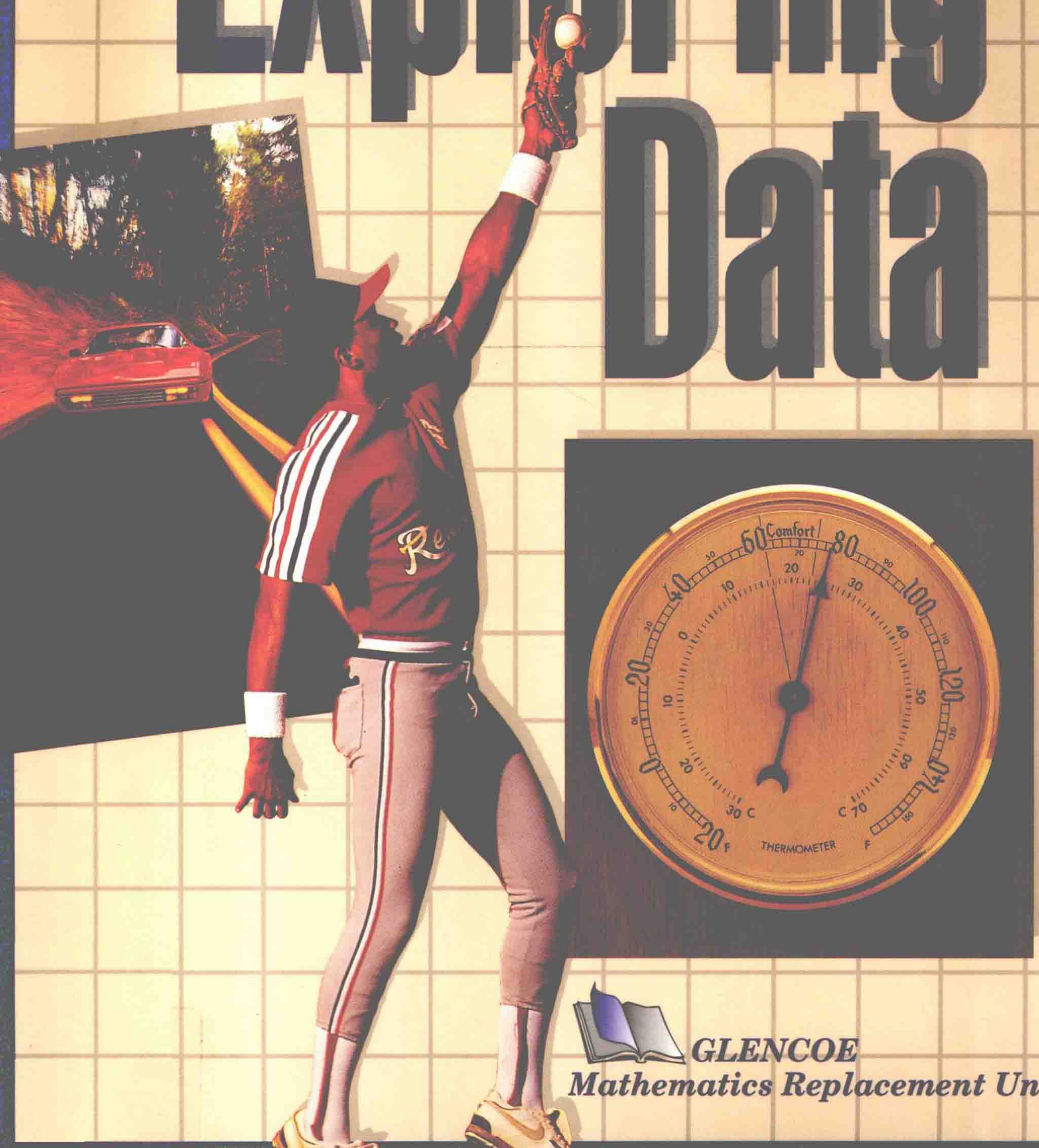
Exploring Data



GLENCOE
Mathematics Replacement Units

Exploring Data

An Alternative Unit for Representing and Analyzing One-Variable Data

GLENCOE
Mathematics Replacement Units

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NEW DIRECTIONS IN THE MATHEMATICS CURRICULUM

Exploring Data is a replacement unit developed to provide an alternative to the traditional method of presentation of selected topics in Pre-Algebra, Algebra 1, Geometry, and Algebra 2.

