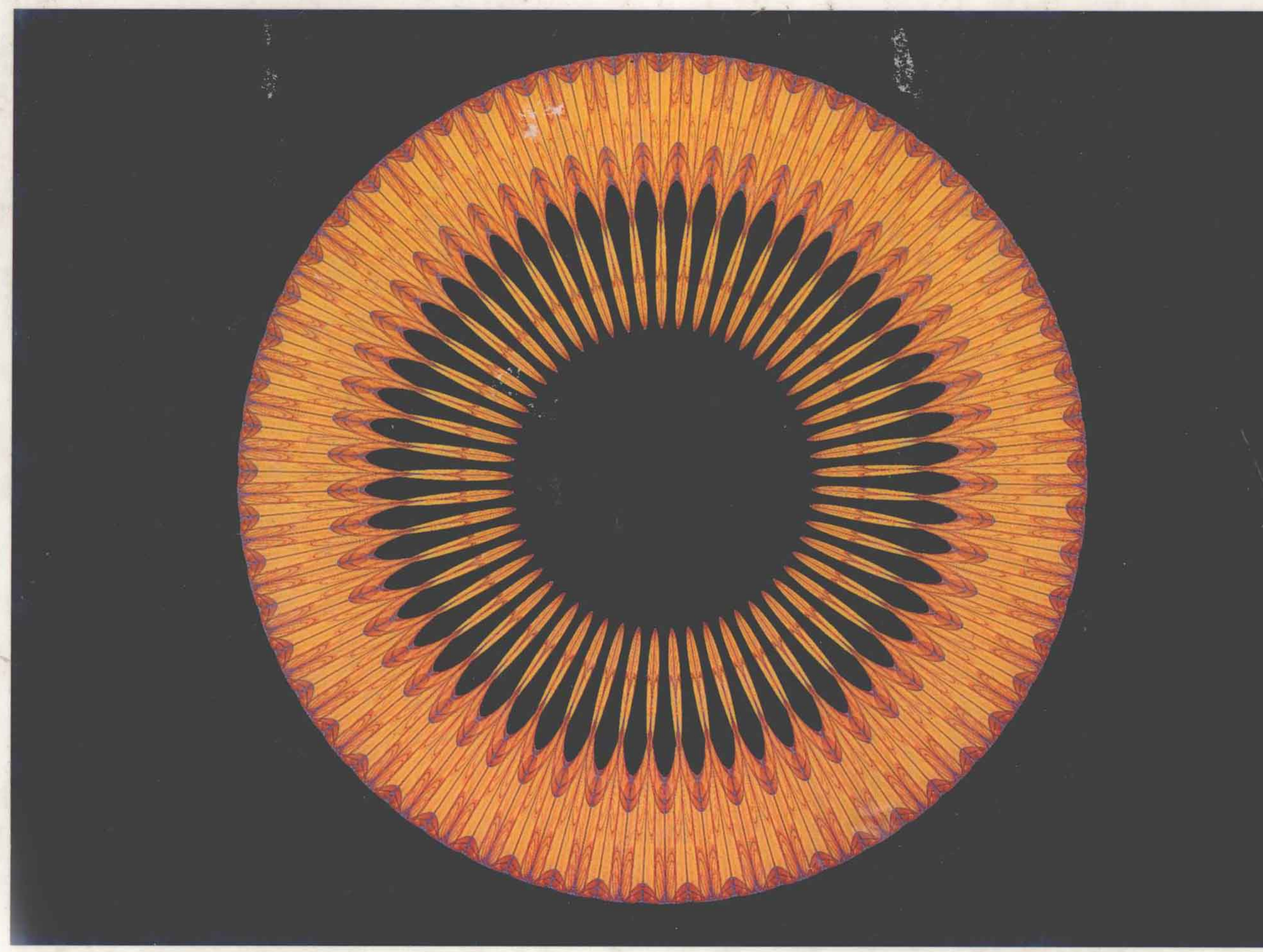
PRECALCULUS

GRAPHING AND DATA ANALYSIS



MICHAEL SULLIVAN • MICHAEL SULLIVAN III

Precalculus

Graphing and Data Analysis

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Library of Congress Cataloging-in-Publication Data
Sullivan, Michael
Precalculus: graphing and data analysis/
Michael Sullivan, Michael Sullivan, III.
p. cm.
Includes index.
ISBN 0-13-778499-6
1. Functions. I. Sullivan, Michael. II. Title.
QA331.3.S928 1998
510—dc21 97-23427
CIP

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Cover Design: *Jeanette Jacobs*
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Manufacturing Buyer: *Alan Fischer*
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Cover Photo: SUNFLOWER in *Symmetry and Chaos,* by Michael Field and Martin Golubitsky, OUP, 1992.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-778499-6

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia, Pty. Limited, *Sydney*
Prentice-Hall Canada Inc. *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
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Simon & Schuster Asia Pte. Ltd., *Singapore*
Editoria Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

For Our Students. . .
Past and Present

Why We Wrote This Book

Times continue to change! In 1987, *Precalculus* was written, a traditional text that developed all the skills a student would need to succeed in college level mathematics courses. This text, now in its 4th edition, remains traditional in its approach. As technology developed, various philosophies regarding the use of incorporating graphing utilities into the curriculum emerged. These philosophies, as they relate to Precalculus, lead to different views regarding the use of technology in the classroom. In 1996, we wrote *Precalculus Enhanced with Graphing Utilities* in response to the call for a text to fully incorporate technology while maintaining strict mathematical content. Through the use of technology, the text allows students to recognize patterns, visualize, explore, and foreshadow concepts from areas such as Calculus. This book, *Precalculus: Graphing and Data Analysis,* was written to address the point of view that technology should not only be used to graph functions and solve equations, but also should be used to analyze data, draw scatter diagrams, and find curves of best fit. Thus, this text incorporates concepts never before discussed in a traditional Precalculus course while still maintaining strict mathematical content. Students are not simply "pushing buttons" on their calculators, but must answer thoughtful questions that force them to utilize mathematics.

