cdot 4 = 6 + 8 = 14$$

(d) $$(2 + 3) \cdot 4 = 5 \cdot 4 = 20$$
$$2 \cdot 4 + 3 \cdot 4 = 8 + 12 = 20$$

Since $a + (b + c)$ and $(a + b) + c$ are always equal we may, without ambiguity, use the symbol $a + b + c$ to denote the real number they represent. Similarly, the notation abc is used to represent either $a(bc)$ or $(ab)c$. An analogous situation exists if four real numbers a, b, c, and d are added. For example, we could consider

$$(a + b) + (c + d), \quad a + [(b + c) + d], \quad [(a + b) + c] + d,$$

and so on. It can be shown that regardless of how the four numbers are grouped, the same result is obtained, and consequently it is customary to write $a + b + c + d$ for any of these expressions. Furthermore, it follows from the Commutative Properties (1.1) that the numbers can be interchanged in any way. For example,

$$a + b + c + d = a + d + c + b = a + c + d + b.$$

We shall justify a manipulation of this type by referring to "commutative and associative properties of real numbers." A similar situation exists for multiplication, where the expression $abcd$ is used to denote the product of four real numbers.

The Distributive Properties (1.5) are useful for finding products of many different types of expressions. The next example provides two illustrations. Others will be found in the exercises.

Example 2 If a, b, c, and d denote real numbers, show that

(a) $a(b + c + d) = ab + ac + ad$

(b) $(a + b)(c + d) = ac + bc + ad + bd$

Solutions Each product may be found by using property (1.5) several times. The reader should supply reasons for each step in the following.

(a)
$$\begin{aligned} a(b + c + d) &= a[(b + c) + d] \\ &= a(b + c) + ad \\ &= (ab + ac) + ad \\ &= ab + ac + ad \end{aligned}$$

(b)
$$\begin{aligned} (a + b)(c + d) &= (a + b)c + (a + b)d \\ &= (ac + bc) + (ad + bd) \\ &= ac + bc + ad + bd \end{aligned}$$

If $a = b$ and $c = d$, then since a and b are merely different names for the same real number, and likewise for c and d, it follows that $a + c = b + d$ and $ac = bd$. This is often called the **substitution principle**, since we may think of replacing a by b and c by d in the expressions $a + c$ and ac. As a special case, using the fact that $c = c$ gives us the following rules.

(1.6)

If $a = b$, then $a + c = b + c$.
If $a = b$, then $ac = bc$.

We sometimes refer to the rules in (1.6) by the statements "any number c may be added to both sides of an equality" and "both sides of an equality may be multiplied by the same number c." These rules constitute two extremely important algebraic manipulations. We shall make heavy use of them in Chapter 2, in conjunction with solving equations.

Properties (1.1)–(1.5) can be used to prove the following results (see Exercises 36 and 37).

(1.7)

$a \cdot 0 = 0$ for every real number a.
If $ab = 0$, then either $a = 0$ or $b = 0$.

The statements in (1.7) imply that $ab = 0$ *if and only if* either $a = 0$ or $b = 0$. The phrase "if and only if," which is used throughout mathematics, always has a two-fold character. Here it means that if $ab = 0$, then $a = 0$ or $b = 0$ and, *conversely*, if

$a = 0$ or $b = 0$, then $ab = 0$. Consequently, if both $a \neq 0$ and $b \neq 0$, then $ab \neq 0$; that is, *the product of two nonzero real numbers is always nonzero.*

The following rules for negatives can also be proved directly from properties (1.1)–(1.5). (See Exercise 41.)

(1.8)
$$\begin{aligned} -(-a) &= a \\ (-a)b &= -(ab) = a(-b) \\ (-a)(-b) &= ab \\ (-1)a &= -a \end{aligned}$$

The operation of **subtraction** (denoted by $-$) is defined by

(1.9)
$$a - b = a + (-b)$$

The next example indicates that the Distributive Properties hold for subtraction.

Example 3 If a, b, and c are real numbers, show that

$$a(b - c) = ab - ac.$$

Solution We shall list reasons after each step as follows.

$$\begin{aligned} a(b - c) &= a[b + (-c)] && (1.9) \\ &= ab + a(-c) && (1.5) \\ &= ab + [-(ac)] && (1.8) \\ &= ab - ac && (1.9) \end{aligned}$$

If $b \neq 0$, then **division** (denoted by $\div$) is defined by

(1.10)
$$a \div b = a\left(\frac{1}{b}\right) = ab^{-1}$$

The symbol a/b is often used in place of $a \div b$, and we refer to it as the **quotient of *a* by *b*** or the **fraction *a* over *b***. The numbers a and b are called the **numerator** and **denominator**, respectively, of the fraction. It is important to note that since 0 has no multiplicative inverse, a/b is not defined if $b = 0$; that is, *division by zero is not permissible.* Also note that

$$1 \div b = \frac{1}{b} = b^{-1}.$$

The following rules for quotients may be established, where all denominators are nonzero real numbers.