The NCTM Board of Directors' Statement on Algebra says,

> "Making algebra count for everyone will take sustained commitment, time and resources on the part of every school district. As a start, it is recommended that local districts–...
> 3. experiment with replacement units specifically designed to make algebra accessible to a broader student population."
> (May, 1994 *NCTM News Bulletin*.)

This unit uses data analysis as a context to introduce and connect broadly useful ideas in statistics and algebra. It is organized around multi-day lessons called investigations. Each investigation consists of several related activities designed to be completed by students working together in cooperative groups. The focus of the unit is on the development of mathematical thinking and communication. Students should have access to computers with statistical software and/or calculators capable of producing graphs and lines of best fit.

About the Authors

Christian R. Hirsch is a Professor of Mathematics and Mathematics Education at Western Michigan University, Kalamazoo, Michigan. He received his Ph.D. degree in mathematics education from the University of Iowa. He has had extensive high school and college level mathematics teaching experience. Dr. Hirsch was a member of the NCTM's Commission on Standards for School Mathematics and chairman of its Working Group on Curriculum for Grades 9-12. He is the author of numerous articles in mathematics education journals and is the editor of several NCTM publications, including the *Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 9-12.* Dr. Hirsch has served as president of the Michigan Council of Teachers of Mathematics and on the Board of Directors of the School Science and Mathematics Association. He is currently a member of the NCTM Board of Directors.

Arthur F. Coxford is a Professor of Mathematics Education and former Chairman of the Teacher Education Program at the University of Michigan, Ann Arbor, Michigan. He received his Ph.D. in mathematics education from the University of Michigan. He has been involved in mathematics education for over 30 years. Dr. Coxford is active in numerous professional organizations such as the National Council of Teachers of Mathematics, for which he was the editor of the 1988 NCTM Yearbook, *The Ideas of Algebra, K-12.* He was also the general editor of the 1993 and 1994 NCTM Yearbooks. Dr. Coxford has served as the president of the Michigan Council of Teachers of Mathematics and as the president of the School Science and Mathematics Association. He is presently the general editor of the 1995 NCTM Yearbook.

CONSULTANTS

Each of the Consultants read all five investigations. They gave suggestions for improving the Student Edition and the Teaching Suggestions and Strategies in the Teacher's Annotated Edition.

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Making Mathematics Accessible to All: First-Year Pilot Teachers

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To the Student

The most often asked question in mathematics classes must be "When am I ever going to use this?" One of the major purposes of *Exploring Data* is to provide you with a positive answer to this question.

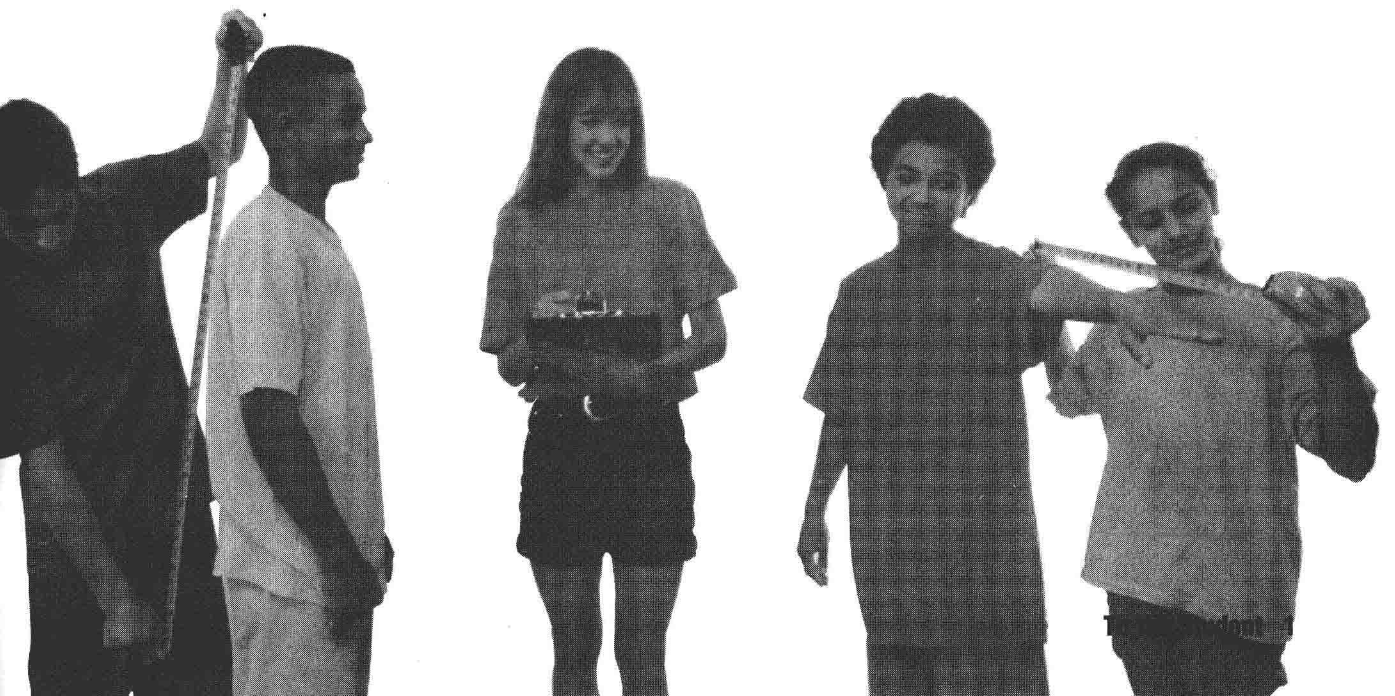
There are several characteristics that this unit has that you may have not experienced before. Some of those characteristics are described below.

