

Bill Cox

Understanding Engineering Mathematics

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Understanding Engineering Mathematics

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Preface

This book contains most of the material covered in a typical first year mathematics course in an engineering or science programme. It devotes Chapters 1–10 to consolidating the foundations of basic algebra, elementary functions and calculus. Chapters 11–17 cover the range of more advanced topics that are normally treated in the first year, such as vectors and matrices, differential equations, partial differentiation and transform methods.

With widening participation in higher education, broader school curricula and the wide range of engineering programmes available, the challenges for both teachers and learners in engineering mathematics are now considerable. As a result, a substantial part of many first year engineering programmes is dedicated to consolidation of the basic mathematics material covered at pre-university level. However, individual students have widely varying backgrounds in mathematics and it is difficult for a single mathematics course to address everyone's needs. This book is designed to help with this by covering the basics in a way that enables students and teachers to quickly identify the strengths and weaknesses of individual students and 'top up' where necessary. The structure of the book is therefore somewhat different to the conventional textbook, and 'To the student' provides some suggestions on how to use it.

Throughout, emphasis is on the key mathematical techniques, covered largely in isolation from the applications to avoid cluttering up the explanations. When you teach someone to drive it is best to find a quiet road somewhere for them to learn the basic techniques before launching them out onto the High Street! In this book the mathematical techniques are motivated by explaining where you may need them, and each chapter has a short section giving typical applications. More motivational material will also be available on the book web-site. Rigorous proof for its own sake is avoided, but most things are explained sufficiently to give an understanding that the educated engineer should appreciate. Even though you may use mathematics as a tool, it usually helps to have an idea of how and why the tool works.

As the book progresses through the more advanced first year material there is an increasing expectation on the student to learn independently and 'fill in the gaps' for themselves – possibly with the teacher's help. This is designed to help the student to develop a mature, self-disciplined approach as they move from the supportive environment of pre-university to the more independent university environment. In addition the book web-site (www.bh.com/companions/0750650982) will provide a developing resource to supplement the book and to focus on specific engineering disciplines where appropriate.

In the years that this book has been in development I have benefited from advice and help from too many people to list. The following deserve special mention however. Dave Hatter for having faith in the original idea and combining drink and incisive comment well mixed in the local pub. Peter Jack for many useful discussions and for the best part of the S(ketch) GRAPH acronym (I just supplied the humps and hollows). Val Tyas for typing

much of the manuscript, exploring the limits of RSI in the process, and coping cheerfully with my continual changes. The late Lynn Burton for initial work on the manuscript and diagrams. She was still fiddling with the diagrams only weeks before she succumbed to cancer after a long and spirited fight. I am especially indebted to her for her friendship and inspiration – she would chuckle at that.

I also benefited from an anonymous reviewer who went far beyond the call of duty in providing meticulous, invaluable comment – It's clear that (s)he is a good teacher. Of course, any remaining errors are my responsibility.

The team at Butterworth-Heinemann did a wonderful job in dealing with a complicated manuscript – sense of humour essential!

Last but not least I must mention the hundreds of students who have kept me in line over the years. I have tried to write the book that would help most of them. I hope they, and their successors, will be pleased with it.

Bill Cox, June 2001

To the Student

Whatever your previous background in mathematics, it is likely that when you begin your engineering studies at university you will need to consolidate your mathematical skills before moving on to new material. The first ten chapters of this book are designed to help with this 'transition' by providing you with individual pathways to quickly **review** your current skills and understanding, then **revise** and **reinforce** where necessary.

Chapters 1–10 have a three-part structure by which you:

- **Review** your present knowledge and skills, with a review test on key topics
- **Revise** as you need to
- **Reinforce** the essential skills that you will need for your particular programme, so that they are there when you need them.

The three sections are linked, so that you can choose your own pathway through the material and focus on your specific needs. **In the review test the single arrows forward you to the corresponding revision section and the review question solution, while the double arrows fast forward you to the reinforcement exercises.**

Some suggestions for working through these chapters may help:

- Use the lists of prerequisites and objectives to get an overview of the chapter – what have you seen before, what needs a reminder, what is completely new?
- Use the review test to establish your current understanding of the various topics.
- Where you are confident you may still be able to learn something from the revision section or polish your skills by working through the reinforcement questions.
- Where you are unsure and need a reminder, go to the relevant revise section for some hints before trying the review question, then consolidate your skills with the reinforcement exercises.
- Where the topic is perhaps new to you, start with the revision section, using the review questions as worked examples and the reinforcement exercises as further practice.

