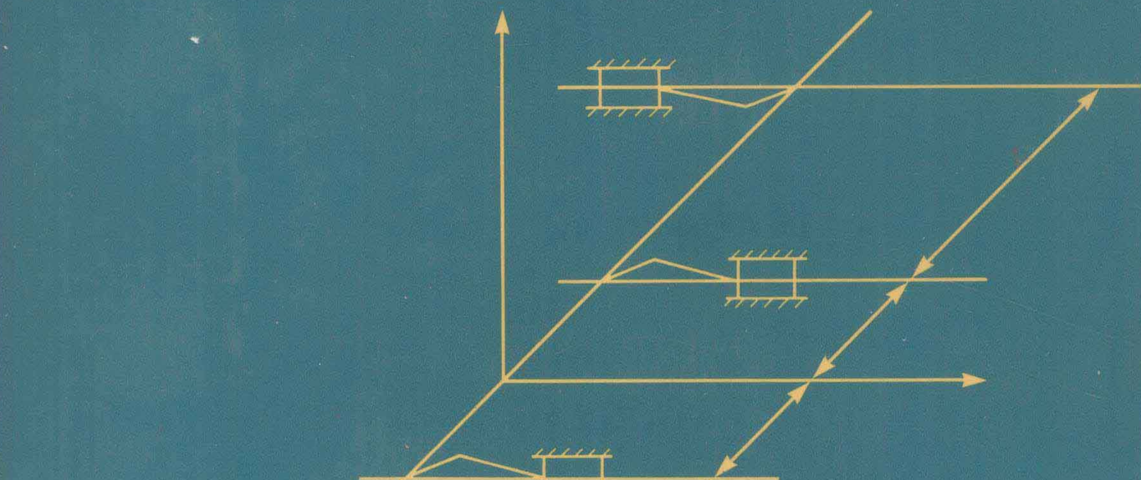
D. G. Gorman W. Kennedy

Applied Solid Dynamics



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Applied Solid Dynamics

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Applied Solid Dynamics

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Preface

The reader, confronted with the preface of yet another textbook on dynamics, might well be forgiven for asking: is such an addition to the already extensive range of texts on this subject really necessary? In our opinion, the answer is yes.

Our experience, gained over many years teaching both undergraduate and postgraduate courses in applied mechanics, has indicated the importance of conveying to students not only the fundamental concepts governing the motion of particles or bodies, but the way in which these fundamentals are associated with practical systems, which at first appear to bear no relationship to any of the theory that has been imparted. Many contemporary textbooks fail to make the successful transition between theory and practice, oversimplifying either or both of these elements and leaving the student stranded somewhere between.

The aim of this book is to help students bridge the gap between theoretical knowledge and practical application, thereby enabling them to approach specific problems with confidence.

Central to this theme is the relationship between rectilinear and rotational systems. Students may understand the basic principles of dynamics when dealing with purely rectilinear systems, but may have difficulty in relating these principles to rotating systems; this difficulty is compounded many times over when the system being analysed possesses both rectilinear and rotational components of motion. In an attempt to overcome these problems, Chapter 1 formulates the concept of dynamically equivalent systems, the use of which enables even the most complex of systems to be represented by a much simpler model—provided certain important criteria are met. The usefulness of this concept is demonstrated in Chapter 2 in the study of the transmission of power through geared systems. In this chapter, also, the reader is introduced to an innovative vector system for the analysis of epicyclic gear transmission.

The transmission of motion by coplanar link mechanisms is investigated in Chapter 3, which also highlights the importance of the simple reciprocating mechanism in relation to the force analysis of bearings and sliding components, and acts as a precursor to the analysis of more complex multicylinder engines in a later chapter.

Chapter 4 builds upon the knowledge imparted in Chapter 3 by demonstrating the effect of intermittent energy transfer in a reciprocating system, and highlights the need for the use of flywheels to act as energy reservoirs in such systems. Attention is also devoted to the general design of flywheels.

Further work on the transmission of power is studied in depth in Chapter 5 where the friction drive, in the form of belts and clutches, is the means of motion and energy transfer. In addition, the manner of energy dissipation, using frictional brake systems, is rigorously examined.