Investigations *Exploring Data* consists of five investigations. Each investigation has one, two, or three related activities. After a class discussion introduces an investigation or activity, you will probably be asked to work cooperatively with other students in small groups as you gather data, look for patterns, and make conjectures.

Projects A project is a long-term activity that may involve gathering and analyzing data. You will complete some projects with a group, some with a partner, and some as homework.

Portfolio Assessment These suggest when to select and store some of your completed work in your portfolio.

Share and Summarize These headings suggest that your class discuss the results found by different groups. This discussion can lead to a better understanding of key ideas. If your point of view is different, be prepared to defend it.



Investigation 1

Patterns

Patterns are all around you. You hear them in the music you listen to. You see them in the clothes you wear. You experience them with the changing seasons. Observing and studying patterns is one way by which you can better understand the world in which you live. The study of patterns is the heart of mathematics.

Activity 1-1 Representing Patterns

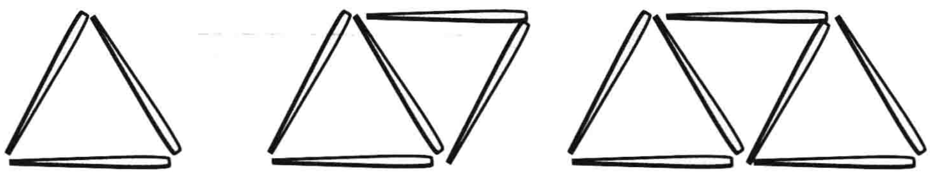
Materials

toothpicks

calculator

PARTNER PROJECT

Study the pattern of shapes below.



1. Use toothpicks to make the next three shapes in the pattern.
2. How many toothpicks did you use to form the fifth and sixth shapes?
3. How many toothpicks would be needed to form the twelfth shape?
4. Write a sentence describing the pattern.
5. If n represents the number of toothpicks in a shape of the pattern, how could you represent the number of toothpicks in the next shape?
6. If the pattern was continued, would there be a shape formed by 68 toothpicks? Explain your answer.
7. Recall that the distance around a shape is called its **perimeter**. If the length of a toothpick is one unit, then the perimeter of the first shape in the pattern above is 3 units.

    a. What is the perimeter of the second shape?

b. The table below shows the relationship of the perimeter to the number of triangles in each shape of the triangle pattern. Copy and complete the table.

