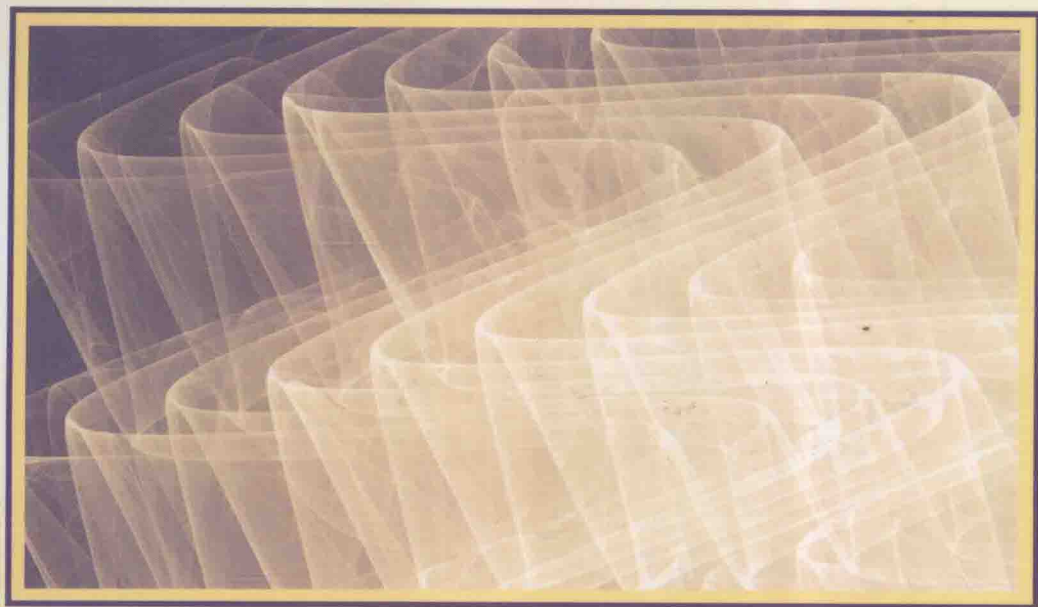


**Automation and Control
Engineering Series**

Modeling and Control of Vibration in Mechanical Systems



**Chunling Du
Lihua Xie**



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Modeling and Control of Vibration in Mechanical Systems

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Preface

This book is primarily intended for researchers and engineering practitioners in systems and control, especially those engaged in the area of modeling and control of vibrations in mechanical structures and systems. The book aims at empowering readers with a clear understanding of characteristics of various vibrations, their effects on system stability and performance, and techniques for rejecting vibrations of different frequency ranges and their limitations. Special attention is given to recently developed vibration modeling and control techniques in high precision systems. Many real-world examples are given to demonstrate the modeling and control techniques.

Vibration exists in a wide spectra of engineering systems such as hard disk drives, automotives, aerospace and aeronautic systems, manufacturing systems, etc. Vibration is undesirable in most engineering applications, lowering system performance, wasting energy and creating unwanted noise. Although the problem of vibration control has been studied for a long time, it remains and indeed becomes more challenging in many applications such as precision engineering and hard disk drives, where an extremely high positioning accuracy is required. Therefore, vibration control has drawn more intensive efforts from researchers and engineering practitioners in recent years. It is our intention in this book to present to readers some of the recent developments in this field.

The book presents the latest results in vibration modeling and advanced control design for vibration attenuation in mechanical actuation systems to achieve high precision positioning performance. It focuses on vibration and disturbance rejections using recently developed control techniques for high precision positioning, and demonstration of the benefits gained from the applications of these techniques. The theoretical developments and principles of control design are elaborated in detail so that the reader can apply the techniques developed to obtain solutions with the help of MATLAB[®]. Examples are presented throughout the book so that the subject can be better understood. A number of simulation and experimental results with comprehensive evaluations are provided in each chapter, except Chapters 1, 4, and 5, which are dedicated to the review of related background knowledge.

The book summarizes a collective research effort which we have had the pleasure to contribute to. Many results reported in the book are due to the collaboration with Guoxiao Guo from Western Digital Corporation, Jianliang Zhang and Jul Nee Teoh from Data Storage Institute (DSI) of Singapore, Youyi Wang from Nanyang Technological University (NTU), and Frank Lewis from the University of Texas at Arlington. The research work contained in this book was mainly performed at DSI and the School of Electrical and Electronic Engineering (EEE) of NTU, Singapore.

