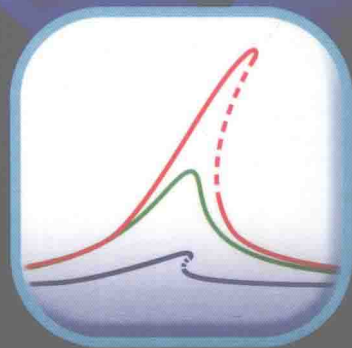
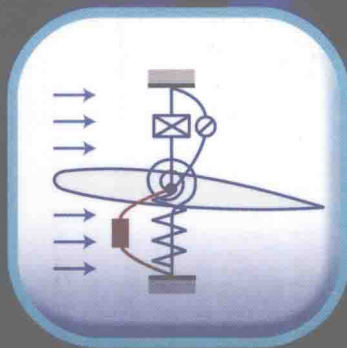
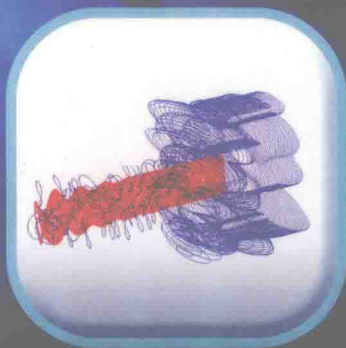


ALPER ERTURK AND DANIEL J. INMAN

# PIEZOELECTRIC ENERGY HARVESTING



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*These oscillations arise freely, and I have determined various conditions, and have performed a great many beautiful experiments on the position of the knot points and the pitch of the tone, which agree beautifully with the theory.*

—Daniel Bernoulli (from a letter to Leonhard Euler)<sup>1</sup>

*We have found a new method for the development of polar electricity in these same crystals, consisting in subjecting them to variations in pressure along their hemihedral axes.*

—Pierre and Paul-Jacques Curie  
(from the paper announcing their discovery)<sup>2</sup>

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<sup>1</sup> In Timoshenko, S.P., 1953, *History of Strength of Materials (with a brief account of the history of theory of elasticity and theory of structures)*, McGraw-Hill, New York.

<sup>2</sup> In Cady, W.G., 1946, *Piezoelectricity: An Introduction to the Theory and Applications of Electromechanical Phenomena in Crystals*, McGraw-Hill, New York.

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# Preface

Energy harvesting from ambient waste energy for the purpose of running low-powered electronics has emerged during the last decade as an enabling technology for wireless applications. The goal of this technology is to provide remote sources of electric power and/or to recharge storage devices, such as batteries and capacitors. The concept has ecological ramifications in reducing the chemical waste produced by replacing batteries and potential monetary gains by reducing maintenance costs. The potential for enabling wireless monitoring applications, such as structural health monitoring, also brings an element of increasing public safety. With the previously mentioned potential as motivation, the area of energy harvesting has captivated both academics and industrialists. This has resulted in an explosion of academic research and new products. The evolution of low-power-consuming electronics and the need to provide wireless solutions to sensing problems have led to an emergence of research in energy harvesting. One of the most studied areas is the use of the piezoelectric effect to convert ambient vibration into useful electrical energy. Most products currently available for harvesting vibrational energy are based on this use of the piezoelectric effect. The focus of this book is placed on detailed electromechanical modeling of piezoelectric energy harvesters for various applications.

The area of vibration-based energy harvesting encompasses mechanics, materials science, and electrical circuitry. Researchers from all three of these disciplines contribute heavily to the energy harvesting literature. Due to the topic being spread over numerous different fields of study, many oversimplifications resulted from early attempts to understand and to develop predictive models. Our hope with the current volume is to provide reliable techniques for precise electromechanical modeling of piezoelectric energy harvesters and to understand the relevant phenomena. The term *energy harvester* is defined in this book as the generator device undergoing vibrations due to a specific form of excitation. The main focus is therefore placed on modeling the electromechanical response of the device for the respective form of excitation rather than investigating the storage components and the power electronics aspects. A brief review of the literature of piezoelectric energy harvesting circuits is also provided.

As far as the prerequisite of the material covered in this book is concerned, we have assumed that the reader is knowledgeable at the level of a BS degree in an engineering curriculum that includes a basic vibrations or structural dynamics course. Fundamental background in ordinary differential equations and partial differential equations is essential. Some of the topics in this book are related to subjects not necessarily covered in most undergraduate engineering curricula, such as random vibrations, nonlinear oscillations, and aeroelastic vibrations. However, references to excellent books and papers are provided as required. These aforementioned topics of vibrations and structural dynamics are coupled with the electrical domain throughout

this text to formulate and/or to investigate various problems for vibration-based energy harvesting using piezoelectric transduction.

