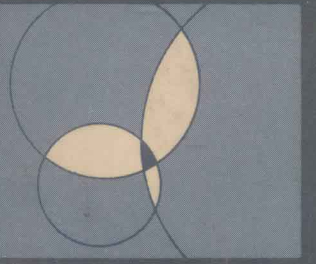
# COLLEGE ALGEBRA



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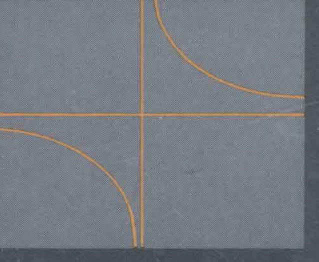

FOURTH EDITION

GORDON FULLER

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*Texas Tech University*

# College Algebra

## 4th Edition

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## 4th Edition

# Preface

The Fourth Edition of COLLEGE ALGEBRA, like the previous ones, has been written for college freshmen who do not have the background in algebra to begin with analytic geometry and calculus. In preparing this edition, every effort has been made to produce a clear, readable text from which students can learn and instructors can teach. Toward that objective, extensive revisions have been made, a few of which are listed:

1. Each chapter ends with a review exercise.

2. The study of quadratic equations is preceded by a brief introduction to complex numbers. The knowledge of such numbers will enable students to solve quadratic equations whose roots are not real.

3. There are separate word-problem sections for the quadratic formula and for the systems of quadratic equations.

4. A section and an exercise are included on linear and quadratic equations with imaginary coefficients.

5. New types of inequalities, such as regional inequalities, are treated.

6. The method of finding approximations to irrational roots of polynomial equations has been simplified.

7. Inverse functions have been introduced, and the treatment of exponential functions has been expanded.

8. The discussion of probability is brief and clear so that instructors can now introduce this fascinating topic to students.

9. New appendices list algebraic symbols and their meanings; the axioms used in college algebra; and major definitions, equations, laws, and properties.

Each topic is illustrated with solved examples which have been designed to clarify the principles involved. The careful organization and explanations help the student learn and comprehend difficult concepts.

Meticulous care has been used in selecting the chapter end problems. Answers for two-thirds of the problems are included in the text, and the answers to problems whose numbers are multiples of three (3, 6, 9, etc.) are in the Instructor's Manual. At appropriate points in the book detailed solutions are given. This should be helpful to those who would like more information than is found in the bare answers.

The necessary numerical tables are printed on the end papers for easy reference. These include powers and roots of numbers, common logarithms of numbers, and values of trigonometric functions.

The book is complete, and has enough material for a class meeting three, four, or five times a week. Some classes may be well served by three class meetings a week for two quarters. The chapters are sufficiently independent to afford suitable material for any given group from the various options.

Special thanks go to the following individuals who read the manuscript and made valuable suggestions: Fr. Micah E. Kozoil, O.S.B., Saint Vincent College, Dr. Paul Matthews, Tulas Junior College, Dr. Walter L. Wilson, Jr., and Dr. Henry C. Miller, Jr., both at University of Alabama at Tuscaloosa.

# Contents

# CHAPTER 1

# *Sets and Operations*

The concept of a set is basic in the study of algebra and in other areas of mathematics. Although sets have long been used in mathematics, this idea has been greatly emphasized in recent years. As a consequence, a new terminology and symbolism relating to sets have been developed and somewhat standardized. This new approach aids in the understanding and deepens the insight as the student studies mathematics. Because of the importance of this new trend, we introduce our study of algebra with an elementary treatment of sets.

## 1–1 SETS

We are accustomed to thinking of a set as a collection of objects or other entities. For example, we understand the meaning of "set of china," "set of chairs," "set of books." In fact, the meaning of a set is so firmly and definitely fixed in our minds that a definition of the word is unnecessary.

Each object of a set is called a *member* or *element* of the set. As with the word "set," we take "member" and "element" as undefined terms.