The standards developed by MAA, AMATYC, and NCTM encourage modeling and connecting mathematics to various disciplines. This text introduces mathematical theory through the use of models developed in other disciplines. The text also expects students to develop their own models either from data or situations and proceed to analyze these models mathematically.

To this end, we begin with a discussion of bivariate data, representing it in scatter diagrams and determining whether the relationship between the variables is linear or nonlinear. Using a graphing utility, LINear REGression techniques are used to find and interpret curves of best fit for linearly related data. As the mathematics evolves, we use a graphing utility to find curves of best fit for quadratic, power, cubic, exponential, logarithmic, logistic, and sinusoidal curves of best fit. Finally, the ideas of the limit and the derivative of a function are introduced and explored. In each instance the data is used not only to find curves of best fit, but also to analyze the functions found using concepts such as local minima and maxima. By comparing the results of their analysis to the actual data, students are able to judge the reasonableness of their answer.

The data itself is truly real, drawn from sources such as the Statistical Abstract of the United States, the U.S. Census Bureau, the U.S. Department of Justice, the Bureau of Labor Statistics, the New York and NASDAQ Stock Exchanges, and so on. Each chapter begins with a popular issue, such as domestic and global use of oil, asking students to analyze and interpret data from web site locations. This data is dynamic, changing on a regular basis. The data is widely based, drawn from disciplines such as physics, biology, chemistry, economics, finance, psychology, and statistics.

Notwithstanding the emphasis on data analysis, we fully utilize the graphers ability to promote visualization, exploration, and foreshadowing of concepts. The technology is interwoven with the mathematics. Doing mathematics by hand using traditional methods and doing mathematics using technology are often combined, such as using the domain of a function to set a viewing window. Many examples are solved both graphically and algebraically. We encourage students to distinguish between solving a problem using the full power of the technology vs. recognizing when the technology is limited or the simple nature of the problem makes a solution by hand the better choice.

About This Book

Content The content of this book contains the topics found in a traditional Precalculus text, except possibly for the chapter devoted to the limit and the derivative of a function. In addition, it contains topics unique to a technology approach: sections on data analysis and curves of best fit appear in chapters dealing with linear functions (LINear REGression), quadratic functions (QUADratic REGression), power functions (POWer REGression), polynomial functions (CUBic REGression), logarithmic and exponential functions (LOGarithmic REGression, EXPonential REGression, LOGISTIC REGression), and trigonometric functions (SINusoidal REGression). Also, many examples and exercises found in this text are ones that cannot be handled using traditional methods.

Organization In recognition of the fact that some students may not be fully prepared for this course, a specially designed Appendix has been included to provide necessary review. The Appendix is referenced at the beginning of the chapter and at the appropriate place within the chapter.

Chapter 1 begins with a discussion of rectangular coordinates, and their use in graphing data in a scatter diagram. Graphing is done early in this chapter (1.2), with emphasis on the graphs of certain key equations. A section on modelling linearly related data appears here. Also included is a discussion of solving equations and inequalities, utilizing both an algebraic and graphing approach. The solution of polynomial and rational inequalities is postponed to Chapter 3, so the power of graphing polynomial and rational functions can be used.

Chapter 2 develops the important concept of a function from a discrete and a continuous point of view, emphasizing tables, graphs, and properties and including discussions on average rate of change, local maxima and minima, and increasing and decreasing functions. The last section focuses on the construction of functions in applications.

Chapter 3 discusses graphs and properties of polynomial and rational functions. Here, data resulting in scatter diagrams lead to curves of best fit that are quadratic, power, or cubic are analyzed. Complex numbers and quadratic equations with a negative discriminant provide motivation for the Fundamental Theorem of Algebra. Many exercises here ask students to draw scatter diagrams, find the appropriate curve of best fit, and analyze the resulting function using techniques introduced in the chapter.

Chapter 4, Exponential and Logarithmic Functions, places emphasis on graphs and properties, utilizing data with scatter diagrams that lead to curves of best fit that are exponential, logarithmic, or logistic.

Chapter 5, Trigonometric Functions, uses the unit circle approach to define the six trigonometric functions and develop properties of these functions. The graphs of the trigonometric functions are then discussed. The chapter concludes with the inverse trigonometric functions.

Chapter 6, Analytic Trigonometry, deals with trigonometric identities and trigonometric equations. A graphing utility is used to solve trigonometric equations for which no algebraic or trigonometric method is available.

Chapter 7, Applications of Trigonometric Functions, deals both with early applications of trigonometry (requiring solving right or oblique triangles) and with modern applications, such as simple harmonic motion, damped motion, and sinusoidal curve fitting.

Chapter 8, Polar Coordinates; Vectors, introduces polar coordinates, with an emphasis on graphing. Included is a discussion of Demoivre's Theorem. Independent of the preceding topics, sections on vectors in the plane, vectors in space, and the dot product are also provided.

