

***Techniques of  
Differentiation  
and Integration:***

***A Program for Self-instruction***

***Herman Meyer***

***Robert V. Mendenhall***



# ***Techniques of Differentiation and Integration:***

***A Program for Self-instruction***

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***Techniques of Differentiation and Integration:***  
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## **Preface**

Because programmed materials have not been widely used in teaching college mathematics, we would like to discuss just what this programmed unit is and what it is not.

First, it is not a substitute for a textbook in the Calculus; it makes no claim to cover the full range of subject matter normally taught in a first course in the Calculus. In particular, it does not deal at all with the development of the theory or its motivations or applications. It does not provide the full range of educational experiences which form the objectives of any course in mathematics; it makes no attempt to develop the student's imagination or originality. Nor does it purport to teach the material it does include with greater efficiency than that attained by a competent teacher and textbook.

Instead, this program is intended as a supplement to a teacher and textbook. Its sole purpose is to remove the preoccupation with the techniques of differentiation and integration from the *classroom* and thus significantly increase the classroom time available for the teaching of the theory and applications of the Calculus. It deals only with that part of a calculus course which is the almost automatic application of rules and concepts, and it provides an avenue for the drill necessary to attain proficiency in these more or less routine techniques.

Since this program is not a calculus textbook, definitions sometimes appear in the body of the material without being specifically identified. The function concept is presumed but not in its general form; domains for functions are not exhibited nor are theorems stated precisely. No hint appears of the existence of functions having no derivatives or antiderivatives, nor of the dependence of some integration techniques on the domain of the function. These iniquities were accepted in order to place sole emphasis here on techniques. We trust that the instructor will exploit the class time gained by this program to present precise formulations of the concepts of the Calculus with increased attention to its intuitive foundations and rich applications.

No standard programming style has been followed. The program is linear (nonbranching) and accounts for individual differences only in that the more talented students can proceed at a faster pace. New techniques are first introduced in considerable detail, with the student guided to solutions. Then he is released for further practice and drill. The format, which places answers on the back of the question page, was selected in order to facilitate the mechanics of proceeding through the program. But it introduces an unfortunate temptation for the student which he must continually resist; even a quick glance at an answer before he adequately attacks a problem

can vitiate both the value of the problem and the development of his own resources.

The existence of programmed material does not release the teacher from pacing the course of study by assignments and examinations. For this reason a detailed table of contents is included. Since the material is relatively self-contained and independent of an accompanying text, it can be initiated at the very beginning of the course and can thus act to motivate the more precise and systematic development of the Calculus.

Nor does the existence of programmed material release the student from responsibility for his own learning. Each problem deserves pencil, paper, and effort to attain a solution *before* the answer is sought. And each error deserves thoughtful analysis and correction. Otherwise, it is quite possible to "proceed" through the program and learn very little indeed.

A short table of integrals is included, and frequent references to this table appear throughout. These are indicated by the integral symbol  $\int$  followed by a number. Thus  $\int 17$  indicates a reference to the integral formula numbered 17. It is assumed, particularly for Chap. 1, that the student has access to mathematical tables giving values of the sine and cosine functions (angles measured in radians), of the exponential and logarithmic (napierian) functions, and of the square, cube, and reciprocal functions.

*Herman Meyer*  
*Robert V. Mendenhall*

## ***To The Student***

This book has been designed and written to assist you to learn the techniques of integration and differentiation. To be as efficient and useful as possible, it employs a self-instructional technique called programming. Since you may be unfamiliar with this type of presentation, the following notes on programming, its use in this book, and the study suggestions may be of value.

### ***Programmed Books***

Essentially, a programmed book is the same as a conventional text in its overall purpose: it attempts to teach certain concepts and to drill in various procedures. But a programmed book is more explicit than most texts in the presentation of ideas, concepts, and manipulations to be learned. By breaking each new idea or skill into a series of logical and carefully graded steps (called "frames") and requiring that you actively respond to each frame, programming increases your participation in learning beyond that normally possible from reading an ordinary text. Actively responding to each frame means that you are not allowed to passively absorb what you read; each frame requires you to do something. The frames are a carefully constructed sequence of items which lead from the simple to the complex and which progressively require that you take more and more responsibility for the operations you are asked to do. You must work a problem and then write the answer to it, decide between alternatives, or otherwise engage in thinking out what the book teaches. The quick knowledge of whether you were right or not in answering a frame keeps you apprised of exactly how much you have learned.