In Chapters 6 and 7 the problems associated with rotational out-of-balance are investigated. This subject is, perhaps, one of the most important aspects of dynamics likely to confront the practising engineer, since in both rotational and reciprocating machines it can often be the major source of vibration. In Chapter 6 a detailed description of the experimental method for determining out-of-balance forces in rotating systems is presented. In Chapter 7 the out-of-balance frame forces and moments associated with a range of positive displacement engines are investigated and recommendations for minimizing these are suggested.

As a natural extension to the work covered in Chapter 3, Chapter 8 expands general plane motion analysis to cover bodies undergoing general space motion, with obvious application to aerospace problems and the kinematic analysis of three-dimensional robotic motion; in addition, examination is made of the related topic of gyrodynamics.

The last five chapters of the book are concerned with vibration theory and the residual effects of this undesirable phenomenon. To some readers this may appear to be a somewhat excessive concentration on this topic; however, in our opinion the extensive coverage of vibration merely reflects the importance of this subject within a whole range of engineering disciplines, particularly in relation to power generation and transmission systems. Vibration theory is introduced at an elementary level in Chapter 9 with an analysis of a single degree of freedom, mass/elastic system performing rectilinear and angular oscillating motions. Once again extensive use is made of the technique of dynamic equivalence in creating simplified mathematical models. The effects of damping, harmonic forcing, transmissibility and seismic excitation are also assessed.

In Chapter 10, systems possessing two degrees of freedom are considered in the absence of damping and external force, but with the added complication of gearing.

The complexity of the system is increased in Chapter 11 with the introduction of multi degree of freedom systems which relate more closely to the practical vibration problems experienced in structural design. Using a simple two degree of freedom system purely as a vehicle, the student is introduced to some of the principles of matrix analysis of such systems. A central theme of this chapter is modal analysis.

In Chapter 12 vibration analysis is extended to cover distributed mass/stiffness systems as opposed to lumped systems. Commencing with the simplest of all distributed systems, namely the stretched wire, analysis proceeds through extensional and torsional vibration of prismatic bars. The lateral vibration of uniform beams is examined together with the effect that rotational motion has on the vibratory response of such components. In addition to the classical analysis, consideration is also given to approximate methods of solution, particularly those that are energy based.

Chapters 9–12 are concerned essentially with the analysis and prediction of the vibrating response of mass/elastic systems, whether such systems are single or multi degree of freedom in nature or are modelled in terms of lumped or distributed parameters. The aim of the practising engineer (and this is consequently of importance to postgraduates and undergraduates as potential engineers) is to reduce, or if possible eliminate completely, the effects of vibration. Chapter 13 is devoted to highlighting some of the ways in which this may be achieved, both by passive system analysis and also in relation to more recently developed active control technology. It is not intended to be an in-depth analysis of the subject but rather an attempt to draw to the reader's attention the range of options open to the engineer in the field of vibratory control.

In the course of the preparation of this text, we were deeply indebted to Julia Shelton, Denis Mudge and Len Bernstein for their proof reading and valuable comments and criticisms; and to Geoff Hancock for many philosophical debates with one of us. The preparation of the manuscript was undertaken by Yvonne Johnson, Marian Parsons and June Neilson—the standard of the finished product being probably the best compliment to their efforts.

Finally, we would like to express our gratitude to our respective families for their patience, understanding and support over the year it took to complete this text; and to the engineering students of Queen Mary College (University of London) and the University of Strathclyde who were our main motivators in this task.

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Contents

1 Introduction

1.1 Historical review

Dynamics can be defined as that branch of science dealing with the study of the motion of systems under the action of forces, and thus it contrasts with *statics* which is concerned with stationary systems under the action of forces. Dynamics can be divided into two main branches, namely dynamics of solids and dynamics of fluids. Under the action of shearing forces, however, fluids react differently from solids in that they continue to deform as long as the shear forces are applied. For that reason, solid dynamics and fluid dynamics are normally treated separately. In this text we shall concern ourselves with dynamics of solids and, in particular, some of its applications to modern systems.

As in all subjects, although we are mainly concerned with their application, it is of interest to learn something of the history and the people involved in the development of the subject.