Chapter 9, Analytic Geometry, discusses conics, including rotation of axes and polar forms. Parametric equations are also discussed, including an example of motion simulation.

Chapter 10, Systems of Equations and Inequalities, discusses both graphing and algebraic approaches. Many examples and exercises here can only be worked using technology.

Chapter 11, Sequences; Induction; Counting; Probability, is mostly traditional since the use of technology usually does not lead to more efficient methods.

Chapter 12, A Preview of Calculus: the Limit and the Derivative of a Function, introduces limits first through the use of Tables and then by graphing. Algebraic techniques for calculating limits follow. One-sided limits and continuity preceed the discussion of the derivative. Applications to velocity of falling objects and instantaneous rates of change are provided.

Instructor Supplements

Instructor Resource Manual

written by Michael Sullivan, Michael Sullivan III and Katy Murphy

ISBN: 0-13-685389-7

Contains complete step-by-step worked out solutions to all the even numbered exercises in the textbook. Also included are strategies for using the "Mission Possible" collaborative learning projects found in each chapter.

Written Test Item File

ISBN: 0-13-685355-2

Features questions that are found within the computerized test bank in a format that can be used for selecting specific problems for testing.

TestPro Computerized Test Generator

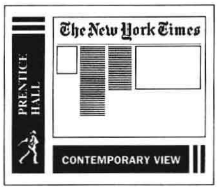
MAC ISBN: 0-13-685272-6

IBM ISBN: 0-13-685264-5

Allows instructors to generate tests from algorithms keyed to the text by chapter, section, and learning objective. Instructors select from thousands of test questions and hundreds of algorithms which generate different but equivalent questions. A user-friendly expression-building toolbar, editing and graphing capabilities are included. Customization toolbars allow for customized headers and layout options which provide instructors with the ability to add or delete workspace or add columns to conserve paper.

Tutorial Videos
ISBN: 0-13-685116-9

A new tutorial videotape series, created exclusively to accompany this text, include 15 minute segments for each section of the text. Each segment uses both traditional and graphical ways of solving mathematical problems. These videos provide an alternative process which can add to your students' success in this course. Included with each set of tapes is a permission letter to duplicate the tapes for your mathematics lab or library.



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A free newspaper supplement from Prentice Hall and the New York Times which includes interesting and relevant articles on mathematics in the world around us. Great for getting students to talk and write about mathematics. This supplement is updated each fall. To request free copies for all of your students, contact your Prentice Hall representative.

Prentice Hall Companion Website
Located at *www.prenhall.com/sullivan* this website offers you and your students additional information, projects and exercises to enrich the learning experience using this text. The Internet "Excursions" found in the beginning of each chapter have duplicate pages on the website with the links to the appropriate data and information-gathering sites described in the projects.

Acknowledgments

Textbooks are written by authors, but evolve from an idea into final form through the efforts of many people. Special thanks to Don Dellen, who first suggested this book and the other books in this series. Don's extensive contributions to publishing and mathematics are well known; we will all miss him dearly.

There are many people we would like to thank for their input, encouragement, patience and support. They have our deepest thanks and appreciation. We apologize for any omissions . . .

James Africh, *College of DuPage*
Steve Agronsky, *Cal Poly State University*
Dave Anderson, *South Suburban College*
Joby Milo Anthony, *University of Central Florida*
James E. Arnold, *University of Wisconsin-Milwaukee*
Agnes Azzolino, *Middlesex County College*
Wilson P. Banks, *Illinois State University*
Dale R. Bedgood, *East Texas State University*
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Mary Voxman, *University of Idaho*
Darlene Whitkenack, *Northern Illinois University*
Chris Wilson, *West Virginia University*
Carlton Woods, *Auburn University*
George Zazi, *Chicago State University*

Recognition and thanks are due particularly to the following individuals for their valuable assistance in the preparation of this edition: Jerome Grant for his support and commitment; Sally Denlow, for her genuine interest and insightful direction; Bob Walters for his organizational skill as production supervisor; Patrice Lumumba Jones for his innovative marketing efforts; Ray Mullaney for his specific editorial comments; the entire Prentice Hall sales staff for their confidence; and to Katy Murphy for checking the answers to all the exercises.

Michael Sullivan
Michael Sullivan, III

PREFACE TO THE STUDENT

As you begin your study of Precalculus, you might feel overwhelmed by the number of theorems, definitions, procedures and equations that confront you. You may even wonder whether you can learn all this material in a single course. For many of you, this may be your last mathematics course, while for others, just the first in a series of many. Don't worry—either way, this text was written with you in mind.

This text was designed to help you—the student, master the terminology and basic concepts of Precalculus. These aims have helped to shape every aspect of the book. Many learning aids are built into the format of the text to make your study of this material easier and more rewarding. This book is meant to be a "machine for learning," one that can help you to focus your efforts and get the most from the time and energy you invest.

This book requires that you have access to a graphing utility: a graphing calculator or a computer software package that has a graphing component. Be sure you have some familiarity with the device you are using before the course begins.