Algorithms applied in magnetic recording systems were implemented at DSI and those in the Stewart platform at the School of EEE, NTU. We would like to express our sincere appreciation to DSI for its supportive environment and vibrant research atmosphere. We are also sincerely grateful to Dr. Ong Eng Hong and the colleagues in Mechatronics and Recording Channel Division of DSI, and EEE, NTU for their support.

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Symbols and Acronyms

\mathcal{R}^n :	n -dimensional real Euclidean space
$\mathcal{R}^{n \times m}$:	set of $n \times m$ real matrices
I_n :	$n \times n$ identity matrix
(A, B, C, D) :	state-space representation of a system
$\left[\begin{array}{c cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array} \right]$:	compact representation of system: $\begin{aligned} x(k+1) &= Ax(k) + B_1w(k) + B_2u(k) \\ z(k) &= C_1x(k) + D_{11}w(k) + D_{12}u(k) \\ y(k) &= C_2x(k) + D_{21}w(k) + D_{22}u(k) \end{aligned}$
$\text{diag}\{A_1, A_2, \dots, A_n\}$:	block diagonal matrix with A_j (not necessarily square), $j = 1, 2, \dots, n$, on the diagonal
X^T :	transpose of matrix X
X^* :	complex conjugate transpose of matrix X
$P \geq 0$:	symmetric positive semidefinite matrix $P \in \mathcal{R}^{n \times n}$
$P > 0$:	symmetric positive definite matrix $P \in \mathcal{R}^{n \times n}$
$P \geq Q$:	$P - Q \geq 0$ for symmetric $P, Q \in \mathcal{R}^{n \times n}$
$P > Q$:	$P - Q > 0$ for symmetric $P, Q \in \mathcal{R}^{n \times n}$
$\bar{\sigma}(X)$:	largest singular value of X
$\text{Trace}(X)$:	trace of X
$\ \cdot\ $:	Euclidean vector norm
$\ w\ _2$:	ℓ_2 -norm of a signal $\{w(k)\}$, i.e., $\sqrt{\sum_{k=0}^{\infty} \ w(k)\ ^2}$.

$\ell_2\{[0, \infty)\}$:	space of square summable sequences on $\{[0, \infty)\}$. The signal $\{w(k)\}$ is said to be from $\ell_2\{[0, \infty)\}$ or simply ℓ_2 if $\ w\ _2 < \infty$.
$\ G\ _2$:	H_2 norm of transfer function G
$\ G\ _\infty$:	H_∞ norm of transfer function G
$Re(\)$:	the real part of a complex number
$Im(\)$:	the imaginary part of a complex number
$\rho(\)$:	spectral radius
<i>AGC</i> :	automatic gain control
<i>AVC</i> :	active vibration control
<i>deg</i> :	degree
<i>det</i> :	determinant
<i>DSA</i> :	Dynamic Signal Analyzer
<i>FFT</i> :	fast Fourier transform
<i>FXLMS</i> :	filtered-X LMS
<i>HDD</i> :	hard disk drive
<i>LDV</i> :	Laser Doppler Vibrometer
<i>LFT</i> :	Linear fractional transformation
<i>LMI</i> :	linear matrix inequality
<i>LMS</i> :	least mean square
<i>LQG</i> :	linear quadratic Gaussian
<i>LTR</i> :	loop transfer recovery
<i>MEMS</i> :	micro electro-mechanical system
<i>MSE</i> :	mean square error
<i>NRRO</i> :	nonrepeatable runout
<i>PES</i> :	position error signal

<i>PID</i> :	proportional-integral-derivative
<i>PLPF</i> :	phase lead peak filter
<i>PZT</i> :	lead zirconate titanate/piezoelectric
<i>RBF</i> :	radial basis function
<i>RMS</i> :	root mean square
<i>RPM</i> :	rotations per minute
<i>RRO</i> :	repeatable runout
<i>SSTW</i> :	self-servo track writing
<i>STW</i> :	servo track writing
<i>TMR</i> :	track misregistration
<i>VCM</i> :	voice coil motor

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