The foundations of dynamics may be said to have been laid down by Descartes, Kepler and Galileo. René Descartes (1596–1650), a French philosopher and mathematician, widely regarded as the founder of modern philosophy, introduced analytical geometry—hence the term 'cartesian coordinate system' with which the reader will be familiar. Johan Kepler (1571–1630), a German astronomer, discovered the three basic laws of planetary motion which were published between 1609 and 1619. Galileo Galilei (1564–1642), an Italian mathematician, astronomer and physicist, discovered the uniform period of the pendulum and demonstrated that different bodies of different weight descended at the same rate. Galileo's studies were, however, seriously hindered as a consequence of his theories contradicting those of Aristotle, therefore leading him into continual conflict with the ecclesiastical Inquisition.

On the basis of the work of these three men, Sir Isaac Newton (1642–1727), an English mathematician, astronomer and physicist, formulated his three Laws (or Axioms) of Motion. Newton related the force that acts on a particle to the momentum change it produced, and both these quantities are vectors. Newton essentially derived his three Laws of Motion so that each Law pertained to the three mutually perpendicular directions in space. These laws form the basis of vectorial mechanics. He acknowledged the impact of the earlier work of Descartes, Kepler and Galileo, when he stated: 'If I have seen a little farther than others it is because I have stood on the shoulders of giants'.

About the same time as Newton was working on his Laws of Motion, a German mathematician, Baron Gottfried Wilhelm von Leibniz (1646–1716) was working in this same field, albeit under quite a different approach. Leibniz related the *vis viva* to 'the work of the force', whereby *vis viva* is twice the kinetic energy and 'the work of the force' is called at the present time the 'work function'. In many cases, the work function is simply the potential energy. Later, however, Joseph Louis Lagrange (1736–1813), a French mathematician, and then Sir William Rowan Hamilton (1805–1865), an Irish mathematician, developed analytical dynamics by regarding Leibniz's ideas as the basis of a principle.

In the twentieth century, Albert Einstein (1879–1955), an American physicist, drew attention to the failure of Newtonian mechanics in extreme situations, namely when speeds close to those of light are involved or for events on a molecular scale; however, Hamiltonian dynamics, when suitably interpreted, can cope with these extreme situations and quantum mechanics, which is by and large attributed to Einstein, also has its foundations in Hamiltonian mechanics. In addition it is interesting to note that the design of modern semiconductors (silicon chips) is, to a large extent, based on the theories of Sir William Rowan Hamilton and Robert Brown (1773–1852), a Scottish botanist whose work on the bombardment of particles by molecules has given rise to the term 'Brownian movement'.

Mechanical engineers are not, in general, concerned with the extreme situations of quantum mechanics, and therefore Newtonian mechanics is more than adequate. Attention will consequently be confined to vectorial or Newtonian dynamics and its application to the design and analysis of mechanisms, vehicles, machines, etc.

1.2 Newton's three Laws of Motion

Because of their importance, we shall quote the original Latin form of the three Laws of Motion as set down by Newton in the first edition of his *Principia* in 1687.

Lex I Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

Lex II Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.

Lex III Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actiones in se mutuo semper esse aequales et in partes contrarias dirigi.

In 1729 Andrew Motte translated the *Principia* into English; in 1934 F. Cajori, University of California Press, revised this translation, whereby (allowing for errors in the translation from Latin) Newton's three Laws of Motion are:

Law 1 Every body continues in a state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it.

Law 2 The change in motion is proportional to the motive force impressed; and it is made in the direction of the straight line in which the force is impressed.

Law 3 To every action there is always opposed an equal reaction; or, the mutual action of two bodies upon each other are always equal, and directed to contrary parts.

The Second Law forms the basis of most analysis of dynamic systems and is usually presented in the more recognizable form:

$$\mathbf{F} = M\mathbf{a}$$

where **F** is the resultant force vector, **a** is the resultant acceleration vector measured in a non-accelerating frame of reference, and M is the mass of the body. Sometimes the Second Law is expressed as the resultant force being equal to the time rate change of momentum with its change in the direction of the force. Both formulations are, however, equally correct when applied to bodies of constant mass. Although, strictly speaking, this law refers to particle motion, it can also be directly applied to rigid bodies (a system of particles bounded by a closed surface that cannot deform) in cases where the motion is purely rectilinear, i.e. where all particles contained within the body move in parallel straight lines. However, it can also be shown to be directly applicable to rotating solid bodies in the form

$$\mathbf{T} = I\boldsymbol{\alpha}$$

where **T** is the resultant torque vector acting on the body about a point fixed in inertial space, $\boldsymbol{\alpha}$ is the angular acceleration vector of the body about the same point, and I is the mass moment of inertia of the body about the point—often referred to as the **polar mass moment of inertia**.