Here are some hints we give our students at the beginning of the course:

1. Take advantage of the feature PREPARING FOR THIS CHAPTER. At the beginning of each chapter, we have prepared a list of topics to review. Be sure to take the time to do this. It will help you proceed quicker and more confidently through the chapter.
2. Read the material in the book before the lecture. Knowing what to expect and what is in the book, you can take fewer notes and spend more time listening and understanding the lecture.
3. After each lecture, rewrite your notes as you re-read the book, jotting down any additional facts that seem helpful. Be sure to do the Now Work Problem x as you proceed through a section. After completing a section, be sure to do the assigned problems. Answers to the Odd ones are in the back of the book.
4. If you are confused about something, visit your instructor during office hours immediately, before you fall behind. Bring your attempted solutions to problems with you to show your instructor where you are having trouble.
5. To prepare for an exam, review your notes. Then proceed through the Chapter Review. It contains a capsule summary of all the important material of the chapter. If you are uncertain of any concept, go back into the chapter and study it further. Be sure to do the Review Exercises for practice.

Remember the two "golden rules" of Precalculus:

1. DON'T GET BEHIND! The course moves too fast, and it's hard to catch up.
2. WORK LOTS OF PROBLEMS. Everyone needs to practice, and problems show where you need more work. If you can't solve the homework problems without help, you won't be able to do them on exams.

We encourage you to examine the following overview for some hints on how to use this text.

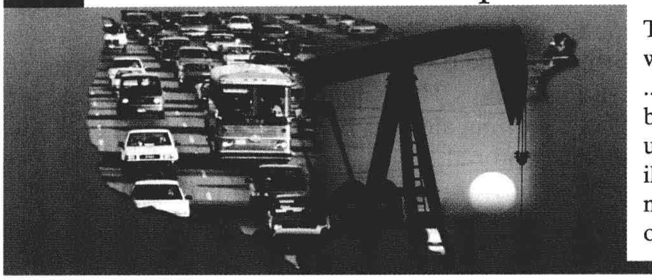
Best Wishes!

Michael Sullivan
Michael Sullivan, III

OVERVIEW

CHAPTER 2

Functions and Their Graphs



The list of concepts for review will help you in two major ways. ... First, it allows you to review basic concepts immeditely before using them in context. Second, it illustrates the natural building of mathematical concepts throughout the course.

Imagine yourself as an expert for the EPA and sitting in Congressional policy meetings having to explain the importance of balancing energy resources with environmental protection. On the following page is the Internet Excursion placing you in exactly that position and asking the tough questions a member of congress might ask. Use the Sullivan website at:

www.prenhall.com/sullivan

to help link you to the Internet resources you will need to answer the questions asked.

PREPARING FOR THIS CHAPTER

Before getting started on this chapter, review the following concepts;

Topics from Algebra and Geometry *(Appendix, Section 1)*

Graphs of Certain Equations (*Example 2, p. 20;* Example 7, p. 26,)

Figure 35, p. 27; Example 9, p. 27

Tests for Symmetry of an Equation (p. 25)

Procedure for Finding Intercepts of an Equation (p. 23)

Linear Curve Fitting *(Chapter 1, Section 1.4)*

OUTLINE

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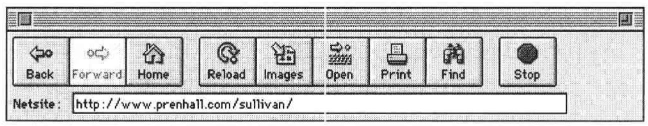

HOW LONG WILL THE OIL LAST?

The global supply of oil and other sources of energy is more than adequate to meet present needs, but most of this supply is outside of the United States. Currently, oil supplies about 40 percent of the world's energy, with the United States the biggest consumer. Since oil is a nonrenewable energy resource, plans must be made for the eventuality of diminishing supply.

Suppose that you were a consultant for the EPA and members of congress asked you to sit on a Energy Policy Steering committee as an expert analyst. You must convince the committee of the importance of environmental concerns in planning the global energy system. You are very aware of the fact that present modes of energy use and production threaten serious environmental deterioration. Your plan is to create a set of what-if functions that will allow your committee to model many possible scenarios for their policy guidelines.

1. $E(t)$, the first function that you create, will allow you to model United States oil consumption over a period of time. Make a scatterplot using the EPA Data on United States energy consumption, per capita from 1950–1990. Since oil provides 40 percent of the energy consumed, you adjust your figures to get oil consumption per capita. Use the LINear REGression tool on your graphing utility to find the linear function $E(t)$ of best fit for modeling the data. Does this seem like a good model?
2. The second function, $P(t)$, models population. From the United States Census Data make a scatterplot, and then use a QUADratic REGression to model the data. Next, let $O(t) = E(t) \cdot P(t)$. What does $O(t)$ model? Graph $O(t)$. Compare to the actual figures from BP Petroleum.
3. Using the function $O(t)$, estimate the area under the graph for the years 1997 until 2000. To do this make a trapezoid, and find its area. Now estimate the areas under $O(t)$ using 10-year intervals. For what year does the total of all the trapezoid areas equal the remaining United States oil reserve?
4. Could this sort of analysis be used to predict when the world's supply of oil will run out? Can your committee think of any methods that might improve your predictions?

Each chapter begins by placing you in an intriguing and relevant situation where you will need to collect information from the Internet, analyze data according to material available within the chapter, and answer thought-provoking questions. All the appropriate links can be found on the "live" pages at: http://www.prenhall.com/sullivan

Perhaps the most central idea in mathematics is the notion of a *function.* This important chapter deals with what a function is, how to graph functions, how to perform operations on functions, and how functions are used in applications.

