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PRODUCTIVE THINKING

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Acknowledgments

THIS book owes much to Clara W. Mayer, Dean of the School of Philosophy and Liberal Arts in the New School for Social Research. But for her untiring effort the manuscript would not have reached its final form; she was intensely interested in its subject matter and deeply devoted to its aims. That she never failed to find time for it notwithstanding her full time activities at the School, was a source of inspiration.

I am very grateful to Dr. S. E. Asch for his fine help in preparing the manuscript; to Mr. and Mrs. Benno Elkan for untiring friendly encouragement; to Mrs. Clara Mond and Mrs. Maria Di Piazza for indefatigable secretarial assistance.

Special thanks are due Dr. Alvin Johnson and my Faculty. I regret that this book, which is limited to some elementary problems, does not fully express the spirit of co-operation in the social sciences which is so alive in our Faculty and which owes so much to Alvin Johnson. Only briefly and occasionally could I touch upon these issues, which mean so much to all of us. I am likewise indebted to my friends whose work is connected with the problems of this book.

To the subjects of my experiments, the adults and the children, I am thankful for having learned so much. And I am very grateful to the distinguished men of science—Einstein above all—who made it possible for me to study intimately, in many conversations, how some of their great achievements in thinking developed.

The book contains only a small portion of the studies and material I have given to students in lectures and seminars. I have reported on parts of it in single lectures at Harvard, Chicago, Bloomington, Ann Arbor, Smith College, Swarthmore, Bryn Mawr, et al.

It is a kind of prolegomena; I hope I shall be able to finish two other books to which this is a mere introduction. In spite of its limitations I hope it may be of use.

MAX WERTHEIMER

New Rochelle, N. Y.

September 23, 1943

Note by the Editors of the First Edition

THE manuscript of *Productive Thinking*, upon which Max Wertheimer had worked during the last seven years, was completed shortly before his death. In preparing the work for publication we found that certain linguistic revisions were necessary. Mere difficulties of expression, it seemed to us, should not be allowed to obscure the meaning which Wertheimer clearly had in mind. The reader may be assured, however, that changes have been made only where the purpose of clarity could not otherwise have been attained. The content and the form of the discussion have not been altered in any place.

All remaining notes and papers of Wertheimer will be deposited with the Graduate Faculty of Political and Social Science, where qualified scholars will have the opportunity to study them.

Valentin Wertheimer assisted his father in some of the final revisions of the manuscript and has been helpful to us in many ways.

We are grateful to Mrs. Ray Borne for the painstaking care with which she saw the manuscript through the press.

S. E. Asch
W. Köhler
C. W. Mayer

1945

Note by the Editor of the Enlarged Edition

THIS edition contains, in addition to the contents of the previous edition, various chapters and fragments that were found among Max Wertheimer's notes and papers. An early version of the Table of Contents, also found among his papers, indicated that he had at one time planned to include the three chapters (Two, Five, and Six) that have been added. Although the finished manuscript of the original book did not make use of them, they and six new appendices are presented here because it seemed to many people that this material should be more widely known.

The manuscripts of the additions seemed less finished than the manuscript of the original edition. Hence some changes in organization and expression were required, but these editorial changes were kept to a minimum. Notes by the present editor are indicated by "—M.W." These persons have read the new portions of this volume, and made many helpful suggestions: Rudolf Arnheim, Solomon E. Asch, Mary Henle, Clara W. Mayer, Lise Wertheimer, and Valentin Wertheimer.

The following note, found among Max Wertheimer's papers, suggests the further work he had intended and may serve as an introduction to some of the additions.

I had limited this book mostly to subject matters which contain summative features as, e.g., the size of an area, the sum of a series, in order to focus the reader on just the gestalt in the thinking process itself; I have discussed only some formally elementary cases of gestalt features in the subject matter. In a second book I will have to deal also with the subject matters that are themselves real gestalten. Whereas methods and tools for piecemeal or structure-blind operations are highly developed, this by comparison is not the case for problems in gestalten, problems characteristic for the development of gestalt theory.

Two books must follow: one on the broader aspects of the psychology of thinking, the other on the problem of gestalt logic in which the traditional issues appear as the Euclidean special case. Here also the features of productive gestalt heuristics will have to be treated.

Boulder, Colorado
April 15, 1958

Michael Wertheimer

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* All material marked with an asterisk appears for the first time in this edition.

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Introduction

WHAT occurs when, now and then, thinking really works productively? What happens when, now and then, thinking forges ahead? What is really going on in such a process?

