Optimization of Structural and Mechanical Systems

Jasbir S Arora
Editor

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PREFACE

This book covers several important topics on the subject of optimization of structural and mechanical systems. Computational optimization methods have matured over the last few years due to the extensive research by applied mathematicians and the engineering community. These methods are being applied to a variety of practical applications. Several general-purpose optimization programs as well as programs for specific engineering applications have become available recently. These are being used to solve practical and interesting optimization problems.

The book covers state-of-the-art in computational algorithms as well as applications of optimization to structural and mechanical systems. Formulations of the problems are covered and numerical solutions are presented and discussed. Topics requiring further research are identified. Leading researchers in the field of optimization and its applications have written the material and provided significant insights and experiences with the applications. The topics covered include:

- Optimization concepts and methods
- Optimization of large scale systems
- Optimization using evolutionary computations
- Multiobjective optimization
- Shape optimization
- Topology optimization
- Design sensitivity analysis of nonlinear structural systems
- Optimal control of structures
- Nonlinear optimal control
- Optimization of systems for acoustics
- Design optimization under uncertainty
- Optimization-based inverse kinematics of articulated mechanisms
- Multidisciplinary design optimization
- mesh free methods for optimization
- Kriging metamodel based optimization,

- Sensitivity-free formulations for structural and mechanical system optimization
- Robust design based on optimization
- Parallel computations for design optimization
- Semidefinite programming for structural optimization.

The book is suitable for advanced courses on optimization of structural and mechanical systems. It is also an invaluable resource for researchers, graduate students, and practitioners of optimization.

I would like to thank all the authors for their diligence and meticulous work in writing their chapters. Without their hard work this book would not be possible. I would also like to thank the staff at World Scientific Publishing Company for their patience and help in finalizing the material for the book.

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CONTENTS

CHAPTER 1

INTRODUCTION TO OPTIMIZATION

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Basic concepts of optimization are described in this chapter. Optimization models for engineering and other applications are described and discussed. These include continuous variable and discrete variable problems. Optimality conditions for the continuous unconstrained and constrained problems are presented. Basic concepts of algorithms for continuous and discrete variable problems are described. An introduction to the topics of multiobjective and global optimization is also presented.

1. Introduction

Optimization is a mature field due to the extensive research that has been conducted over the last about 60 years. Many types of problems have been addressed and many different types of algorithms have been investigated. The methodology has been used in different practical applications and the range of applications is continuously growing. Some of the applications are described in various chapters of this book. *The purpose of this chapter is to give an overview of the basic concepts and methods for optimization of structural and mechanical systems.* Various optimization models are defined and discussed. Optimality conditions for continuous variable optimization problems are presented and discussed. Basic concepts of algorithms for continuous variable and discrete variable optimization problems are described. Topics of multiobjective and global optimization are also introduced. The material of the chapter is available in many textbooks on optimization.[1-7] It is derived from several recent publications of the author and his co-workers.[7-34]

2. Optimization Models

Transcription of an optimization problem into a mathematical formulation is a critical step in the process of solving the problem. If the formulation of the problem as an optimization problem is improper, the solution for the problem is most likely going to be unacceptable. For example, if a critical constraint is not included in the formulation, then most likely, that constraint is going to be violated at the optimum point. Therefore special attention needs to be given to the formulation of the optimization problem.

Any optimization problem has three basic ingredients:

- *Optimization variables*, also called *design variables* denoted as vector $\mathbf{x}$.
- *Cost function*, also called the objective function, denoted as $f(\mathbf{x})$.
- *Constraints* expressed as equalities or inequalities denoted as $g_i(\mathbf{x})$.

The variables for the problem can be continuous or discrete. Depending on the types of variables and functions, we obtain continuous variable, discrete variable, differentiable and nondifferentiable problems. These models are described next; for more details and practical applications of the models, various references can be consulted.[7,9-12,14,16,25-30]

2.1. *Optimization Models: Continuous Variables*

Any continuous variables optimization problem can be transcribed into a *standard nonlinear programming (NLP) model* defined as minimization of a cost function subject to equality constraints and inequality constraints expressed in a "≤" form as Problem P.[7]

Problem P. Find the optimization variable vector $\mathbf{x} = [x_1\ x_2\ \ x_n]^T$ to minimize a cost function $f(\mathbf{x})$ subject to equality and inequality constraints:

$$g_j(\mathbf{x}) = 0,\ j = 1 \text{ to } p \tag{1}$$

$$g_j(\mathbf{x}) \le 0,\ j = p+1 \text{ to } m \tag{2}$$

where n is the number of variables, p is the number of equality constraints, and m is the total number of constraints. Note that the *explicit lower* and *upper bounds* on the variables are included in Eq. (2). However, for efficient numerical calculations the simple form of these constraints is exploited.