### ***How To Use This Book***

Since the organization of this programmed book is unusual, leaf through the pages quickly and note the following features:

#### *The use of right- and left-hand pages*

Notice that in the main part of the book, the right-hand page contains the *frames* of the program. Numbered by chapters, they follow a normal sequence through each chapter. Notice also that the back of each right-hand page contains answers (and discussion) to *just* those frames appearing on its front. The answers are keyed to their frames by identical numbers.

#### *Use of color*

As a help in comparing answers, the answer page has been printed in color so that the visual contrast between frames and answers will keep their functions separate in your mind.

### *Information panels*

These panels introduce discussions of related materials and ideas into the flow of frames and, like the frames, always appear on the right-hand page. As you will note, the panels are preceded and followed by colored rules, thus separating them from the other parts of the page.

### **Study Suggestions**

The following suggestions should be helpful to you in learning how best to use this book:

1. Read and work the problem presented in each frame (right-hand page only).
2. After having arrived at your *own* answer, write it out in complete form and turn the page to consult the answer and discussion (left-hand page only). If your answer does not agree with the one given, try to find out why, correct your mistake, and then go ahead.
3. An obvious final point: do not consult the answers *before* working the problem and do not look ahead at future answers. Learning is an activity; you must *do* something *yourself*. If you are tempted to browse through the answers given on the left-hand page, use the marker provided in the book to cover the answers below the one you are consulting.

The following graph figures are for your use while working certain sections of the program. They have been grouped in the front of the book for your convenience in referring to them.

