MESOSCOPIC PHENOMENA IN SOLIDS

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Oh, how many of them there
are in the fields!
But each flowers in its
own way –
In this is the highest achievement
of a flower!

Matsuo Bashó
1644–1694

PREFACE TO THE SERIES

Our understanding of condensed matter is developing rapidly at the present time, and the numerous new insights gained in this field define to a significant degree the face of contemporary science. Furthermore, discoveries made in this area are shaping present and future technology. This being so, it is clear that the most important results and directions for future developments can only be covered by an international group of authors working in cooperation.

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The General Editors of the Series,

V.M. Agranovich A.A. Maradudin

PREFACE

The physics of disordered systems has enjoyed a resurgence of interest in the last decade. New concepts such as weak localization, interaction effects, Coulomb gap, etc., have been developed for the transport properties of metals and insulators (Lee and Ramakrishnan 1985, Altshuler and Lee 1988). With the fabrication of smaller and smaller samples and the routine availability of low temperatures, new physics has emerged from the studies of small devices. The new field goes under the name 'mesoscopic physics' and has developed rapidly both experimentally and theoretically. This book is designed to review the current status of the field.

In the theory of systems with quenched disorder, the conventional wisdom is that physical measurements on a single sample are well described by the ensemble averaged quantities. Each member of the ensemble is characterized by the same set of macroscopic parameters such as size, shape, impurity concentration, etc., but at the same time, each sample has its own realization of disorder which distinguishes it from other members of the ensemble. This approach of ensemble averaging gives a 'macroscopic' description of the system, and is very successful in describing systems of sufficiently large size. The reason is that such systems are believed to be self-averaging, i.e., the typical sample to sample difference δX in any physical property X becomes much smaller than the ensemble averaged value $\langle X \rangle$ as the sample size L increases. It is clear that the macroscopic approach becomes less and less adequate when the sample size decreases and δX becomes measurable. At the same time it appears that a 'microscopic' approach which describes a given sample starting from a concrete realization of disorder would be limited to numerical simulation. The lesson we have learned in the past few years is that the macroscopic approach breaks down on a surprisingly large length scale in disordered systems. Furthermore, it has been possible to develop an intermediate ('mesoscopic') approach which gives a systematic description of sample specific properties. Imry (1986) has provided an excellent summary of the early work on the general subject.

The mesoscopic approach is a statistical one in which the observable X is treated as a random variable characterized not only by its ensemble-averaged value $\langle X \rangle$, but also by its variance $\langle (\delta X)^2 \rangle$ and higher-order moments $\langle (\delta X)^n \rangle$. ($\langle \cdots \rangle$ means ensemble average and $\delta X \equiv X - \langle X \rangle$.) The full solution of a problem in this approach means the evaluation of the distribution function, i.e. the probability density of finding a sample with a given X in the ensemble.

At first sight, the experimental study of a particular sample has little to do with the statistics of an ensemble. However, we have learned that by tuning some external parameters, mesoscopic systems exhibit properties which vary in a way which is specific and reproducible for a given sample. These properties may be called 'fingerprints' of the sample. An early example is observation of the conductance G in narrow samples of silicon metal-oxide-semiconductor inversion layers as a function of gate voltage (Fowler et al. 1982). As shown in fig. 1, the conductance at low temperatures exhibits strong fluctuations as a function of gate voltage which looks random but are nevertheless reproducible and sample specific. Another example is the 'magneto-fingerprint' in metallic samples (Umbach et al. 1984), i.e. sample-specific dependence of the conductance of an external magnetic field H. From the statistical point of view, the quantity which depends on the external variable, e.g. $G(H)$, is a random function which can be characterized by the auto-correlation function $\langle G(H_1)G(H_2)\rangle - \langle G(H_1)\rangle\langle G(H_2)\rangle$. If this correlation vanishes at large enough $|H_1 - H_2|$, the same sample at two different magnetic fields can be considered as two independent members of the ensemble. It is this observation which allows us to relate the fluctuations of a given sample to the statistics of sample-to-sample fluctuations.