(1.11)

$$\frac{a}{b} = \frac{c}{d} \quad \text{if and only if } ad = bc$$

$$\frac{a}{b} = \frac{ad}{bd}$$

$$\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$$

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Example 4 Find (a) $\frac{2}{3} + \frac{9}{5}$ (b) $\frac{2}{3} \cdot \frac{9}{5}$ (c) $\frac{2}{3} \div \frac{9}{5}$

Solutions Using (1.11) we have

(a) $$\frac{2}{3} + \frac{9}{5} = \frac{(2 \cdot 5) + (3 \cdot 9)}{3 \cdot 5} = \frac{10 + 27}{15} = \frac{37}{15}$$

(b) $$\frac{2}{3} \cdot \frac{9}{5} = \frac{2 \cdot 9}{3 \cdot 5} = \frac{18}{15} = \frac{6 \cdot 3}{5 \cdot 3} = \frac{6}{5}$$

(c) $$\frac{2}{3} \div \frac{9}{5} = \frac{2}{3} \cdot \frac{5}{9} = \frac{2 \cdot 5}{3 \cdot 9} = \frac{10}{27}$$

The **positive integers** 1, 2, 3, 4, ... may be obtained by adding the real number 1 successively to itself. The negatives, $-1, -2, -3, -4, \ldots$, of the positive integers are referred to as **negative integers**. The **integers** consist of the totality of positive and negative integers together with the real number 0.

Observe that by the Distributive Properties, if a is a real number then

$$a + a = (1 + 1)a = 2a$$

and

$$a + a + a = (1 + 1 + 1)a = 3a.$$

Similarly the sum of four a's is $4a$, the sum of five a's is $5a$, and so on.

If a, b, and c are integers and $c = ab$, then a and b are called **factors**, or **divisors**, of c. For example the integer 6 may be written as

$$6 = 2 \cdot 3 = (-2)(-3) = 1 \cdot 6 = (-1)(-6).$$

Hence $1, -1, 2, -2, 3, -3, 6$, and -6 are factors of 6.

A positive integer p different from 1 is **prime** if its only positive factors are 1 and p. The first few primes are 2, 3, 5, 7, 11, 13, 17, and 19. One of the reasons for the importance of prime numbers is that every positive integer a different from 1 can be expressed in one and only one way (except for order of factors) as a product of primes. (The proof of this result will not be given in this book.) As examples, we have

$$12 = 2 \cdot 2 \cdot 3, \quad 126 = 2 \cdot 3 \cdot 3 \cdot 7, \quad 540 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5.$$

A real number is called a **rational number** if it can be written in the form a/b, where a and b are integers and $b \neq 0$. Real numbers that are not rational are called **irrational**. The ratio of the circumference of a circle to its diameter is an irrational real number and is denoted by π. It is often approximated by the decimal 3.1416 or by the rational number 22/7. We use the notation $\pi \approx 3.1416$ to indicate that π is *approximately equal* to 3.1416. To cite another example, a real number a such that $a^2 = 2$, where a^2 denotes $a \cdot a$, is not rational. There are two such irrational numbers denoted by the symbols $\sqrt{2}$ and $-\sqrt{2}$.

Real numbers may be represented by decimal expressions. Decimal representations for rational numbers either terminate or are nonterminating and repeating. For example, it can be shown by long division that a decimal representation for 7434/2310 is 3.2181818..., where the digits 1 and 8 repeat indefinitely. The rational number 5/4 has the terminating decimal representation 1.25. Decimal representations for irrational numbers may also be obtained; however, they are always nonterminating and nonrepeating. The process of finding decimal representations for irrational numbers is usually difficult. Often some method of successive approximation is employed. For example, the device learned in arithmetic for extracting square roots can be used to find a decimal representation for $\sqrt{2}$. Using this technique we successively obtain the approximations 1, 1.4, 1.41, 1.414, 1.4142, and so on.

Sometimes, it is convenient to use the notation and terminology of sets. **A set** may be thought of as a collection of objects of some type. The objects are called **elements** of the set. Capital letters $A, B, C, R, S, \ldots$ will often be used to denote sets. Lower-case letters $a, b, x, y, \ldots$ will represent elements of sets. Throughout our work $\mathbb{R}$ will denote the set of real numbers, and $\mathbb{Z}$ the set of integers. If every element of a set S is also an element of a set T, then S is called a **subset** of T. For example, $\mathbb{Z}$ is a subset of $\mathbb{R}$. Two sets S and T are said to be equal, written $S = T$, if S and T contain precisely the same elements. The notation $S \neq T$ means that S and T are not equal.

EXERCISES

In each of Exercises 1–10, justify the equality by stating only one of the properties (1.1)–(1.5).

1 $(4 \cdot 5) \cdot 4 = 4 \cdot (4 \cdot 5)$

2 $3 \cdot (4 + 5) = (4 + 5) \cdot 3$

3 $(4 \cdot 5) \cdot 4 = 4 \cdot (5 \cdot 4)$

4 $(4 + 5) \cdot 3 = 4 \cdot 3 + 5 \cdot 3$

5 $3 \cdot (5 + 0) = 3 \cdot 5$

6 $3 + (-3) = 0$

7 $1 \cdot (2 + 3) = 2 + 3$

8 $(1 + 2) + 1 = 1 + (1 + 2)$

9 $(1/4)4 = 1$

10 $0 \cdot 1 = 0$

Use properties (1.1)–(1.5) to find the products in Exercises 11–20, where all letters represent real numbers.