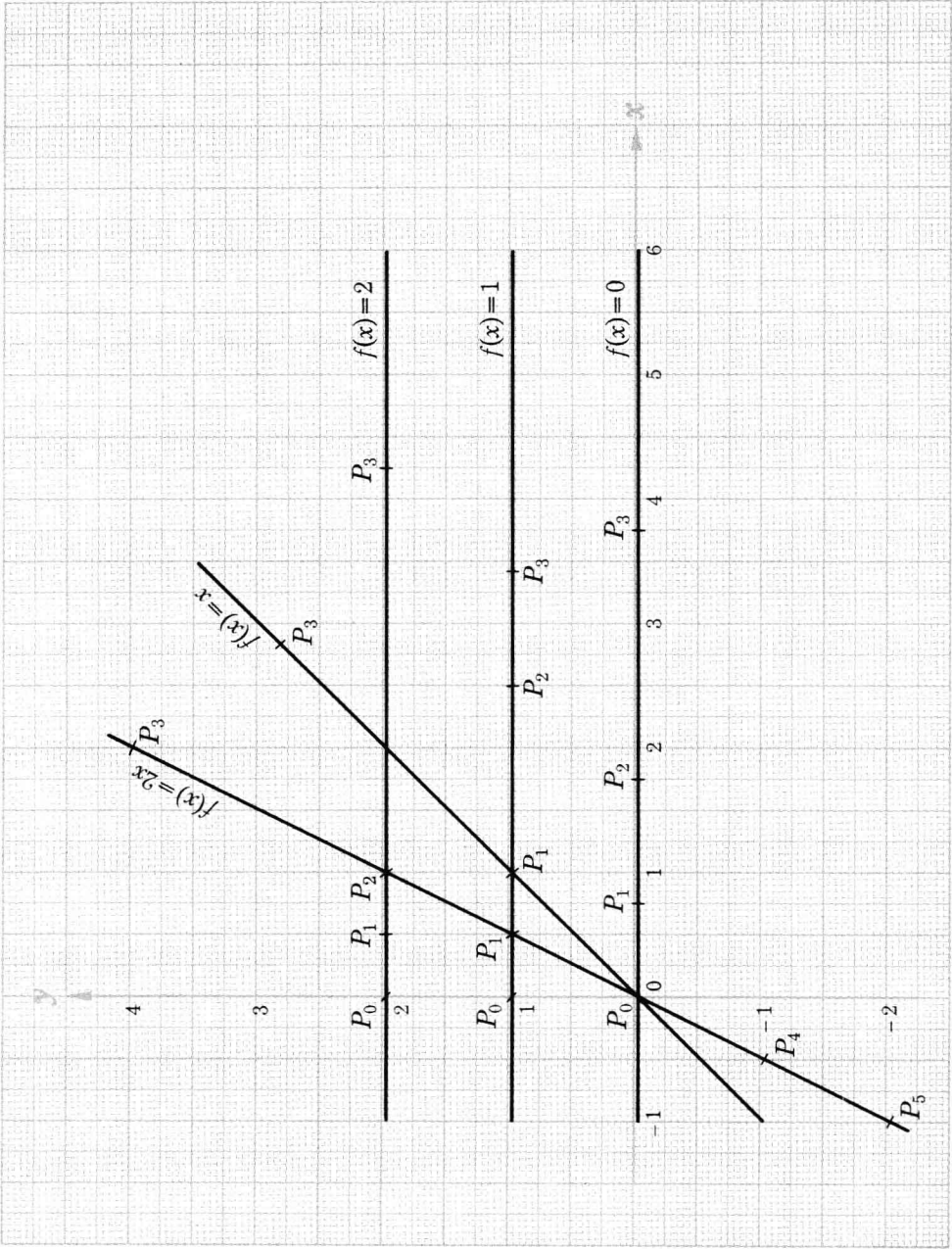


Figure I