Numerous kinds of sets arise in mathematics. The following are examples, where each set is denoted by a capital letter.

$A$ = the set of vowels in the English alphabet

$B$ = the positive integers less than 6*

*We assume the existence of the numbers 1, 2, 3, 4, and so on indefinitely. This set of numbers is referred to as the *counting numbers*, the *natural numbers*, and the *positive integers*.

$C =$ the set of positive odd integers less than 50*
$D =$ the set of states of the United States
$E =$ the set of all numbers between 3 and 4
$F =$ the set of all angles having measures between $0°$ and $20°$

Customarily, the members of sets are specified in the following ways:

1. The members of a set are listed, if practicable, and enclosed within braces. This is called the *list*, or *roster*, *method* of specification.
2. A description of the set is enclosed within braces. This is called the *description*, or *rule*, *method* of specification.

Using these methods, we specify the above sets with the following notations.

$A = \{a, e, i, o, u\}$
$B = \{1, 2, 3, 4, 5\}$
$C = \{1, 3, 5, \ldots, 49\}$
$D = \{x \mid x \text{ is a state of the United States}\}$
$E = \{x \mid 3 < x < 4\}$
$F = \{\theta \mid 0° < \theta < 20°\}$

The three dots in set $C$ indicate that the missing odd integers are members of the set. The vertical bars in $D$, $E$, and $F$ may be read "such that." The Greek letter $\theta$ in set $F$ stands for an arbitrary member of the set, and the symbols $\langle$ and $\rangle$ mean, respectively, "is less than" and "is greater than." We suggest the following wording for sets $D$, $E$, and $F$.

$D$ is the set of states $x$ such that $x$ is a state of the United States.
$E$ is the set of numbers $x$ such that $x$ is a number between 3 and 4.
$F$ is the set of angles $\theta$ such that $\theta$ is greater than $0°$ and less than $20°$.

The elements of each set $A$ to $F$ are definitely determined. We can decide at once if a given object is or is not a member of the set. A set that meets this condition is said to be *well defined.*

Although we expressed sets $A$, $B$, and $C$ by the roster method, we can also express them by the description method. Thus for set $A$ we write

$A = \{x \mid x \text{ is a vowel of the English alphabet}\}$

Set $D$ can be expressed by the roster method but not conveniently. Sets $E$ and $F$ cannot be expressed by the roster method.

*If $n$ is an integer, $2n$ is called an *even* integer and $2n + 1$ is called an *odd* integer.

If the elements of a set can be listed from the first element through the last element, the set is said to be *finite*. A set which is not finite is said to be *infinite*. Sets $A$, $B$, $C$, and $D$ above are finite; sets $E$ and $F$ are infinite.

EXAMPLE 1. Express the set of months in the year having 31 days by (a) the roster method, (b) the description method.

*Solution.*

{Jan, Mar, May, Jul, Aug, Oct, Dec}

$\{x \mid x \text{ is a month of the year having 31 days}\}$

We use the symbol $\in$ to indicate that a given element is a member of a particular set. Thus, referring to sets $A$ and $F$ above, we write

$i \in A$ and $10° \in F$

The symbol $\in$ may be read "is a member of" or "belongs to." The symbol $\notin$ is read "is not a member of." For example, $b \notin A$ and $21° \notin F$.

The members of a set may be variables or constants as expressed in the following definition.

DEFINITION 1–1. A symbol, usually a letter, which may stand for any member of a specified set of objects is called a *variable*. If the set has only one member, the symbol is called a *constant*.

The letters $x$ and $\theta$ in sets $D$, $E$, and $F$ are variables.

## 1–2 RELATED SETS

In this section we shall point out ways in which two sets may be related.

DEFINITION 1–2. Two sets $A$ and $B$ are said to be *equal* ($A = B$) if each element of set $A$ is an element of set $B$ and each element of set $B$ is an element of set $A$.

The sets $A = \{x, y, z\}$ and $B = \{y, z, x\}$, for example, are equal.