The word *function* apparently was introduced by René Descartes in 1637. For him, a function simply meant any positive integral power of a variable *x*. Gottfried Wilheim von Leibniz (1646–1716), who always emphasized the geometric side of mathematics, used the word function to denote any quantity associated with a curve, such as the coordinates of a point on the curve. Leonhard Euler (1707–1783) employed the word to mean any equation or formula involving variables and constants. His idea of a function is similar to the one most often used today in courses that precede calculus. Later, the use of functions in investigating heat flow equations led to a very broad definition, due to Lejeune Dirichlet (1805–1859), which describes a function as a rule or correspondence between two sets. It is his definition that we use here.

2.1 FUNCTIONS

1 Determine Whether a Relation Represents a Function
2 Identify the Graph of a Function
3 Find the Value of a Function
4 Find the Domain of a Function
5 Obtain Information from/about the Graph of a Function

In Section 1.1, we said a **relation** is a correspondence between two variables, say x and y, and can be written as a set of ordered pairs (x, y). When relations are written as ordered pairs (x, y), we say x is related to y. Often, we are interested in specifying the type of relation (such as an equation) that might exist between the two variables. For example, when variables appear to be linearly related (based on the scatter diagram), we find a line of best fit. In Section 1.4, we found that fertilizer and crop yield appeared to be linearly related and the line of best fit was $y = 0.717x + 4.786$, where x represents the amount of fertilizer and y represents the crop yield. Thus, if we

Begin each section by reading the learning objectives. These objectives will help you organize your studies and prepare for class.

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EXAMPLE 1 Vertical Shift Down

Use the graph of $f(x) = x^2$ to obtain the graph of $h(x) = x^2 - 4$.

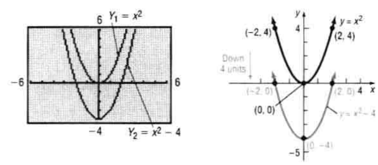
Solution Table 5 lists some points on the graphs of $f = Y_1$ and $h = Y_2$. The graph of h is identical to that of f, except that it is shifted down 4 units. See Figure 38.

TABLE 5

X	Y1	Y2
-3	9	5
-2	4	0
-1	1	-3
0	0	-4
1	1	-3
2	4	0
3	9	5

Y2=X²-4

FIGURE 38



Increase your understanding, visualize, discover, explore, and solve problems using a graphing utility.

Horizontal Shifts

EXPLORATION On the same screen, graph each of the following functions:

$Y_1 = x^2$
$Y_2 = (x - 1)^2$
$Y_3 = (x - 3)^2$
$Y_4 = (x + 2)^2$

What do you observe?

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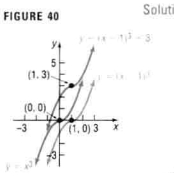
Now work Problem 31.

Vertical and horizontal shifts are sometimes combined.

EXAMPLE 2 Combining Vertical and Horizontal Shifts

Graph the function $f(x) = (x - 1)^3 + 3$.

FIGURE 40



Solution We graph f in steps. First, we note that the rule for f is basically a cube function. Thus, we begin with the graph of $y = x^3$, shown in blue in Figure 40. Next, to get the graph of $y = (x - 1)^3$, we shift the graph of $y = x^3$ horizontally 1 unit to the right. See the graph shown in red in Figure 40. Finally, to get the graph of $y = (x - 1)^3 + 3$, we shift the graph of $y = (x - 1)^3$ vertically up 3 units. See the graph shown in green in Figure 40. Note the points that have been plotted on each graph. Using key points such as these can be helpful in keeping track of just what is taking place.

In Example 2, if the vertical shift had been done first, followed by the horizontal shift, the final graph would have been the same. (Try it for yourself.)

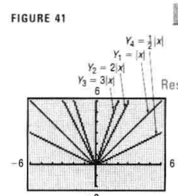
Now work Problem 43.

Compressions and Stretches

EXPLORATION On the same screen, graph each of the following functions:

$Y_1 = |x|$
$Y_2 = 2|x|$
$Y_3 = 3|x|$
$Y_4 = \frac{1}{2}|x|$

FIGURE 41



Result Figure 41 illustrates the graphs. You should have observed the following pattern. The graphs of $Y_2 = 2|x|$ and $Y_3 = 3|x|$ can be obtained from the graph of $Y_1 = |x|$ by multiplying each y-coordinate of $Y_1 = |x|$ by factors of 2 and 3, respectively. This is sometimes referred to as a vertical *stretch* using factors of 2 and 3.

The graph of $Y_4 = \frac{1}{2}|x|$ can be obtained from the graph of $Y_1 = |x|$ by multiplying each y-coordinate by $\frac{1}{2}$. This is sometimes referred to as a vertical *compression* using a factor of $\frac{1}{2}$.

"Now Work"

Many examples end with "Now Work Problems." The problems suggested here are similar to the corresponding examples and provide a great way to check your understanding as you work through the chapter. The solutions to all "Now Work" problems can be found in the back of the text as well as in the student solutions manual.

OVERVIEW

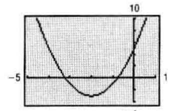
stretching, the shape of the graph of a quadratic function will look like one of the parabolas in Figure 5.

EXAMPLE 1 Graphing a Quadratic Function

Graph the function $f(x) = 2x^2 + 8x + 5$. Find the vertex and axis of symmetry.