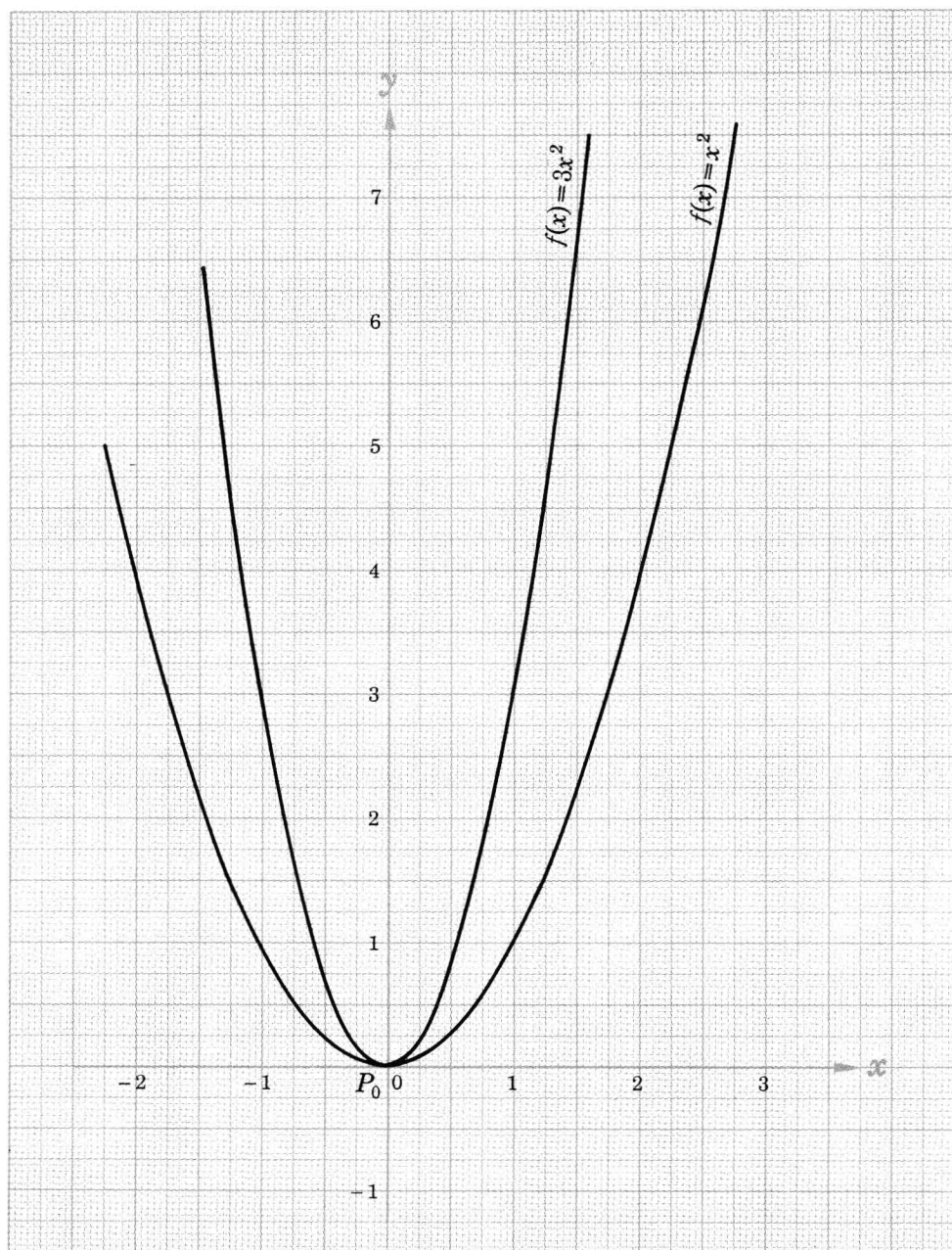


Figure II

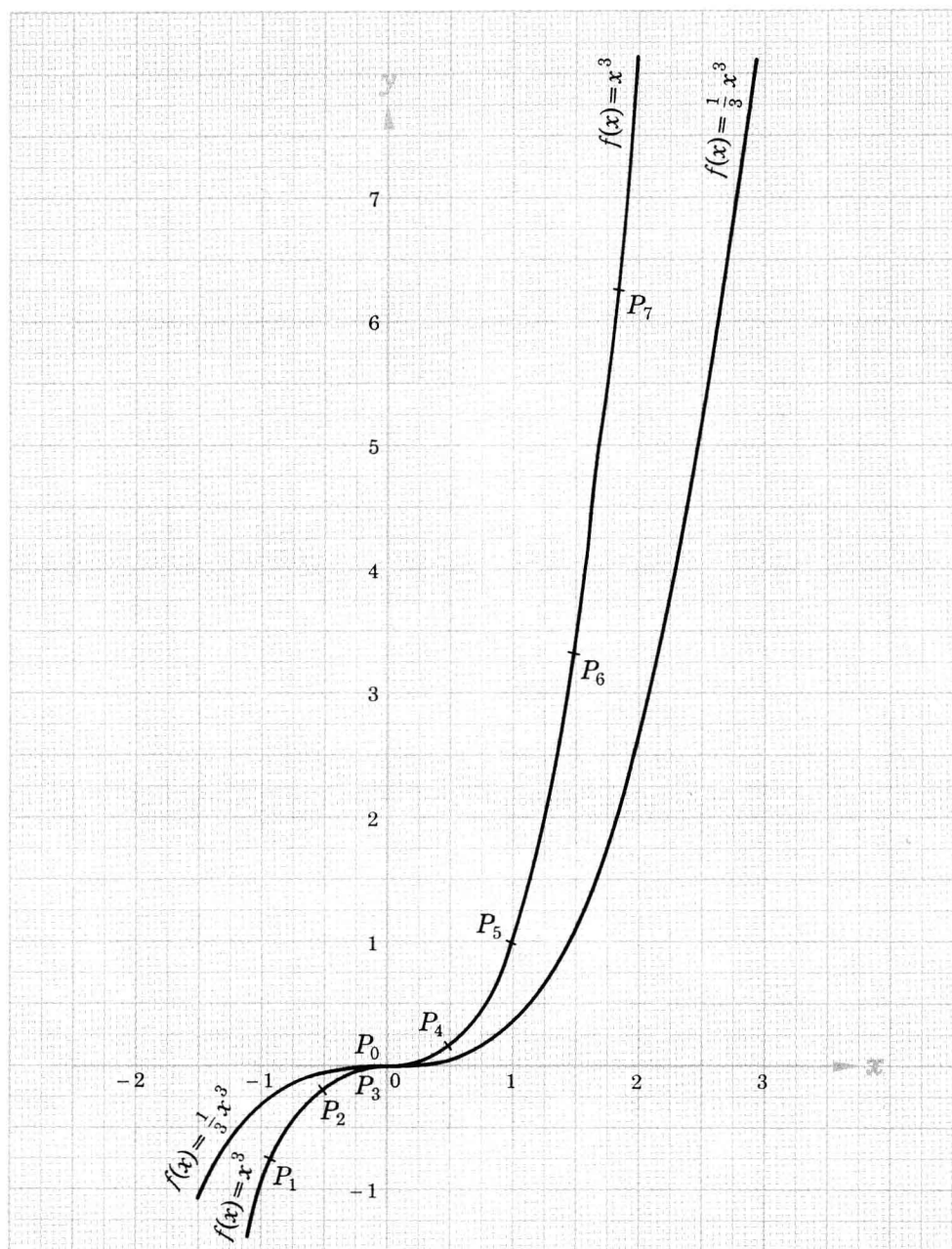