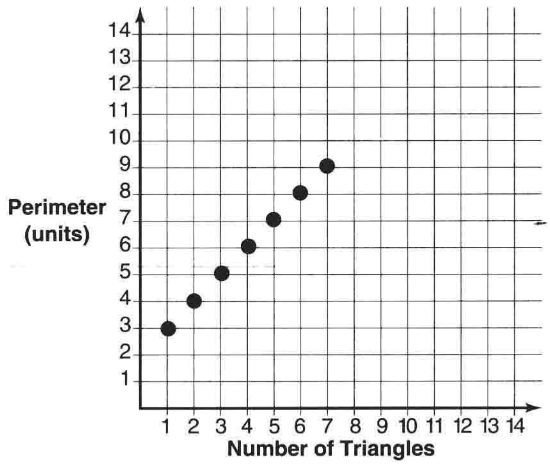
Number of triangles	1	2	3	4	5	6	7	8
Perimeter in units	3	4						

c. Write in words a description of how the perimeter of a shape in the pattern is related to the number of triangles.

Share & Summarize

d. If n represents the number of triangles in a shape of the pattern, how could you represent the perimeter of the shape? Be prepared to share your findings with the class.

8. The relationship between the perimeter of a shape and the number of triangles in the shape is represented by the graph shown below.



Graphing Calculator Activity

You can learn how to use a graphing calculator to plot points in Activity 1 on page 68.

a. The lowest point on the graph indicates that the first shape in the pattern has a perimeter of 3 units. Of the points shown, what does the highest point on the graph represent?

b. Describe how to locate the point that represents the perimeter of a shape consisting of 8 triangles.

c. Describe any pattern you see in the graph.

d. Determine a method for using the pattern in the graph to find the perimeter of the shape that consists of 10 triangles. Write instructions that a classmate could use with the graph to determine the perimeter of the same shape.

Homework Project

9. As you saw in Exercise 7, a table is useful for organizing data and helps in the search for patterns.

a. The table below shows the relationship of the number of toothpicks to the number of triangles in each shape of the triangle pattern. Copy and complete the table.

Number of triangles	1	2	3	4	5	6
Number of toothpicks	3	5				

b. If a shape consists of 8 triangles, how many toothpicks would it contain? Verify your answer by forming the shape.

c. Write in words a description of how the number of toothpicks in a shape of the pattern is related to the number of triangles.

d. If n represents the number of triangles in a shape of the pattern, how could you represent the number of toothpicks the shape contains? Check to see that your answer works with the data in the table.

e. If a shape consists of 65 triangles, how many toothpicks would it contain?

Share & Summarize

f. If a shape in the pattern was formed using 201 toothpicks, how many triangles are in the shape? Be prepared to share your findings with the class.

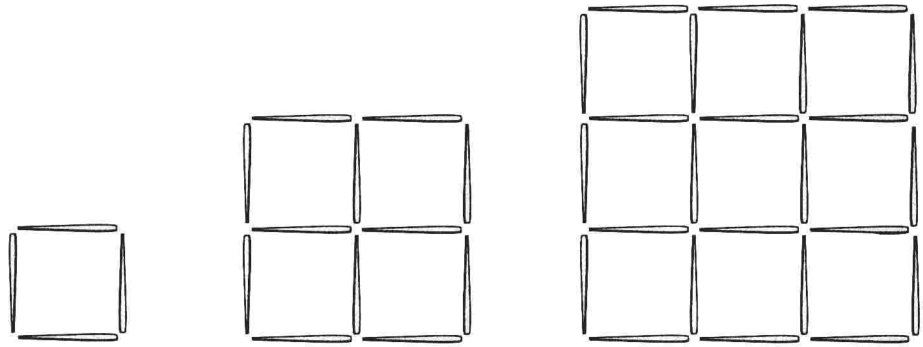
10. Draw a graph that shows the relationship between the number of toothpicks and the number of triangles in a shape of the pattern. Use the data from your table in Exercise 9.

a. What pattern do you see in the graph?

b. Describe and explain any similarities or differences between this graph and the graph in Exercise 8.

c. Use the pattern in the graph to determine the number of toothpicks of the shape that consists of 8 triangles. Is this the same number as your answer to Exercise 9b?

11. Study the pattern of growing squares below.



a. Describe the length of a side of each shape of the pattern.

b. Draw the next shape for the pattern.

c. If the length of a toothpick is 1 unit, then the smallest square has an area of 1 square unit. Copy and complete the table below.

Number of toothpicks on one side of the shape	**1**	**2**	**3**	**4**	**5**	**6**
Area in square units		4				
Perimeter in units		8				
Total number of toothpicks		12				

d. If n represents the number of toothpicks on the side of a shape, how could you represent the area of the shape?