Graphing Solution: Before graphing, notice the leading coefficient, 2, is positive and, therefore, the graph will open up and the vertex will be the lowest point. Now graph f. See Figure 6. We observe the graph does in fact open up. To estimate the vertex of the parabola, we use the MINIMUM command. The vertex is $(-2, -3)$. Therefore, the axis of symmetry is the line $x = -2$.

FIGURE 6

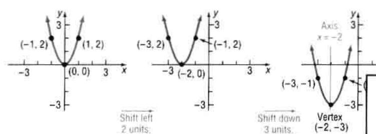


Algebraic Solution: We begin by completing the square* on the right side:

$$\begin{aligned} f(x) &= 2x^2 + 8x + 5 \\ &= 2(x^2 + 4x) + 5 && \text{Factor out the 2 from } 2x^2 + 8x. \\ &= 2(x^2 + 4x + 4) + 5 - 8 && \text{Complete the square of } 2(x^2 + 4x)\text{. Notice that (2) the factor of 2 requires that 8 be added and subtracted.} \\ &= 2(x + 2)^2 - 3 \end{aligned}$$

The graph of f can be obtained in three stages, as shown in Figure 7. Now compare this graph to the graph in Figure 5(a). The graph of $f(x) = 2x^2 + 8x + 5$ is a parabola that opens up and has its vertex (lowest point) at $(-2, -3)$. Its axis of symmetry is the line $x = -2$.

FIGURE 7



*Refer to the Appendix, Section 2, for a review.

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Procedures, both algebraic and technical, are clearly expressed throughout the text.

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MISSION POSSIBLE

Predicting The Future of Olympic Track Events

Some people think that women athletes are beginning to "catch up" to men in the Olympic Track Events. In fact, researchers at UCLA published data in 1992 that seemed to indicate that within 65 years the top men and women runners would be able to compete on an equal basis. Other researchers disagreed. Here are some data on the 200-meter run with winning times for the Olympics in the year given.

	Men's Time in Seconds	Women's Time in Seconds
1948	21.1	24.4
1952	20.7	23.7
1956	20.6	23.4
1960	20.5	24.0
1964	20.3	23.0
1968	19.83	22.5
1972	20.00	22.40
1976	20.23	22.37
1980	20.19	22.03
1984	19.80	21.81
1988	19.75	21.34
1992	19.73	21.72

(You may notice that the introduction of better timing devices meant more accurate measures starting in 1968 for the men and 1972 for the women.)

1. To begin your investigation of the UCLA conjecture, make a graph of these data. To get the kind of accuracy you need, you should use graph paper. You will need to use only the first quadrant. On your x-axis place the Olympic years from 1948 to 2048, counting by 4's. On your y-axis place the numbers from 15 to 25, which represent the seconds; if you're using graph paper, allow about four squares per one second. Use X's to represent the men's times on the graph and O's to represent the women's times. These should form what we call a *scatter plot*.
2. Using a ruler or straightedge, draw a line that represents roughly the slope and direction indicated by the men's scores. Do the same for the women's scores. Write a sentence or two explaining why you think your line is a good representation of the scatter plot.
3. Next find the equation for each line, using your graph to estimate the slope and y-intercept for each. Try to be as accurate as possible, remembering your units and using the points where the line crosses intersections of the grid. The slope will be in seconds per year.
4. Do your two lines appear to cross? In what year do they cross? If you solve the two equations algebraically, do you get the same answer?
5. Some graphing calculators enable you do this problem in a statistics mode. They will plot the individual points that you type in and find a line that represents the data. Use your graphing calculator to find out whether its equation matches yours.
6. Make a group decision about whether you think that women's times will "catch up" to men's times in the future. Write two to three sentences explaining why you believe they will or will not.

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"Mission Possible"
Learn to "think beyond the box." Collaborate with fellow students to solve these extended mathematical situations based on relevant "real-world" data.

Logistic Curve Fitting

Logistic growth models can be used to model situations where the value of the dependent variable is limited. Many real-world situations conform to this scenario. For example, the population of the human race is limited by the availability of natural resources such as food, shelter, and so forth. When the value of the dependent variable is limited, a logistic growth model is often appropriate.

EXAMPLE 3

Fitting a Curve to a Logistic Growth Model

The data on the left, obtained from R. Pearl ("The Growth of Population," *Quarterly Review of Biology* 2 (1927): 532–548) represents the amount of yeast biomass after t hours in a culture.

Time (in hours)	Yeast Biomass	Time (in hours)	Yeast Biomass
0	9.6	10	513.3
1	18.3	11	559.7
2	29.0	12	594.8
3	47.2	13	629.4
4	71.1	14	640.8
5	119.1	15	651.1
6	174.6	16	655.9
7	257.3	17	659.6
8	350.7	18	661.8
9	441.0		

(a) Using a graphing utility, draw a scatter diagram of the data with time as the independent variable.
(b) Using a graphing utility, fit a logistic growth model to the data.
(c) Using a graphing utility, graph the function found in (b) on the scatter diagram.
(d) What is the predicted carrying capacity of the culture?
(e) Use the function found in (b) to predict the population of the culture at $t = 19$ hours.

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Many examples and exercises connect real-world situations to mathematical concepts.

Learning to work with models is a skill that transfers to many disciplines.