The First Law is a consequence of the Second Law, since there can be no acceleration when the resultant force is zero and therefore the body will either remain at rest or continue to move with constant velocity.

The Third Law defines the rules regarding action and reaction between connected bodies and as such sets out the guidelines for the construction of 'free body diagrams' to which the Second Law is then applied. By means of a practical example, let us now demonstrate how the Second and Third Laws are applied.

1.2.1 Basic vehicle dynamics problem

Consider the case of a rear wheel drive automobile as shown in Figure 1.1 where the *applied* torque of magnitude T_Q at the rear wheels is the driving torque produced by the engine, and F_d is the magnitude of the *applied* aerodynamic drag force acting on the vehicle. We shall assume at this stage that any energy losses from the system, due to heat dissipation, are negligible.

Figure 1.2 shows the free body diagrams of the car body, wheels and road, neglecting any frictional torque at the wheel bearings and any rolling resistance forces (forces at the wheels due to air pressure variations within the tyres).

By inspecting the directions of the forces and torques on each of the free body diagrams, the reader will note that the values and directions of the *actions and reactions*

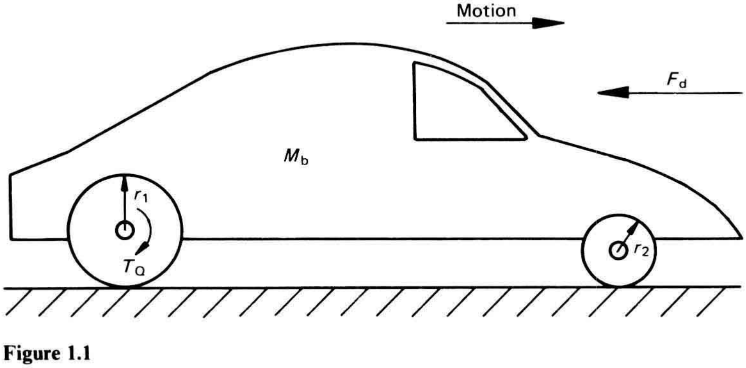

Figure 1.1

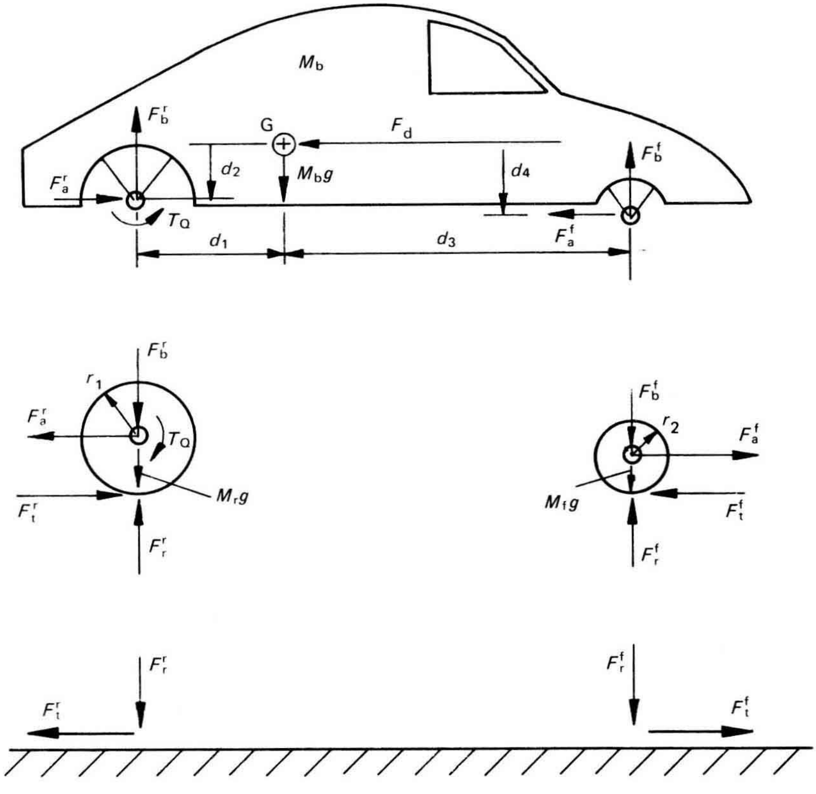

Figure 1.2

are in compliance with the Third Law. Consider now the car body, of mass M_b and centre of mass at the point G, and let us assume that the only motion is the forward rectilinear motion and that the drag force can be taken to act at G. Therefore, from the First and Second Laws, we have

$$F_b^r + F_b^f - M_b g = 0 \qquad \text{(vertical equilibrium)}$$

$$d_1 F_b^r - d_2 F_a^r - d_3 F_b^f + d_4 F_a^f - T_Q = 0 \qquad \text{(angular equilibrium)}$$

$$F_a^r - F_a^f - F_d = M_b \frac{dv}{dt} \qquad \text{(horizontal rectilinear motion)} \tag{1.1}$$

where v is the forward speed of the car body and dv/dt represents the acceleration. Similarly, for the rectilinear motion of the wheels, we have

$$-M_r g + F_r^r - F_b^r = 0 \qquad \text{and} \qquad -M_f g + F_r^f - F_b^f = 0$$