Share & Summarize

e. If n represents the number of toothpicks on the side of a shape, how could you represent the perimeter of the shape? Be prepared to share your findings with the class.

12. Extension Refer to the pattern of squares in Exercise 11.

a. If p represents the perimeter of a shape in the pattern, how could you represent the perimeter of the next shape in the pattern?

b. If n represents the number of toothpicks on the side of a shape, how could you represent the total number of toothpicks in the shape?

GROUP PROJECT

13. Extension Use toothpicks to design a pattern of triangles. Draw the shapes in the pattern. Investigate possible relationships between the number of toothpicks on a side of each shape and its perimeter and the total number of toothpicks. Write a summary of your findings.

Activity 1-2 Using Patterns

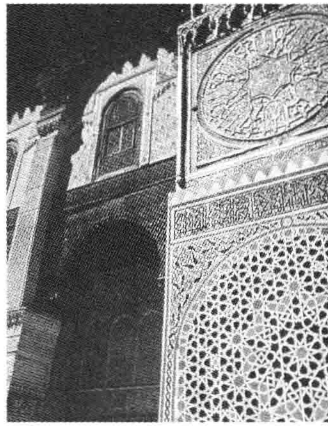
Materials

tracing paper

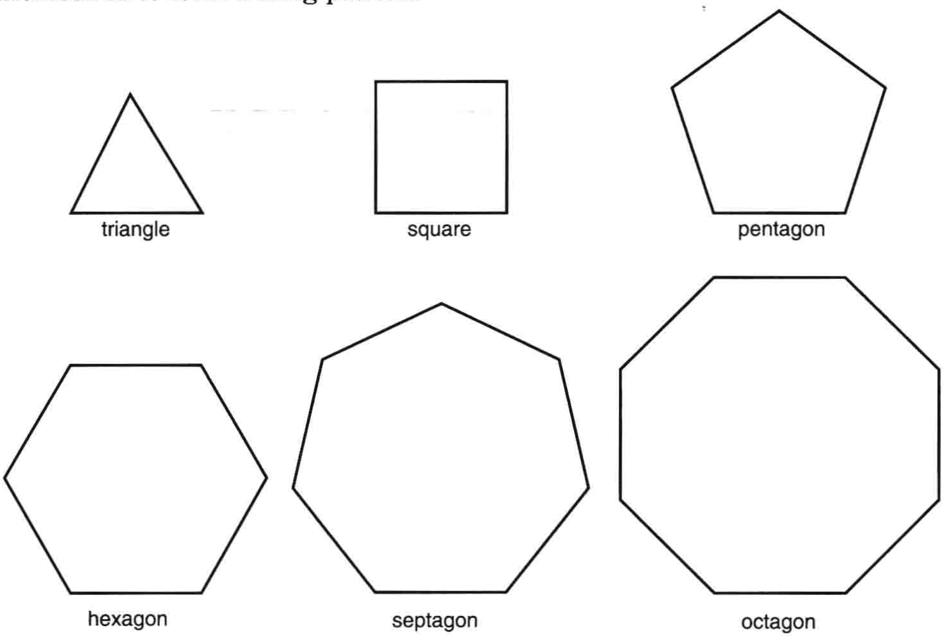
calculator

pattern blocks

Careers such as that of graphics artists and designers may center around the use of patterns. The study of mathematical patterns is often helpful in their work. A **tiling pattern** is an arrangement of polygon shapes that completely covers a flat surface without overlapping or leaving gaps. The floor or ceiling in your classroom may be covered with a square tiling pattern.



Jackie Washington, a commercial interior designer, is searching for polygon shapes that can be manufactured and used as floor tiles. Because ease of installation is a factor, she is considering only **regular polygons.** In a regular polygon, all sides are the same length, and all angles are the same size. What regular polygons could she use repeatedly by themselves to form a tiling pattern?



PARTNER PROJECT

1. For each polygon above:

- **a.** Mark a point P on a sheet of tracing paper.
- **b.** Carefully trace the polygon so that P is one of the vertices.
- **c.** Determine whether repeated tracings of the polygon will cover the region around point P with no overlaps or gaps.
- **d.** Check if the pattern can be extended by repeated tracings to tile a floor.
- **e.** Copy and complete the table at the top of the next page.