374 **Chapter 4** Exponential and Logarithmic Functions

Solution (a) See Figure 54.
(b) A graphing utility fits a logistic growth model of the form $y = \frac{c}{1 + ae^{-bx}}$ by using the LOGISTIC regression option. See Figure 55. The logistic growth function of best fit to the data is

$$y = \frac{663.0}{1 + 71.6e^{-0.5470x}}$$

where y is the amount of yeast biomass in the culture and x is the time.
(c) See Figure 56.
(d) Based on the logistic growth function found in (b), the carrying capacity of the culture is 663.
(e) Using the logistic growth function found in (b), the predicted amount of yeast biomass at $t = 19$ is

$$y = \frac{663.0}{1 + 71.6e^{-0.5470(19)}} = 661.5$$

FIGURE 54

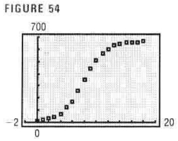


FIGURE 55

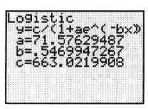
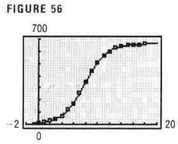


FIGURE 56



Now work Problem 13.

Choosing the Best Model

We have discussed seven different models thus far that can be used to explain the relation between two variables, x and y (see Table 8).

TABLE 8

Model	Functional Form	Section
1. Linear	$y = ax + b$	1.4
2. Quadratic	$y = ax^2 + bx + c$	3.1
3. Power	$y = ax^b$	3.2
4. Cubic	$y = ax^3 + bx^2 + cx + d$	3.3
5. Exponential	$y = ab^x$	4.8
6. Logarithmic	$y = a + b \ln x$	4.8
7. Logistic	$y = \frac{c}{1 + ae^{-bx}}$	4.8

How can we be certain the *model* we used is the "best" *model* to explain this relation? For example, why did we use an exponential model in Example 1 of this section?

Unfortunately, there is no such thing as *the* correct model. Modeling is not only a science but also an art form. Selecting an appropriate model requires experience and skill in the field in which you are modeling. For ex-

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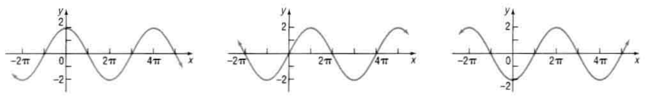
OVERVIEW

7.5 EXERCISES

In Problems 1–10, determine the amplitude and period of each function without graphing.

1. $y = 2 \sin x$
2. $y = 3 \cos x$
3. $y = -4 \cos 2x$
4. $y = -\sin \frac{1}{2}x$
5. $y = 6 \sin \pi x$
6. $y = -3 \cos 3x$
7. $y = -\frac{1}{2} \cos \frac{3}{2}x$
8. $y = \frac{4}{3} \sin \frac{2}{3}x$
9. $y = \frac{5}{3} \sin(-\frac{2\pi}{3}x)$
10. $y = \frac{9}{5} \cos(-\frac{3\pi}{2}x)$

In Problems 11–20, match the given function to one of the graphs (A)–(J).



(A) (B) (C)

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Each end-of-section exercise set begins with visual and concept-based problems, starting you out with the basics of the section.

87. Explain in your own words how you would use your calculator to find the value of $\cot^{-1} 10$.

88. Consult three books on calculus and write down the definition in each of $y = \sec^{-1} x$ and $y = \csc^{-1} x$. Compare these with the definitions given in this book.

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Challenge Yourself!
Critical Thinking and Writing Questions really get you thinking.

74. Advertising and Sales Revenue A marketing firm wishes to find a function that relates the sales S of a product and A, the amount spent on advertising the product. The data are obtained from past experience. Advertising and sales are measured in thousands of dollars.

Advertising Expenditures, A	Sales, S
20	335
22	339
22.5	338
24	343
24	341
27	350
28.3	351

(a) Does the relation defined by the set of ordered pairs (A, S) represent a function?
(b) Using a graphing utility, draw a scatter diagram of the data.
(c) Using a graphing utility, find the line of best fit relating advertising expenditures and sales.
(d) Interpret the slope.
(e) Express the relationship found in (c) using function notation.
(f) What is the domain of the function?
(g) Predict sales if advertising expenditures are $25,000.

75. Distance and Time Using the data below, find a function that relates the distance, s, in miles, driven by a Ford Taurus, and t, the time the Taurus has been driven.
(a) Does the relation defined by the set of ordered pairs (t, s) represent a function?
(b) Using a graphing utility, draw a scatter diagram of the data.

Time (Hours), t	Distance (Miles), s
0	0
1	30
2	55
3	83
4	100
5	150
6	210
7	260
8	300

(c) Using a graphing utility, find the line of best fit relating time and distance.
(d) Interpret the slope.
(e) Express the relationship found in (c) using function notation.
(f) What is the domain of the function?
(g) Predict the distance the car is driven after 11 hours.

76. High School vs. College GPA An administrator at Southern Illinois University wants to find a function that relates a student's college grade point average G to the high school grade point average, x. She randomly selects 8 students and obtains the following data:

High School GPA, x	College GPA, G
2.73	2.43
2.92	2.97
3.45	3.63
3.78	3.81
2.56	2.83
2.98	2.81
3.67	3.45
3.10	2.93

(a) Does the relation defined by the set of ordered pairs (x, G) represent a function?
(b) Using a graphing utility, draw a scatter diagram of the data.
(c) Using a graphing utility, find the line of best fit relating high school GPA and college GPA.
(d) Interpret the slope.
(e) Express the relationship found in (c) using function notation.
(f) What is the domain of the function?
(g) Predict a student's college GPA if her high school GPA is 3.23.