$$F_t^r - F_a^r = M_r \frac{dv}{dt} \tag{1.2}$$

$$F_a^f - F_t^f = M_f \frac{dv}{dt} \tag{1.3}$$

where M_r and M_f are the mass of the rear and front wheels respectively. Now, for the rotational motion of the wheels,

$$T_Q - r_1 F_t^r = I_r \frac{d\Omega_r}{dt}$$

$$r_2 F_t^f = I_f \frac{d\Omega_f}{dt}$$

where I_r and I_f are the polar mass moments of inertia of the rear and front wheels respectively about their axes of rotation. If at this stage we make the important assumption that no slip occurs at the interface between the wheels and the road, i.e. $\Omega_r = v/r_1$ and $\Omega_f = v/r_2$, then these two latter equations can be rearranged to give

$$T_Q/r_1 - F_t^r = I_r/r_1^2 \frac{dv}{dt} \tag{1.4}$$

and

$$F_t^f = I_f/r_2^2 \frac{dv}{dt} \tag{1.5}$$

Summing equations 1.1, 1.2 and 1.3, we have

$$F_t^r - F_t^f - F_d = (M_b + M_r + M_f) \frac{dv}{dt} \tag{1.6}$$

and if we now make the substitution

$$\frac{dv}{dt} = \frac{ds}{dt} \cdot \frac{dv}{ds} = v \frac{dv}{ds} \tag{1.7}$$

where s is the instantaneous rectilinear displacement of the vehicle, then equation 1.6 can be rewritten as

$$\int (F_t^r - F_t^f - F_d)\, ds = \int (M_b + M_r + M_f) v\, dv \tag{1.8}$$

The left-hand side of equation 1.8 represents the work done by the net resultant force acting on the vehicle, whilst the right-hand side represents the resulting change in kinetic energy associated with the *rectilinear* motion of the vehicle. If we now sum equations 1.4 and 1.5 and make the substitution described by equation 1.7, then

$$\int (T_Q/r_1 - F_t^r + F_t^f)\, ds = \int (I_r/r_1^2 + I_f/r_2^2) v\, dv \tag{1.9}$$

that is, the work done by the net resultant torque acting on the wheels is equal to the change in kinetic energy associated with the *rotational* motion of the wheels.

Equations 1.8 and 1.9 illustrate what is often referred to as the **Principle of Work and Kinetic Energy**, the former as applied to the rectilinear motion only and the latter to the rotational motion only. If we now sum equations 1.8 and 1.9 we have

$$\int (T_Q/r_1 - F_d)\, ds = \int (M_b + M_r + M_f + I_r/r_1^2 + I_f/r_2^2) v\, dv \tag{1.10}$$

Equation 1.10 can be considered as the equation describing the rectilinear motion