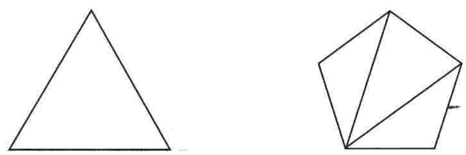
Regular Polygon	Does the polygon form form a tiling pattern?	How many copies of the polygon fit around a point?
Triangle		
Square	yes	4
Pentagon		
Hexagon		
Septagon		
Octagon		

f. Which of the regular polygons given could be used to form a tiling pattern?

g. What do you think determines whether a regular polygon can be used to form a tiling?

There are many other possible regular polygons. For example, a regular polygon could have 10 sides, 24 sides, or 100 sides. In order to determine which polygons could be used as floor tiles, it would be helpful if Ms. Washington had information about the size of the angles of various polygons.

2. Recall that the sum of the angles of a triangle is 180 degrees. How could you use this fact to find the sum of the angles of the pentagon shown below on the right?



$\angle 1 + \angle 2 + \angle 3 = 180^\circ$

3. In a regular pentagon, all angles are **congruent**. Angles are congruent if they have the same measurement. What is the measure of any one angle of the pentagon? Explain how you determined the measure. How could you check your solution?

Share & Summarize

4. Copy and complete the table below using the polygons at the beginning of this activity. Be prepared to share your findings with the class.

Regular Polygon	Number of Sides	Number of Triangles	Sum of Angles	Measure of One Angle
Triangle				
Square				
Pentagon	5	3	540°	108°
Hexagon				
Septagon				
Octagon				

a. What is the least number of triangles into which a polygon of n-sides could be separated?

b. Using patterns from the chart, what would be the sum of the angles of a regular polygon that has 20 sides? What would be the measure of any one of its angles?

c. Explain in words how you would find the sum of the angles and the measure of one angle for a regular polygon with 75 sides.

d. If a regular polygon has n sides, how could you represent the sum of its angles? Explain your reasoning.

e. If a regular polygon has n sides, how could you find the measure of one of its angles?

5. Copy the table below and use the results summarized in the two previous tables to complete it.

Regular Polygon	Is the measure of one angle a factor of 360?	Does the polygon tile?
Triangle		
Square	yes	yes
Pentagon		
Hexagon		
Septagon		
Octagon		

6. A regular decagon has 10 sides. Could this shape be used repeatedly to tile a floor? Explain your reasoning.

7. Under what condition(s) can a regular polygon be used to form a tiling?

Share & Summarize

8. Journal Entry Of all the regular polygons possible, which ones can be used to form tiling patterns? Be prepared to explain your reasoning to the class.

HOMEWORK PROJECT

9. Make a tiling that consists of repeated use of two regular polygons. Draw a sketch of your pattern.

Share & Summarize

10. Journal Entry Make a list of different tiling patterns you see over the next week. Which tiling patterns seem to be most common? What might explain this? Be prepared to share your findings in class.