77. Effect of Gravity on Earth If a rock falls from a height of 20 meters on Earth, the height H (in meters) after x seconds is approximately

$$H(x) = 20 - 4.9x^2$$

(a) Use a graphing utility to graph the function H.
(b) What is the height of the rock when $x = 1$ second? $x = 1.1$ seconds? $x = 1.2$ seconds? $x = 1.3$ seconds?
(c) When is the height of the rock 15 meters? When is it 10 meters? When is it 5 meters?
(d) When does the rock strike the ground?

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Construct your own mathematical models from data and experience how the math you learn in class applies to the world around you.

CHAPTER REVIEW

THINGS TO KNOW

Definitions

Angle in standard position	Vertex is at the origin; initial side is along the positive x-axis
Degree (1°)	$1° = \frac{1}{360}$ revolution
Radian (1)	The measure of a central angle whose rays subtend an arc whose length is the radius of the circle
Unit circle	Center at origin; radius = 1 Equation: $x^2 + y^2 = 1$
Trigonometric functions	$P = (a, b)$ is the point on the unit circle corresponding to $\theta = t$ radians;

$\sin t = \sin\theta = b$ $\cos t = \cos\theta = a$

$\tan t = \tan\theta = \frac{b}{a}, \quad a \neq 0$ $\cot t = \cot\theta = \frac{a}{b}, \quad b \neq 0$

$\csc t = \csc\theta = \frac{1}{b}, \quad b \neq 0$ $\sec t = \sec\theta = \frac{1}{a}, \quad a \neq 0$

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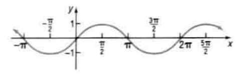
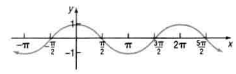
Properties of the Trigonometric Functions

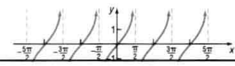
$y = \sin x$ Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Periodic: period $= 2\pi$ (360°)
Odd function



$y = \cos x$ Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Periodic: period $= 2\pi$ (360°)
Even function



$y = \tan x$ Domain: $-\infty < x < \infty$, except odd multiples of $\pi/2$ (90°)
Range: $-\infty < y < \infty$
Periodic: period $= \pi$ (180°)
Odd function



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The chapter review is a great place to check your understanding of the chapter material. Start with "Things to Know." Check your understanding of the concepts listed there. Next, make sure you know "How To" solve the items within that section. "Fill in the Blanks" determines your comfort with vocabulary. "True False" is a stickler for knowing definitions. If you are uncertain of any concept, go back into the chapter and study it further. Be sure to do the Review Exercises for practice. These reviews are for your success in this course—Make good use of them.

HOW TO

Convert an angle from radian measure to degree measure

Convert an angle from degree measure to radian measure

Find the value of each of the remaining trigonometric functions if the value of one function and the quadrant of the angle are given

Use the theorem on cofunctions of complementary angles

Graph the trigonometric functions, including transformations

Use reference angles to find the value of a trigonometric function

Use a calculator to find the value of a trigonometric function

Find the exact value of certain inverse trigonometric functions

Use a calculator to find the approximate values of inverse trigonometric functions

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FILL-IN-THE-BLANK ITEMS

1. Two rays drawn with a common vertex form a(n) ______________. One of the rays is called the ______________ ______________; the other is called the ______________ ______________.
2. In the formula $s = r\theta$ for measuring the length s of arc along a circle of radius r, the angle θ must be measured in ______________.
3. 180 degrees = ______________ radians.
4. Two acute angles whose sum is a right angle are called ______________.
5. The sine and ______________ functions are cofunctions.
6. An angle is in ______________ ______________ if its vertex is at the origin and its initial side coincides with the positive x-axis.
7. The reference angle of 135° is ______________.
8. The sine, cosine, cosecant, and secant functions have period ______________; the tangent and cotangent functions have period ______________.

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TRUE/FALSE ITEMS

T F 1. In the formula $s = r\theta$, r is the radius of a circle and s is the arc subtended by a central angle θ, where θ is measured in degrees.

T F 2. $|\sin\theta| \le 1$

T F 3. $1 + \tan^2\theta = \csc^2\theta$

T F 4. The only even trigonometric functions are the cosine and secant functions.

T F 5. $\tan 62° = \cot 38°$

T F 6. $\sin 182° = \cos 2°$

…hs of $y = \tan x$, $y = \cot x$, $y = \sec x$, and $y = \csc x$ have infinitely many vertical asymptotes.

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REVIEW EXERCISES

In Problems 1–4, convert each angle in degrees to radians. Express your answer as a multiple of π.

1. 135° 2. 210° 3. 18° 4. 15°

In Problems 5–8, convert each angle in radians to degrees.

5. $3\pi/4$ 6. $2\pi/3$ 7. $-5\pi/2$ 8. $-3\pi/2$

In Problems 9–30, find the exact value of each expression. Do not use a calculator.

9. $\tan\frac{\pi}{4} - \sin\frac{\pi}{6}$ 10. $\cos\frac{\pi}{3} + \sin\frac{\pi}{2}$ 11. $3\sin 45° - 4\tan\frac{\pi}{6}$

12. $4\cos 60° + 3\tan\frac{\pi}{3}$ 13. $6\cos\frac{3\pi}{4} + 2\tan\left(-\frac{\pi}{3}\right)$ 14. $3\sin\frac{2\pi}{3} - 4\cos\frac{5\pi}{2}$

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