If we look for answers in books, we often find apparently easy ones. But confronted by actual processes of this kind—when one has just had a creative idea, however modest the issue, when one has begun really to grasp an issue, when one has enjoyed a clean, productive process of thought—those answers often seem to cover up the real problems rather than to face them squarely. The flesh and blood of what has happened seem to be lacking in those answers.

Surely in the course of your life you have been curious about a lot of things, sometimes seriously. Have you been equally serious about what this thing called thinking may be? There are, in this world of ours, eating, thunderstorms, blossoms, crystals. Various sciences deal with them; they attempt by great effort to get real understanding, to grasp what these things really are. Are we equally serious when we ask what productive thinking is?

There are fine cases. You can find them often, even in daily life. If you have had your eyes open, you have probably encountered somewhere in your life—if nowhere else, then in children—this surprising event, the birth of a genuine idea, of a productive development, the transition from a blind attitude to understanding in a productive process. If you have not been fortunate enough to experience it yourself, you may have encountered it in others; or you may—fascinated—have glimpsed it when reading good books.

Many are of the opinion that men do not like to think; that they will do much to avoid it; that they prefer to repeat instead. But in spite of many factors that are inimical to real thinking, that suffo-

cate it, here and there it emerges and flourishes. And often one gets the strong impression that men, even children, long for it.

What really takes place in such processes? What happens if one really thinks, and thinks productively? What may be the decisive features and the steps? How do they come about? Whence the flash, the spark? What are the conditions, the attitudes, favorable or unfavorable to such remarkable events? What is the real difference between good and bad thinking? And in connection with all these questions: how improve thinking? your thinking? thinking itself? Suppose we were to make an inventory of basic operations in thinking—how would it look? What, basically, is at hand? Could the basic operations themselves be enlarged and improved, and thus be made more productive?

For more than two thousand years some of the best brains in philosophy, in logic, in psychology, in education, have worked hard to find real answers to these questions. The history of these efforts, the brilliant ideas brought forward, the hard work done in research and in theoretical discussion, present on the whole a rich, dramatic picture. Much has been achieved. In a large number of special questions solid contributions to understanding have been made. At the same time there is something tragic in the history of these efforts. Again and again when great thinkers compared the ready answers with actual, fine thinking, they were troubled and deeply dissatisfied—they felt that what had been done had merits, but that in fact it had perhaps not touched the core of the problem at all.

The situation is still somewhat of this kind. To be sure, many books deal with these questions as if, fundamentally, everything were settled—in one way or another. For there are basically different ideas about what thinking is, each with serious consequences for behavior, for education. When observing a teacher we may often realize how serious the consequences of such ideas about thinking can be.

Although there are good teachers, with a natural feeling for what genuine thinking means, the situation in schools is often not good. How teachers act, how a subject matter is taught, how textbooks are written, all this is widely determined by two traditional views about the nature of thinking: the view of traditional logic and the view of association theory. These two views have their merits. To a degree

they seem adequate to certain types of thought processes, to certain jobs in thinking; but it is at least an open question whether the way in which they interpret thinking does not cause serious hindrance, an actual impairment of genuine abilities.

This book has been written because the traditional views have ignored important characteristics of thought processes, because in many other books those views are taken for granted without real investigation, because in such books the discussion of thinking runs largely in mere generalities, and because, for the most part, the gestalt view is only superficially known. Much is at stake and it seems proper to bring these neglected issues to the fore, to examine the traditional views, to discuss the crucial problems in concrete instances of fine, productive thought, and in doing so, to give the gestalt interpretation of thinking.

In a number of chapters (One-Six) elementary, seemingly obvious examples will be used for the discussion. We shall face the basic theoretical issues in direct contact with the concrete material. Several methods of experimental discussion will contribute to clarification. We shall study how thinking actually proceeds and what the nature of the process is as a whole, as well as in its parts, steps, and operations. In contrast to poorer ways of thinking the reader may enjoy the way in which fine, though modest, productive processes occur in children.

We shall see that what is going on in these processes is far from being adequately handled by the tools and views of the two traditional approaches. We shall recognize characteristics and operations that have been ignored because they are intrinsically foreign to the customary conceptions. We shall see how such factors work in the achievement of thinking.

In Chapter Seven we shall describe a modest example from human life, which seems to touch the essence of profound human issues.

We shall also give in Chapters Four, Eight, Nine, and Ten several descriptions and interpretations of greater thought processes, concluding with the story of the thought process that led Einstein to the discovery of the relativity theory. In the final chapter we shall formulate our general conclusions.