Activity 1-3 Generalizing Patterns

Materials

centimeter cubes

calculator

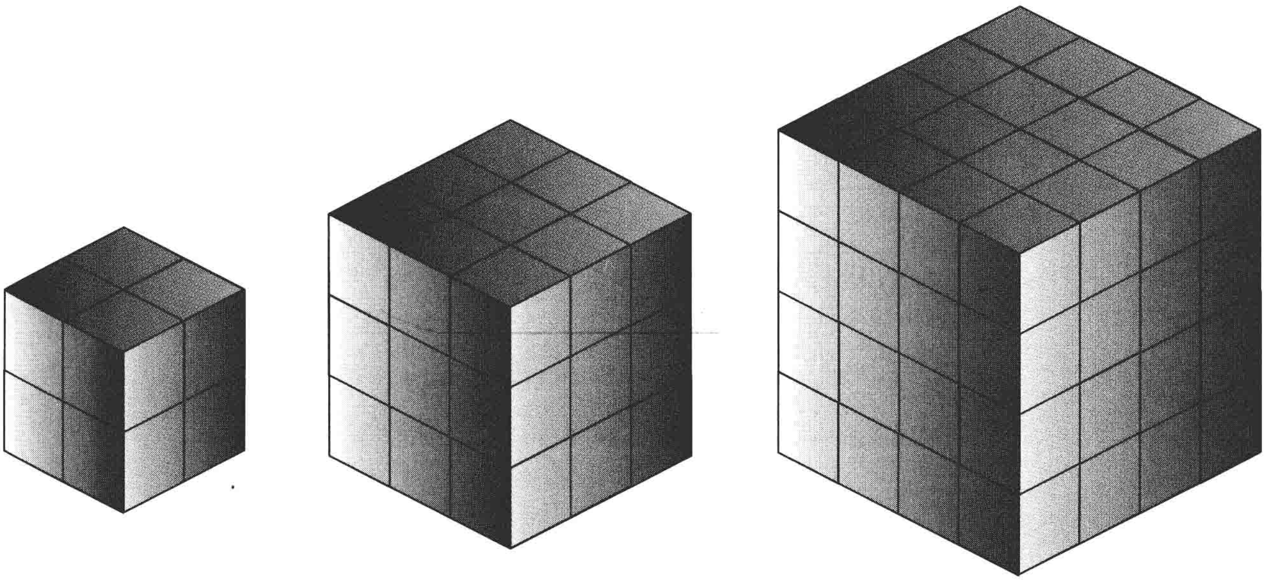
In the last part of Activity 1-1, you investigated patterns associated with square shapes formed from smaller square shapes. Suppose now that 1,000 small cubes have been stacked together and glued to form a larger cube. How many cubes would be along each edge?

If this large cube is dropped in a bucket of paint and completely submerged, how many of the small cubes will have paint on three faces? on two faces? on one face? on no faces?

In investigating these questions, you may find it helpful to collect data for specific cases and then look for a pattern.

PARTNER PROJECT

1. Copy the table below. Use the figures below and centimeter cubes as necessary to complete the table.



		Number of cubes with paint on:			
Number of cubes along an edge	**Total number of cubes**	**3 faces**	**2 faces**	**1 face**	**0 faces**
2					
3					
4					
5					

2. Write a short paragraph describing the location on the large cube of the smaller cubes with 3 painted faces, with 2 painted faces, with 1 painted face, and with 0 painted faces.

3. Suppose a large cube is formed that has six small cubes on each edge. Use your response to Exercise 2 and patterns in the table to determine:

   a. the total number of cubes in this large cube

   b. the number of cubes with paint on 3 faces; on 2 faces; on 1 face; on 0 faces

Share & Summarize

4. Repeat Exercises 3a and 3b using a large cube having 10 small cubes along each edge. Be prepared to share your findings with the class.

5. Repeat Exercises 3a and 3b using a large cube having 20 small cubes on each edge.

6. Suppose n represents the number of small cubes along each edge of a large cube that is built and painted as described on page 9. How could you represent:

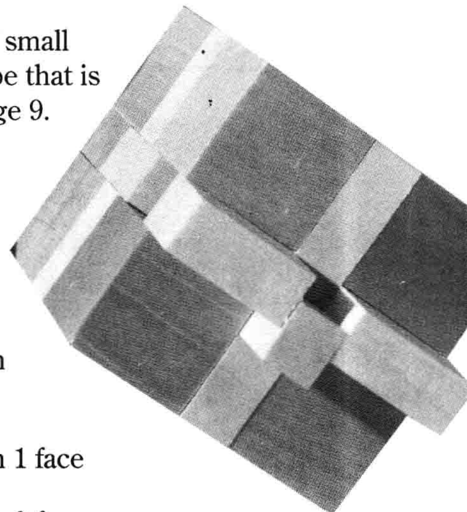
   a. the total number of cubes

   b. the number of cubes with paint on 3 faces

   c. the number of cubes with paint on 2 faces

   d. the number of cubes with paint on 1 face

   e. the number of cubes with paint on 0 faces



HOMEWORK PROJECT

7. Use your generalized patterns in Exercise 6 to extend the table for 7, 8, and 9 cubes along each edge of larger cubes.

8. Could you build a large cube for which the entry in the "2 faces" column is 516? Explain.

9. Could you build a large cube for which the entry in the "1 face" column is 861? Explain.

10. A large cube was built for which the entry in the "0 faces" column was 571,787. What was the total number of small cubes used?