DEFINITION 1–3. If it is possible to pair each element of a set $A$ with exactly one element of a set $B$ and each element of $B$ with exactly one element of $A$, then we say the elements of the sets can be arranged in a *one-to-one correspondence*.

**Definition 1–4.** Two sets $A$ and $B$ are said to be *equivalent* ($A \longleftrightarrow B$) if their elements can be put into a one-to-one correspondence.

According to this definition the infinite set of positive integers and the infinite set of positive even integers are equivalent. The equivalence of the sets $A = \{1, 2, 3, \ldots\}$ and $B = \{2, 4, 6, \ldots\}$ becomes evident from the following method of pairing the elements:

$(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), \ldots$

**Definition 1–5.** If each element of a set $A$ is an element of set $B$, then $A$ is called a *subset* of $B$. If $A$ is a subset of $B$ and if $B$ has one or more elements not belonging to $A$, then $A$ is a *proper* subset of $B$.

To indicate that $A$ is a subset of $B$, we write $A \subseteq B$, which is read "$A$ is contained in $B$" or "$B$ contains $A$." This notation indicates that every element of $A$ is an element of $B$, but it does not indicate whether or not $A$ is a proper subset. To show that $A$ is a proper subset of $B$, we write $A \subset B$.

EXAMPLE 1. The following four subsets are some, but not all, of the subsets of $\{a, b, c, d\}$.

$$\begin{aligned} \{b\} &\subset \{a, b, c, d\} \\ \{b, c\} &\subset \{a, b, c, d\} \\ \{b, c, d\} &\subset \{a, b, c, d\} \\ \{a, b, c, d\} &\subseteq \{a, b, c, d\} \end{aligned}$$

The first three subsets are proper subsets. The fourth subset illustrates the fact that *a set is a subset of itself*; this is a consequence of the definition of a subset.

Thus far we have mentioned sets which contain one or more elements. Conditions may be specified, however, such that a set has no element. For example, the set of blondes in a group of five brunettes has no element. A set which contains no element is called the *null*, or *empty*, set. The symbol $\varnothing$ is used to stand for the empty set. Thus,

$$\begin{aligned} \{x \mid x = 1 \text{ and } x = 2\} &= \varnothing \\ \{x \mid x \text{ is an integer between 5 and 6}\} &= \varnothing \end{aligned}$$

Since $\varnothing$ has no element, we can conclude that each element of $\varnothing$ belongs to any set $A$, and therefore $\varnothing \subseteq A$. In other words, the empty set is a subset of every set.

EXAMPLE 2. If $E = \{2, 5, 8\}$, identify each of the following statements as true or false.

1. $\{2\} \in E$
2. $\varnothing \in E$
3. $1 = \{1\}$
4. $2 \subset E$
5. $\varnothing \subset E$

*Solution.*

1. False. The set $\{2\}$ is not an element of $E$
2. False. $\varnothing$ is a set, not an element of $E$
3. False. An element of a set is not equal to the set
4. False. 2 is an element of $E$, not a subset
5. True. $\varnothing$ is a subset of all sets

EXAMPLE 3. Use the description method to specify the squares of the first five natural numbers.

*Solution.* $\{x \mid x$ is the square of a natural number less than 6$\}$

## EXERCISE 1–1

Enclose within braces (a) a list of the elements of each set and (b) a description of the set.

1. The natural numbers less than 7
2. The even natural numbers between 2 and 13
3. The months of the year having 30 days
4. The first three presidents of the United States
5. The first four natural numbers multiplied by 3
6. The squares of the first four natural numbers
7. The months of the year that are spelled with less than six letters
8. The natural numbers between 18 and 23

Use the description method to designate each of the following sets.

9. $\{2, 4, 8, 16\}$
10. {even natural numbers}
11. {odd natural numbers}
12. {odd natural numbers times 3}
13. $\{1, 8, 27, 64\}$