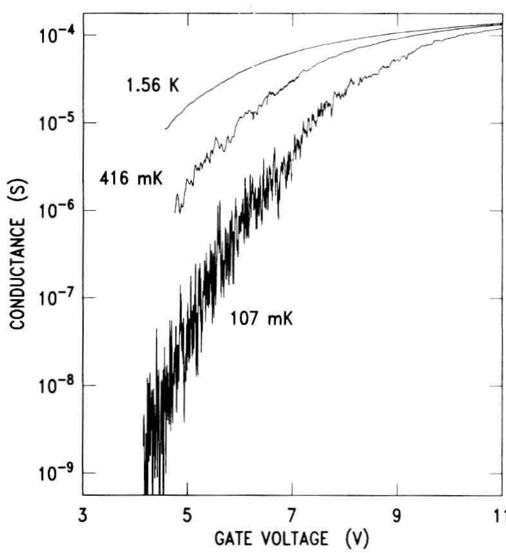

Fig. 1. The conductance of a narrow Si MOSFET (10 μm long and approximately 0.05 μm wide) as a function of gate voltage for three temperatures. The structures that develop at low temperature are reproducible and sample specific. Data are similar to those reported by Fowler et al. (1982) and are from the same sample.

Much of the recent work on mesoscopic systems has focused on the effect of quantum mechanical coherence of the electron wavefunction which leads to surprisingly large fluctuation effects. There are also examples where quantum coherence is not important but nevertheless sample specific fluctuations remain. This topic is sometimes called 'incoherent mesoscopics' and is reviewed in chapter 8 by Raikh and Ruzin. Here we give a qualitative discussion of the physics behind the quantum coherent regime as an introduction to the more detailed treatment given by various review chapters in this book.

We believe that for a large enough sample size L, the relative magnitude of fluctuations $\sqrt{\langle \delta X^2 \rangle}/\langle X \rangle$ is inversely proportional to the volume of the sample $V = L^d$ (d is the effective number of dimensions):

$$\frac{\langle (\delta X)^2 \rangle}{\langle X \rangle^2} \sim \left(\frac{L_c}{L}\right)^d. \tag{1}$$

The reason for this belief is the following. We can try to split our sample into N smaller pieces which (1) give additive contributions to X, and (2) are statistically independent from the point of view of mesoscopic fluctuations. Under these conditions $\langle (\delta X)^2$ is proportional to N while $\langle X \rangle^2 \propto N^2$ and eq. (1) is valid with L_c equal to the size of the smallest pieces which remain statistically independent.

In incoherent mesoscopics, the problem can be treated classically and L_c is of the order of the length scale of spatial inhomogeneity and is generally of microscopic scale. On the other hand, when quantum interference effects are taken into account L_c is determined by the phase coherence length which can diverge when the temperature T tends to zero and can easily become much larger than the length of the quenched spatial inhomogeneity.

The mechanism of the appearance of the mesoscopic fluctuations due to quantum interference can be clarified by a following simple example. Consider the propagation of some kind of wave in a random medium from the source at point B to the detector at point C. If there are several possible trajectories, the total probability to get from B to C equals

$$W = \left| \sum_i A_i \right|^2, \tag{2}$$

where i labels the trajectory and A_i denotes the probability amplitude for this trajectory. A_i is complex: $A_i = \sqrt{W_i} \exp \phi_i$, where W_i is the classical probability of the ith trajectory and ϕ_i is its phase. It is clear that W differs from the classical probability $W_{cl} = \sum_i W_i$ by the sum of the interference terms:

$$W - W_{cl} = 2 \sum_{ij} \sqrt{W_i W_j} \cos(\phi_i - \phi_j). \tag{3}$$

Equation (3) can be shown to vanish upon ensemble averaging, provided that the distance between B and C is much larger than the wavelength and that we can neglect the effects of weak localization (interference which appears for self-intersecting trajectories between different directions of going around the loop). This is because $|\phi_i - \phi_j|$ is random and much larger than 2π, so that $\langle \cos(\phi_i - \phi_j) \rangle = 0$.

The situation changes when we consider the mean square of the probability. It is clear from eq. (3) that since $\langle \cos^2(\phi_i - \phi_j) \rangle = \frac{1}{2}$

$$\langle W^2 \rangle = \langle W \rangle^2 + 2 \sum_{ij} W_i W_j. \tag{4}$$

Therefore, quantum interference leads to a difference between $\langle W^2 \rangle$ and $\langle W \rangle^2$, i.e. it leads to mesoscopic fluctuations. It is also clear that this effect is strongly nonlocal (it is determined by the phases of the trajectories) and this is the reason why eq. (1) is not valid.