The *feasible set* for the problem is defined as a collection of all the points that satisfy the constraints of Eqs. (1) and (2). It is also called the *constraint set*, and is denoted as S:

$$S = \{\mathbf{x} \mid g_j(\mathbf{x}) = 0,\ j = 1 \text{ to } p;\ g_j(\mathbf{x}) \le 0,\ j = p+1 \text{ to } m\} \tag{3}$$

Thus the Problem P can be written simply as

$$\underset{\mathbf{x}\in S}{\text{minimize}}\ f(\mathbf{x}) \tag{4}$$

It is important to note that the feasible set for a problem may be empty if there are too many constraints on the problem or if there are conflicting constraints. In general, this is difficult to determine before the problem is solved. Only after a numerical algorithm fails to find a feasible point for the problem, we can conclude that the set S is empty.[21] In that case the problem formulation needs to be examined to relax some of the constraints, or eliminate conflict in the constraints. In addition, it is difficult to know, in general, if there is a solution to the Problem P. However, the question of *existence of a solution* can be answered with certain assumptions about the problem functions. It turns out that if $f(\mathbf{x})$ is *continuous* on a *nonempty feasible set S*, all constraint functions are continuous, and all inequalities contain their boundary points (i.e., expressed as "$\le$" and not simply as "$<$"), then there is a solution for Problem P. When these requirements are satisfied, a robust numerical algorithm is guaranteed to converge to a solution point.

If there are no constraints on the variables, the set S is the entire design space and the problem is called an *unconstrained optimization problem.* If all the functions are linear in terms of the variables, the Problem P is called a *linear programming* (LP) problem. If the cost function is quadratic and the constraints are linear, the problem is called a *quadratic programming* (QP) problem.

An inequality constraint $g_i(\mathbf{x}) \le 0$ is said to be *active* at a point $\mathbf{x}$ if it is satisfied as an equality at that point, i.e., $g_i(\mathbf{x}) = 0$. It is said to be *inactive* if it has negative value at that point, and *violated* if it has positive value. An *equality* constraint is always either *active* or *violated* at any point.

In some applications, several objective functions need to be optimized simultaneously. These are called *multiobjective* optimization problems. They are usually transformed into Problem P by combining all the objective functions to form a composite scalar objective function. Several approaches to accomplish this objective are summarized in a later section.[7,32,35-37]

When a gradient-based optimization method (discussed in a later section) is used to solve Problem P, the cost and constraint functions are assumed to be twice differentiable.

2.2. *Optimization Models: Mixed Variables*

In many practical applications of optimization, discrete variables occur naturally in the problem formulation. For example,

- plate thickness must be selected from the available dimensions,[7]
- material properties must correspond to the available materials,[7,25]
- structural members must be selected from a catalog,[14,26,29]
- number of reinforcing bars in a concrete member must be an integer,[28]
- diameter of rods must be selected from the available sizes,[7,28]
- number of bolts must be an integer,[27]
- number of strands in a prestressed member must be an integer.[28]

Discrete variables must be treated properly in numerical optimization procedures. A mixed continuous-discrete variable optimization problem is defined next as Problem MP.

Problem MP. A general mixed discrete-continuous variable nonlinear optimization problem is defined by modifying Problem P to minimize the cost function $f(\mathbf{x})$ subject to the constraints of Eqs. (1) and (2) with the additional requirement that each discrete variable be selected from a specified set:

$$x_i \in D_i\,,\ D_i = \left(d_{i1}, d_{i2}, \ldots., d_{iqi}\right);\ i = 1 \text{ to } n_d \tag{5}$$

where n_d is the number of *discrete design variables*, D_i is the set of discrete values for the ith variable, q_i is the number of available discrete values for the ith variable, and d_{ik} is the kth discrete value for the ith variable. Note that the foregoing problem definition includes *integer variable* as well as 0-1 *variable* (on-off variables, binary variables) problems. If the problem has only *continuous variables*, and the functions f and g_j are twice continuously differentiable, we obtain the Problem P. Many discrete variable optimization problems have nondifferentiable functions; therefore gradient-based methods cannot be used to solve such problems. However, methods that do not require gradients of functions are available to solve such problems.

It is also important to note that the discrete variable optimization problems usually require considerably more computational effort compared to the continuous variable problems. This is true even though the number of feasible points with discrete variables is finite and they are infinite with continuous variables.