The remaining chapters (11–17), covering material appropriate to typical first-year courses in engineering mathematics, are designed to support you in developing independent learning skills for more advanced study. The order of material and the structure of these chapters are at first supportive with many examples, then gradually progress to a more concise and mature format. The focus is on the key material, and the text contains leading

problems that encourage you to develop ideas for yourself. The structure of the book is therefore designed not only to ease the transition to university, but also to develop your independent learning skills and prepare you for the style of more advanced textbooks.

Each chapter has a number of 'Applications' exercises that provide illustrations of typical engineering applications, bring together the different topics of the chapter, or prepare the way for later material. Some are simple, while others provide significant and challenging projects.

The book is the core of a larger educational resource of web-based material enabling you to broaden and deepen your studies. The book web-site (www.bh.com/companions/0750650982) provides advice on learning mathematics, solutions to all of the reinforcement and applications exercises, develops some topics more thoroughly, and provides relevant examples and illustrations from different engineering disciplines.

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Number and Arithmetic

In this chapter we review the key features of elementary numbers and arithmetic. The topics covered are those found to be most useful later on.

Prerequisites

It will be helpful if you know something about:

- simple types of numbers such as integers, fractions, negative numbers, decimals
- the concepts of 'greater than' and 'less than'
- elementary arithmetic: addition, subtraction, multiplication and division
- powers and indices notation, $2^3 = 2 \times 2 \times 2$, for example
- how to convert a simple fraction to a decimal and vice versa

Objectives

In this chapter you will find:

- different types of numbers and their properties (particularly zero)
- the use of inequality signs
- highest common factors and lowest common denominators
- manipulation of numbers (BODMAS)
- handling fractions
- factorial ($n!$) and combinatorial (${}^n C_r$ or $\binom{n}{r}$) notation
- powers and indices
- decimal notation
- estimation of numerical expressions

Motivation

You may need the material of this chapter for:

- numerical manipulation and calculation in engineering applications
- checking and using scientific formulae
- illustrating and checking results used later in mathematics
- statistical calculations
- numerical estimation and 'back of an envelope' calculations

A note about calculators

Calculators obviously have their place, particularly in applied mathematics, numerical methods and statistics. However, they are very rarely needed in this chapter, and the skills it aims to develop are better learnt without them.

1.1 Review

1.1.1 Types of numbers

► 5 27 ►►

A. For each number choose one or more descriptions from the following: (a) integer, (b) negative, (c) rational number (fraction), (d) real, (e) irrational, (f) decimal, (g) prime.

(i) is done as an example

- | | | |
|-----------------------|--------------------|---------------------|
| (i) -1 (a, b, c, d) | (ii) $\frac{1}{2}$ | (iii) 0 |
| (iv) 7 | (v) $\frac{23}{5}$ | (vi) $-\frac{3}{4}$ |
| (vii) 0.73 | (viii) 11 | (ix) 8 |
| (x) $\sqrt{2}$ | (xi) -0.49 | (xii) π |

B. Which of the following descriptions apply to the expressions in (i)–(x) below?

- | | | |
|-------------------------|-----------------------------------|----------------------|
| (a) infinite | (b) does not exist | (c) negative |
| (d) zero | (e) finite | (f) non-zero |
| (i) 0×1 (d, e) | (ii) $0 + 1$ | (iii) $\frac{1}{0}$ |
| (iv) $2 - 0$ | (v) 0^2 | (vi) $0 - 1$ |
| (vii) $\frac{0}{0}$ | (viii) $3 \times 0 + \frac{3}{0}$ | (ix) $\frac{0^3}{0}$ |
| (x) $\frac{2}{2}$ | | |

1.1.2 Use of inequality signs

► 7 27 ►►

Express symbolically:

- (i) x is a positive, non-zero, number ($x > 0$)
- (ii) x lies strictly between 1 and 2
- (iii) x lies strictly between -1 and 3
- (iv) x is equal to or greater than -2 and is less than 2
- (v) The absolute value of x is less than 2.