This example gives also an idea of how 'fingerprints' appear. Consider, e.g. the magnetic field dependence of W. The autocorrelation function will be determined by

$$\langle \cos[\phi_i(H_1) - \phi_j(H_1)] \cos[\phi_i(H_2) - \phi_j(H_2)] \rangle.$$

This is equal to $\frac{1}{2}$ at $H_1 = H_2$. On the other hand, $(\phi_i - \phi_j)$ depends on the magnetic field: change in $(\phi_i - \phi_j)$ when the magnetic field goes from H_1 to H_2 equals simply 2π times the change in the magnetic flux through the area between these two trajectories in the units of the quantum magnetic flux $\Phi_0 = hc/2e$. Therefore, everytime the total flux through the system changes by more than Φ_0 we get a new member of the ensemble. At the same time, it means that the typical scale of 'magnetofingerprint' in terms of the magnetic flux through the sample is Φ_0. The Fermi energy dependence of the conductance can also be understood in a very similar way which gives as a typical energy scale of the fingerprint $E_c \sim h/\tau_{tr}$ where τ_{tr} is the typical time it takes an electron to pass through the system.

The consideration cited above does not give the amplitude of the mesoscopic fluctuations. Direct calculation (Altshuler 1985, Lee and Stone 1985) gives an unexpected result which is known now as 'universal conductance fluctuations': for any size and any shape of the sample with metallic conductivity (i.e. $\langle G \rangle \gg e^2/h$) the variance of the conductance fluctuations is of the order of $(e^2/h)^2$. It is possible to understand this result in terms of energy levels statistics (Altshuler and Shklovskii 1986, Wigner 1951, 1955, 1958, Dyson 1962, Dyson and Mehta 1963) in disordered systems. The average conductance in units of e^2/h is known to be equal to the number of electron energy levels in an energy interval with a width E_c centered on a Fermi level (the so-called Thouless number, see Thouless 1974). On the other hand, the energy levels statistics predicts that typical fluctuations of the number of the levels in any interval is of the order of unity. (Altshuler and Shklovskii 1986, Wigner 1951, 1955, 1958, Dyson 1962, Dyson and Mehta 1963). This immediately leads to the universal conductance fluctuations.

Returning to the coherent wave propagation in a disordered medium, it is worth mentioning that this physical picture leads to a very high sensitivity of, say, conductance to a small change of random potential. Consider, for instance, the result of the shift of a single impurity in a sample with metallic conductivity where the electron motion is diffusive. All of the trajectories which passed through this impurity will change their phases. The typical length of the trajectory is L^2/l (L and l are the sample size and the mean free path respectively) and its 'thickness' can be taken as electron–impurity scattering cross-section $S = (lc_{\text{im}})^{-1}$, where c_{im} is the impurity concentration. Hence the probability for a trajectory to pass through a given point is the ratio between the 'trajectory volume' $L^2 S/l$ and the sample volume V. Since the change of the conductance ΔG is of the order of e^2/h times the fraction of the trajectories which are affected by the shift, this change equals to

$$\Delta G \sim \frac{e^2}{h}\frac{L^2}{l^2}\frac{1}{N_{\text{im}}}, \tag{5}$$

where $N_{\text{im}} = Vc_{\text{im}}$ is the total number of impurities in a sample. Equation (5) means, for instance, that in two dimensions a shift of a single impurity leads to a change in the conductance which is independent of sample size. It also follows from eq. (5) that after a shift of $n_0 = N_{\text{im}} l^2/L^2 \ll N_{\text{im}}$ impurities $\Delta G \sim e^2/h$ which means that the phase interference is changed completely and we get a member of the ensemble completely different from the original one.

Most of the chapters in this book are devoted to the development of the above-mentioned ideas. The experimental observations of conductance fluctuations and the Aharonov–Bohm oscillations in disordered metals is reviewed by Washburn in the first chapter. The theory developed to describe these fluctuations and its further development is reviewed by Spivak and Zyuzin in chapter 2. These ideas which were originally developed for the propagation of electron waves in disordered metals have been successfully applied to the propagation of classical waves such as light and accoustic waves through a random medium. In light scattering experiments, the incident and final light beams can be controlled whereas in conductance measurements the incident and transmitted beams are summed over. Thus, the propagation of light waves permits an even more detailed study of the interference and correlation effects. These studies have already led to new insights into the correlation among speckle patterns and a new experimental tool called 'diffusion wave spectroscopy'. These developments are reviewed by Stephen in chapter 3. Chapters 4 and 5 contain reviews of the theoretical and experimental work on low frequency noise in small disordered systems by Feng and Giordano. We have described the sensitivity of the interference effects to the motion of a single impurity in a disordered metal and these articles describe the application of these ideas to the problem of $1/f$ noise in strongly disordered metals. As we already mentioned, in chapter 8 Raikh and Ruzin review fluctuations in the transmittancy through random barriers via classical processes which involve thermal activation. The physics in this regime is quite different from the physics of quantum interference which dominates most of the other chapters in this book, but the phenomenon of sample specific fluctuations is the common ground. Finally, in chapters 9 and 10 theoretical work concerning the distribution of fluctuation quantities like the conductance is discussed. In chapter 9, Stone, Mello, Muttalib and Pichard give a discussion in terms of random matrix theory while in chapter 10, Altshuler, Kravtsov and Lerner discuss the distribution function and its implication for relaxation processes as well.