11 $a(b + 3) + 2(b + 3)$

12 $c(d + 1) + 5(d + 1)$

13 $(a + 2)(b + 3)$

14 $(c + 5)(d + 1)$

15 $2x(y + 2) - 3(y + 2)$

16 $2p(6q + 5) - 3(6q + 5)$

17 $(2x - 3)(y + 2)$

18 $(2p - 3)(6q + 5)$

19 $(4r + 5)(3s + 6)$

20 $(3u + 7)(4v - 1)$

In each of Exercises 21–30, write the expression as a rational number whose numerator is as small as possible.

21 $\frac{3}{4} + \frac{5}{3}$

22 $\frac{1}{5} + \frac{3}{2}$

23 $\frac{5}{6} \cdot \frac{2}{3}$

24 $\frac{4}{9}\left(\frac{1}{2} + \frac{3}{4}\right)$

25 $\frac{1}{2} + \frac{1}{4} + \frac{1}{6}$

26 $\frac{3}{-2} \cdot \frac{-5}{6}$

27 $\frac{13}{4} \div \left(\frac{3}{2} + 1\right)$

28 $\frac{2}{5} + \frac{7}{4} + \frac{3}{2}$

29 $\frac{5}{7}\left(\frac{3}{2} - \frac{5}{6}\right)$

30 $\frac{10}{11} \cdot \frac{11}{10}$

31 Show, by means of examples, that the operation of subtraction on $\mathbb{R}$ is neither commutative nor associative.

32 Show that the operation of division, as applied to nonzero real numbers, is neither commutative nor associative.

In Exercises 33–40, all letters denote real numbers and no denominators are zero.

33 Prove or disprove: $\frac{1}{a} + \frac{1}{b} = \frac{1}{a + b}$

34 Prove or disprove: $\frac{a}{b + c} = \frac{a}{b} + \frac{a}{c}$

Prove the rules in Exercises 35–40.

35 $a = -a$ if and only if $a = 0$.

36 $a \cdot 0 = 0$ (*Hint:* Write $a \cdot 0 = a \cdot (0 + 0) = a \cdot 0 + a \cdot 0$ and then add $-(a \cdot 0)$ to both sides.)

37 If $ab = 0$, then either $a = 0$ or $b = 0$.

38 $\frac{-a}{-b} = \frac{a}{b}$

39 $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

40 $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$ (*Hint:* Write $\frac{a}{b}+\frac{c}{b}=ab^{-1}+cb^{-1}$ and use the Distributive Properties.)

41 Prove (1.8).

3 COORDINATE LINES

It is possible to associate the set of real numbers with the set of all points on a line l in such a way that for each real number a there corresponds one and only one point, and conversely, to each point P on l there corresponds precisely one real number. Such an association between two sets is referred to as a **one-to-one correspondence**. We first choose an arbitrary point O, called the **origin**, and associate with it the real number 0. Points associated with the integers are then determined by laying off successive line segments of equal length on either side of O as illustrated in Figure 1.1. The points corresponding to rational numbers such as 23/5 and $-1/2$ are obtained by subdividing the equal line segments. Points associated with certain irrational numbers, such as $\sqrt{2}$, can be found by geometric construction. For other irrational numbers such as π, no construction is possible. However, the point corresponding to π can be approximated to any degree of accuracy by locating successively the points corresponding to 3, 3.1, 3.14, 3.141, 3.1415, 3.14159, and so on. It can be shown that to every irrational number there corresponds a unique point on l and, conversely, every point that is not associated with a rational number corresponds to an irrational number.

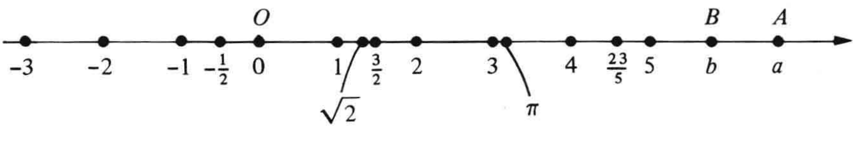

Figure 1.1

The number a that is associated with a point A on l is called the **coordinate** of A. An assignment of coordinates to points on l is called a **coordinate system** for l, and l is called a **coordinate line**, or a **real line**. A direction can be assigned to l by taking the **positive direction** along l to the right and the **negative direction** to the left. The positive direction is noted by placing an arrowhead on l as shown in Figure 1.1.

The numbers which correspond to points to the right of 0 in Figure 1.1 are called **positive real numbers**, whereas those which correspond to points to the left of 0 are **negative real numbers**. The real number 0 is neither positive nor negative. The set of positive real numbers is **closed** relative to addition and multiplication; that is, if a and b are positive, then so is the sum $a+b$ and the product ab.

Note that if a is positive, then $-a$ is negative. Similarly, if $-a$ is positive, then $-(-a)=a$ is negative. A common error is to think that $-a$ is always a negative number; however, this is not necessarily the case. For example, if