Several configurations ranging from conventional cantilevers to more sophisticated devices exhibiting nonlinear phenomena are modeled and tested for vibration-based energy harvesting. Both analytical and approximate analytical distributed-parameter electromechanical models are presented along with several case studies for experimental validation. Guidelines are provided for experimental modal analysis of a piezoelectric energy harvester in a laboratory environment.

The electromechanical response of a piezoelectric energy harvester and the amount of power it generates are completely dependent on the nature of the ambient energy. We consider harmonic as well as non-harmonic forms of ambient excitation. Moving-load excitations, transient vibrations, periodic inputs, and airflow-induced vibrations are also discussed. Each chapter ends with a notes section, which provides additional discussions, and references for further reading.

We are indebted to financial support for our research and experiments performed in energy harvesting. Specifically, some of the results presented here are referred from publications funded by energy harvesting grants from the Air Force Office of Scientific Research (under the programs and encouragement of Dr. “Les” Lee) and the National Institute of Standards and Technology (under the direction of Dr. Jean-Louis Staudenmann). We have also enjoyed collaboration with several colleagues on this subject, such as Dr. Carlos De Marqui Jr. (University of Sao Paulo, Sao Carlos, Brazil) on energy harvesting from aeroelastic vibrations and Dr. Didem Ozevin (University of Illinois, Chicago) along with the MISTRAS Group, Inc. Products and Systems Division on the energy harvesting potential of bridges as well as the bridge strain data. In addition, we have had the pleasure and support of communications with Professor Earl Dowell, Dr. Brian Mann, Mr. Sam Stanton (Duke University), Professor Ephraim Garcia (Cornell University), Professor Yi-Chung Shu (National Taiwan University), Professor Niell Elvin (The City College of New York), Professor Dane Quinn (University of Akron), Dr. Mohammed Daqaq (Clemson University), Dr. Steve Burrow, Dr. David Barton (University of Bristol, UK), Professor Sondipon Adhikari, Professor Michael Friswell (Swansea University, UK), Dr. Andres Arrieta (TU Darmstadt, Germany), and Dr. Ho-Yong Lee (Ceracomp Co. Ltd., Korea). Professor Shashank Priya, Mr. Steve Anton, Ms. Na Kong, Mr. Justin Farmer, and other colleagues and graduate students in the Center for Intelligent Material Systems and Structures at Virginia Tech helped to form an atmosphere of discovery and collegiality, without which this effort would not be possible. In particular, we are indebted to Ms. Beth Howell for all of her help as our program manager. We are also indebted to Mr. Neville Hankins for copyediting and Ms. Shalini Sharma for typesetting the book. Lastly, we would like to thank the energy harvesting community at large for its contributions through the literature and many discussions at conferences and workshops for forming an intellectually stimulating environment.

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# 1

## Introduction to Piezoelectric Energy Harvesting

*This chapter provides an introduction to vibration-based energy harvesting using piezoelectric transduction. Following a summary of the basic transduction mechanisms that can be used for vibration-to-electricity conversion, the advantages of piezoelectric transduction over the other alternatives (particularly electromagnetic and electrostatic transductions) are discussed. Since the existing review articles mentioned in this chapter present an extensive review of the literature of piezoelectric energy harvesting, only the self-charging structure concept that uses flexible piezoceramics and thin-film batteries is summarized as a motivating example of multifunctional aspects. The focus is then placed on summarizing the literature of mathematical modeling of these devices for various problems of interest, ranging from exploiting mechanical nonlinearities to aeroelastic energy harvesting. Along with historical notes, the mathematical theory of linear piezoelectricity is briefly reviewed in order to derive the constitutive equations for piezoelectric continua based on the first law of thermodynamics, which are later simplified to reduced forms for use throughout this text. An outline of the remaining chapters is also presented.*

### 1.1 Vibration-Based Energy Harvesting Using Piezoelectric Transduction

Vibration-based energy harvesting has received growing attention over the last decade. The research motivation in this field is due to the reduced power requirement of small electronic components, such as the wireless sensor networks used in passive and active monitoring applications. The ultimate goal in this research field is to power such small electronic devices by using the vibrational energy available in their environment. If this can be achieved, the requirement of an external power source as well as the maintenance costs for periodic battery replacement and the chemical waste of conventional batteries can be reduced.

As stated by Williams and Yates [1] in their early work on harvesting vibrational energy for microsystems, the three basic vibration-to-electric energy conversion mechanisms are the