Figure III

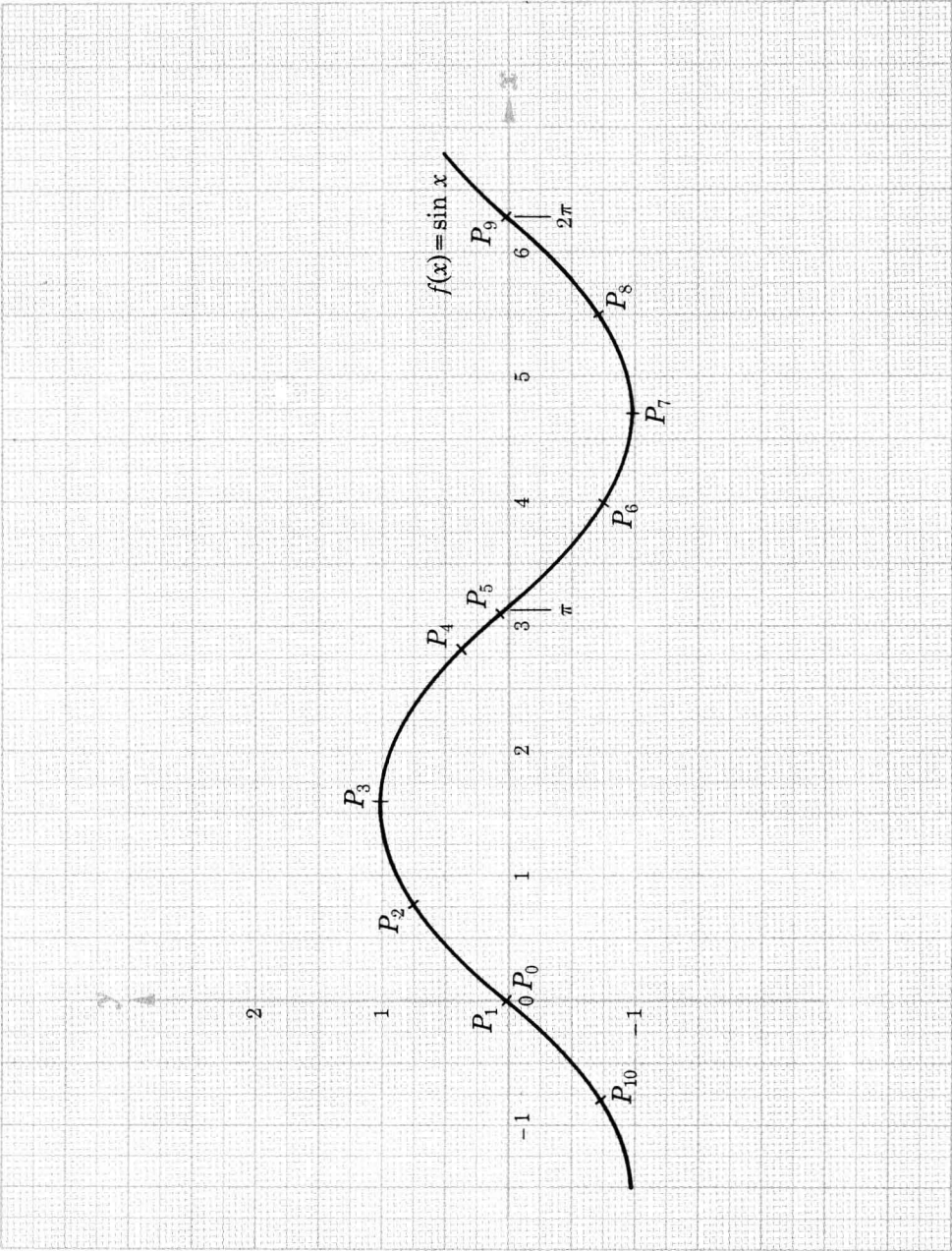


Figure IV