1.1.3 Highest common factor and lowest common multiple

▶ 8 28 ▶▶

A. Express in terms of prime factors

- | | | |
|---------------------------|---------|-----------|
| (i) 15 ($= 3 \times 5$) | (ii) 21 | (iii) 60 |
| (iv) 121 | (v) 405 | (vi) 1024 |
| (vii) 221 | | |

B. Find the highest common factor (HCF) of each of the following sets of numbers

- | | | |
|-----------------|------------------|--------------|
| (i) 24, 30 (6) | (ii) 27, 99 | (iii) 28, 98 |
| (iv) 12, 54, 78 | (v) 3, 6, 15, 27 | |

C. Find the lowest common multiple (LCM) of each of the following sets of numbers

- | | | |
|---------------|-------------|--------------|
| (i) 3, 7 (21) | (ii) 3, 9 | (iii) 12, 18 |
| (iv) 3, 5, 9 | (v) 2, 4, 6 | |

1.1.4 Manipulation of numbers

▶ 10 28 ▶▶

Evaluate

- | | | |
|---|--------------------------|-----------------------------------|
| (i) $2 + 3 - 7 (= -2)$ | (ii) $4 \times 3 \div 2$ | (iii) $3 + 2 \times 5$ |
| (iv) $(3 + 2) \times 5$ | (v) $3 + (2 \times 5)$ | (vi) $18 \div 2 \times 3$ |
| (vii) $18 \div (2 \times 3)$ | (viii) $-2 - (4 - 5)$ | (ix) $(4 \div (-2)) \times 3 - 4$ |
| (x) $(3 + 7) \div 5 + (7 - 3) \times (2 - 4)$ | | |

1.1.5 Handling fractions

▶ 12 28 ▶▶

A. Simplify

- | | | |
|--|-------------------------------------|--|
| (i) $\frac{4}{6} \left(= \frac{2}{3} \right)$ | (ii) $\frac{18}{9}$ | (iii) $\frac{7}{3} \times \frac{4}{7}$ |
| (iv) $\frac{7}{5} \times \frac{3}{14}$ | (v) $\frac{3}{4} \div \frac{4}{5}$ | (vi) $\frac{1}{2} + \frac{1}{3}$ |
| (vii) $\frac{1}{2} - \frac{1}{3}$ | (viii) $\frac{4}{15} - \frac{7}{3}$ | (ix) $1 + \frac{1}{2} + \frac{1}{3}$ |
| (x) $\frac{2}{3} - \frac{3}{4} + \frac{1}{8}$ | | |

B. If the numbers a and b are in the ratio $a : b = 3 : 2$ and $a = 6$, what is b ?

1.1.6 Factorial and combinatorial notation – permutations and combinations

▶ 16 29 ▶▶

A. Evaluate

(i) $3! (= 6)$ (ii) $6!$ (iii) $\frac{24!}{23!}$ (iv) $\frac{12!}{9! 3!}$

- B. (i) Evaluate (a) ${}^3C_2 (= 3)$ (b) 6C_4 (c) 6P_3
 (ii) In how many ways can two distinct letters be chosen from ABCD?
 (iii) How many permutations of the letters ABCDE are there?

1.1.7 Powers and indices

▶ 18 29 ▶▶

A. Reduce to simplest power form.

(i) $2^3 2^4 (= 2^7)$ (ii) $3^4 / 3^3$ (iii) $(5^2)^3$
 (iv) $(3 \times 4)^4 / (9 \times 2^3)$ (v) $16^2 / 4^4$ (vi) $(-6)^2 (-\frac{3}{2})^3$
 (vii) $(-ab^2)^3 / a^2 b$ (viii) $2^2 (\frac{1}{2})^{-3}$

B. Express in terms of simple surds such as $\sqrt{2}$, $\sqrt{3}$, etc.

(i) $\sqrt{50} (= 5\sqrt{2})$ (ii) $\sqrt{72} - \sqrt{8}$ (iii) $(\sqrt{27})^3$
 (iv) $\left(\frac{\sqrt{2}\sqrt{3}}{4}\right)^2$ (v) $\frac{\sqrt{3}\sqrt{7}}{\sqrt{84}}$ (vi) $\frac{\sqrt{3} + 2\sqrt{2}}{\sqrt{3} - \sqrt{2}}$
 (vii) $\left(\frac{3^{1/3} 9^{1/3}}{27}\right)^2$

1.1.8 Decimal notation

▶ 22 30 ▶▶

A. Express in decimal form

(a) $\frac{1}{2}$ (b) $-\frac{3}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{7}$

B. Express as fractions

(a) 0.3 (b) 0.67 (c) 0.6 (d) 3.142

C. Write the following numbers in scientific notation, stating the mantissa and exponent.

(i) 11.00132 (ii) 1.56 (iii) 203.45 (iv) 0.0000321

D. Write the numbers in C to three significant figures.