There are two reviews which are not connected directly to the mesoscopic fluctuations though they also deal with the small systems: chapter 6 is devoted to the effects of Coulomb interaction in the tunneling through small junctions, and chapter 7 is a review of the experimental results on the ballistic transport through a perfect conductor. Both of these fields have developed rapidly in the past few years. In the case of ballistic transport, it has been found to be possible to describe transport properties in terms of the transmission coefficients between different current and voltage leads using the formalism developed by Büttiker and Landauer (Büttiker 1988, Landauer 1970). The transport problem then reduced to a wave-guide problem and it is possible to do microscopic theory using computer simulation. At the same time, these considerations have led to deeper understanding of the quantum Hall effect and its dependence on sample size and lead geometry. Fascinating new phenomena such as the observation of a discrete step in the conductance through a restriction have been discovered (van Wees et al. 1988, Wharam et al. 1988) and the possibility of creating interference phenomena with tailor-made geometries is endless. Chapter 7 serves as an introduction to this topic even though a thorough review of the current status of this rapidly growing field is beyond the scope of this book.

A second important topic deals with the importance of Coulomb repulsion in small systems. Since the Coulomb energy scales inversely with sample size, it clearly will play an increasingly dominant role as the sample size is decreased. One manifestation of the Coulomb repulsion is the phenomenon of Coulomb blockade in tunnel junctions and this topic is reviewed in chapter 6. Again, this topic has seen exciting new developments recently which are too new to be reviewed in this book. An example is the observation of periodic conductance peaks as a function of gate voltage (Meirav et al. 1990). It is clear that Coulomb repulsion is playing a dominant role and the study of the interplay between

Coulomb repulsion and quantum interference will surely occupy many researchers in the years to come.

January, 1991

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References

Altshuler, B.L., 1985, Pis'ma Zh. Eksp. & Teor. Fiz. **51**, 530 [JETP Lett. **41**, 648].
Altshuler, B.L., and B.I. Shklovskii, 1986, Zh. Eksp. & Teor. Fiz. **91**, 220 [Sov. Phys.–JETP **64**, 127].
Altshuler B.L., and P.A. Lee, 1988, Phys. Today (December) p. 2.
Büttiker, M., 1988, IBM J. Res. Dev. **32**, 317.
Dyson, F.J., 1962, J. Math. Phys. **3**, 140, 157, 166.
Dyson, F.J., and M.L. Mehta, 1963, J. Math. Phys. **4**, 701.
Fowler, A.B., A. Hartstein and R.A. Webb, 1982, Phys. Rev. Lett. **48**, 196.
Imry, Y., 1986, in: Directions in Condensed Matter Physics, eds. G. Grinstein and G. Mazenko (World Scientific, Singapore) p. 101.
Landauer, R., 1970, Philos. Mag. **21**, 863.
Lee, P.A., and T.V. Ramakrishnan, 1985, Rev. Mod. Phys. **57**, 287.
Lee, P.A., and A.D. Stone, 1985, Phys. Rev. Lett. **55**, 1623.
Meirav, U., M. Kastner and S.J. Wind, 1990, Phys. Rev. Lett. **65**, 771.
Thouless, D.J., 1974, Phys. Rep. **13**, 93.
Umbach, C.P., S. Washburn, R.B. Laibowitz and R.A. Webb, 1984, Phys. Rev. B **30**, 4048.
van Wees, B.J., H. van Houten, C.W.J. Beenakker, J.G. Williamson, L.P. Kouwenhoven, D. van der Marel and C.T. Foxon, 1988, Phys. Rev. Lett. **60**, 848.
Wharam, D.A., T.J. Thornton, R. Newbury, M. Pepper, H. Ajmed, J.E.F. Frost, D.G. Hasko, D.C. Peacock, D.A. Ritchie and G.A.C. Jones, 1988, J. Phys. C. **21**, L209.
Wigner, E.P., 1951, Ann. Math. **53**, 36.
Wigner, E.P., 1955, Ann. Math. **62**, 548.
Wigner, E.P., 1958, Ann. Math. **67**, 325.

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