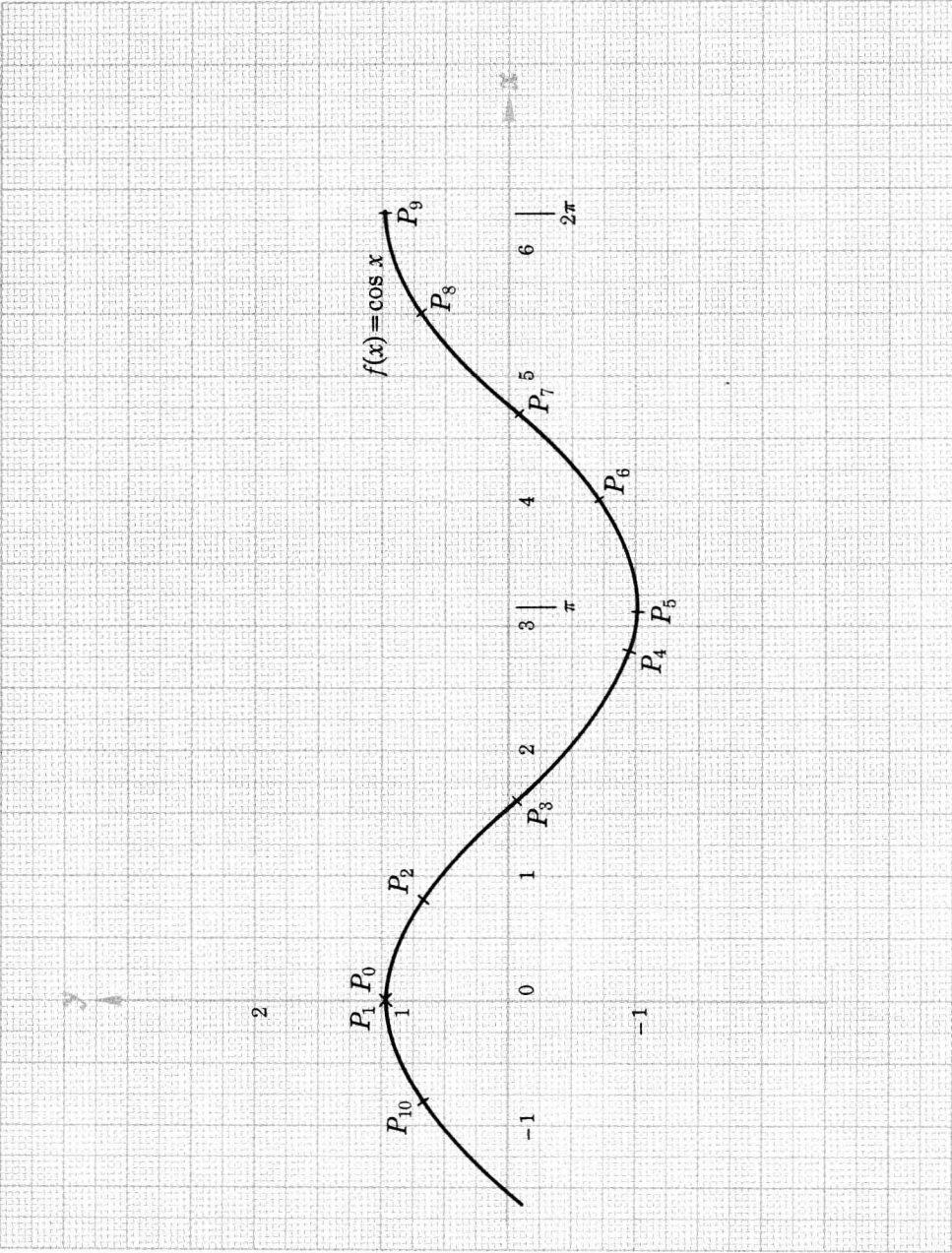


Figure V

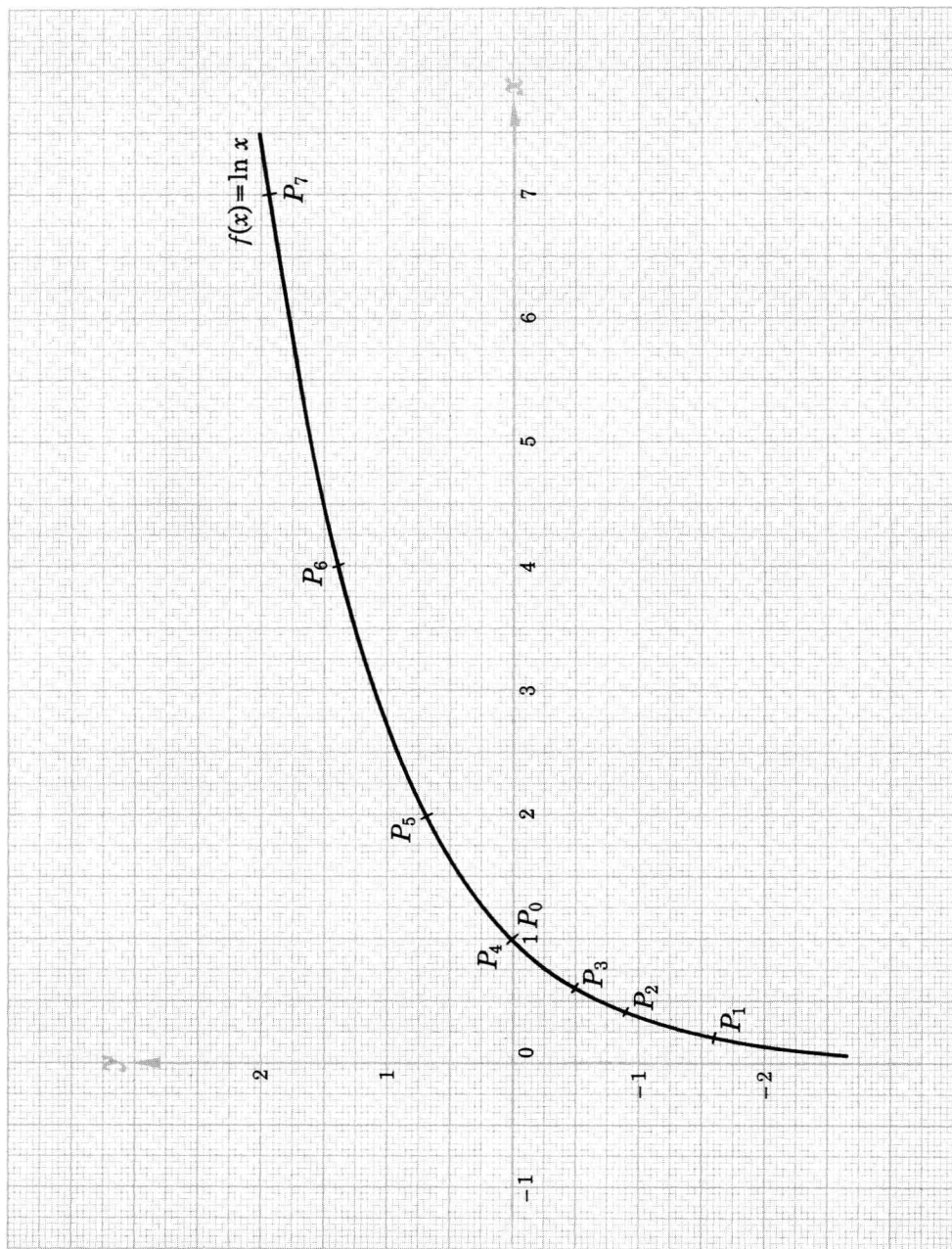


Figure VI

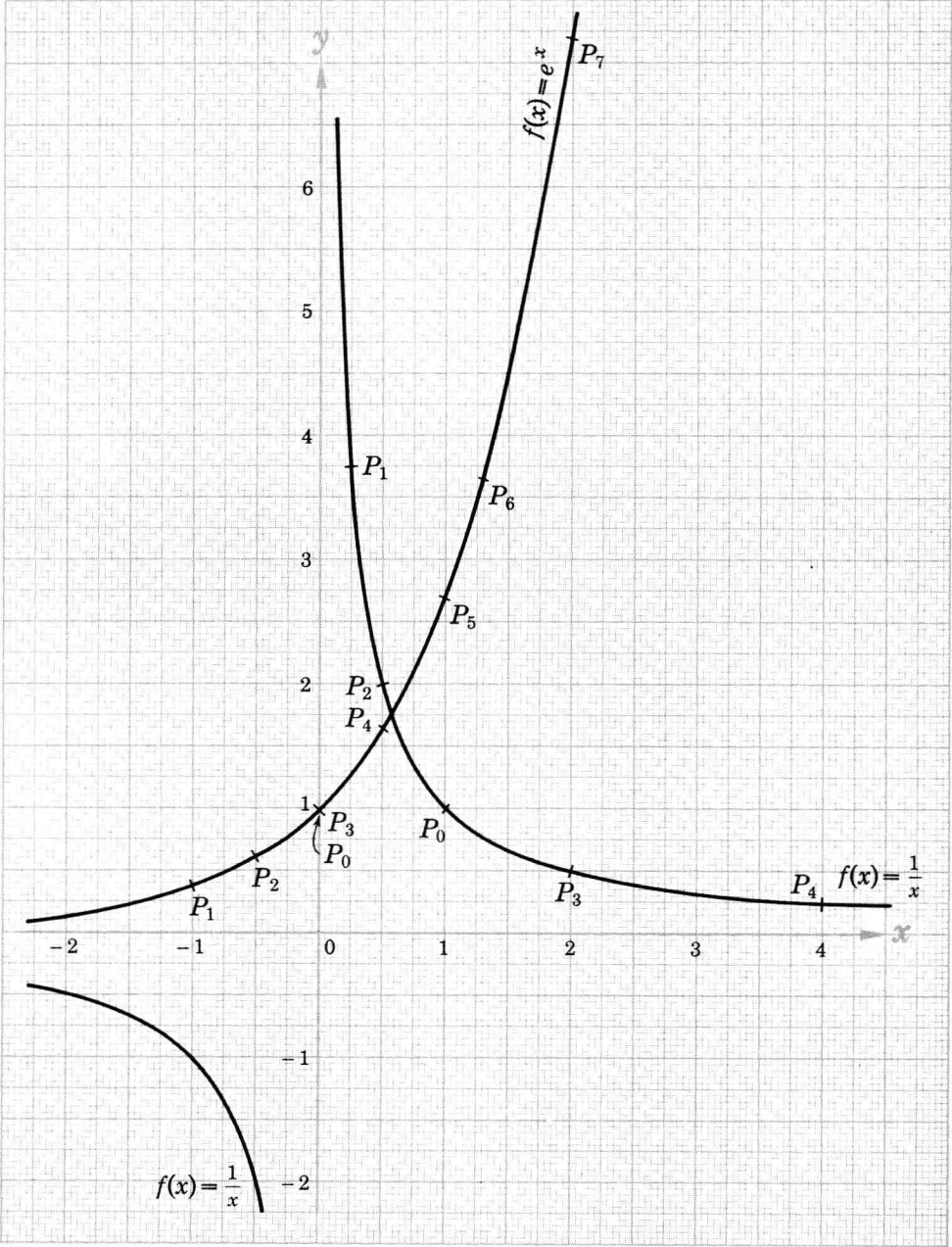


Figure VII

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