



# LABOR MARKETS AND BUSINESS CYCLES

Robert Shimer

# Labor Markets and Business Cycles

Robert Shimer



*This work is published in association with the  
Centre de Recerca en Economia Internacional (CREI)*

Princeton University Press  
Princeton and Oxford

Copyright © 2010 by Princeton University Press

Published by Princeton University Press,  
41 William Street, Princeton, New Jersey 08540

In the United Kingdom: Princeton University Press,  
6 Oxford Street, Woodstock, Oxfordshire OX20 1TW

All Rights Reserved

Shimer, Robert.  
Labor markets and business cycles / Robert Shimer.  
p. cm. - (CREI lectures in macroeconomics)

Includes bibliographical references and index.

ISBN 978-0-691-14022-3 (hardcover : alk. paper)

1. Labor market. 2. Business cycles. I. Title.

HD5706.S445 2009

331.12-dc22 2009047840

British Library Cataloging-in-Publication Data is available

This book has been composed in LucidaBright using T<sub>E</sub>X  
Typeset and copyedited by T&T Productions Ltd, London

Printed on acid-free paper. ©

press.princeton.edu

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

# Introduction

**Series Editor: Hans-Joachim Voth**

**Advisory Board: Antonio Ciccone, Jordi Galí, Jaume Ventura**

The Center for Research in International Economics (CREI) aims to deepen our understanding of the global forces that shape modern economies. CREI was founded in 1994 with support from the Generalitat de Catalunya and Universitat Pompeu Fabra (UPF). It is dedicated to generating research of the highest quality, in all areas of macroeconomics—ranging from growth, international finance, business cycles, the study of labor markets, and monetary economics to trade, development, and international economic history.

The CREI Lectures in Macroeconomics will present new work by young but already distinguished scholars, whose recent contributions have already had a substantial impact on the profession. Authors will be prominent contributors to areas of economics that have attracted a good deal of attention recently. The goal is that scholars delivering the CREI Lectures offer a synthesis of their thinking on one of the key research challenges facing the profession. Books in this series are aimed at graduate students and researchers in macroeconomics, broadly defined.

# Preface

The goal of this book is to consolidate, extend, and provide a new perspective on recent research that uses search frictions and wage rigidities to explain the cyclical dynamics of labor markets. Since the working paper versions of Shimer (2005), Hall (2005), and Costain and Reiter (2008) first circulated in 2002 and 2003, there has been a profusion of research in this area, but the underlying question is as old as macroeconomics: why do employment and unemployment fluctuate so much at business cycle frequencies?

Lucas and Rapping's (1969) theory of intertemporal substitution in labor supply is the starting point for any modern analysis of employment fluctuations, including the Real Business Cycle (RBC) model and the New Keynesian model. The key assumption is that workers decide how much to work at each point in time, taking as given the prevailing wage. To the extent that labor supply is elastic, hours of work fluctuate with movements in the wage.

While models based on intertemporal substitution in labor supply are qualitatively consistent with the movement of hours of work over the business cycle, they run into at least two problems. First, in a frictionless environment, the marginal rate of substitution between consumption and leisure should be equal to the marginal product of labor, after adjusting for labor and consumption taxes. When they looked at data, Parkin (1988), Rotemberg and Woodford (1991, 1999), Hall (1997), Mulligan (2002), and Chari et al. (2007) found that this relationship does not hold. In chapter 1, I reaffirm this finding, verifying that there is a wedge between the marginal rate of substitution and the marginal product of labor, the *labor wedge*, and that the wedge varies cyclically. During almost every recession, the labor wedge increases sharply. From the perspective of a frictionless model, there are two ways to interpret this finding: recessions may be times when labor income taxes and consumption taxes rise, discouraging workers from supplying labor; or they may be times when the disutility of work increases. In a reduced-form model, both would dissuade workers from working, causing countercyclical increases in the measured labor wedge. But unfortunately neither possibility is empirically tenable.

The second problem with the frictionless model is that, in an environment where workers can decide how much to work at each point in

time, it is possible to generate movements in hours worked but impossible to generate unemployment, i.e., nonemployed workers who would like to work at the prevailing wage. This omission potentially has important implications for welfare, since a worker who cannot find a job at the prevailing wage but would like to have one is, by revealed preference, worse off than if she simply chose not to work at that wage. It potentially also has important consequences for the positive analysis of business cycles, since most cyclical movements in the aggregate number of hours worked are accounted for by movements between employment and unemployment, not by movements in hours worked by employed workers.

Equilibrium search-and-matching models provide an ideal laboratory for understanding unemployment and have been used extensively for this purpose.<sup>1</sup> The models build on the idea that it takes workers time to find a job. Thus a worker entering the labor market or a worker who loses her job necessarily experiences a spell of unemployment. Moreover, unemployed workers are worse off than employed workers because they are unable to work until they find a job. In this sense, search and matching provides a theory of unemployment, not just of nonemployment.

Search-and-matching models also often assume that firms must expend resources in order to find a suitable worker. A matching function determines the number of workers and firms that meet as a function of the unemployment rate and firms' recruiting effort. Fluctuations in the profitability of hiring a worker, possibly due to fluctuations in aggregate productivity, induce fluctuations in recruiting. When firms recruit harder, unemployed workers find jobs faster, pulling down the unemployment rate. Thus search-and-matching models naturally generate movements in unemployment duration, which are an important component of the observed fluctuations in unemployment at business cycle frequencies.

But the question remains whether search-and-matching models are quantitatively consistent with the observed behavior of labor market outcomes. There is a good reason to expect that they are not. Recall that a competitive labor market model cannot explain all of the observed fluctuations in the labor wedge. Viewed through the lens of a frictionless model, recessions look like periods when the labor wedge rises, reducing labor supply.

---

<sup>1</sup> Important papers in the search-and-matching literature include Lucas and Prescott (1974), Pissarides (1985), and Mortensen and Pissarides (1994). For a thorough textbook treatment of the matching model, see Pissarides (2000).

Now consider introducing a labor adjustment cost into a competitive model, making it costly for firms to increase their employment level. This will directly lower the volatility of employment. Firms will increase employment by less during expansions because hiring is costly. They will also be less willing to reduce employment during recessions in order to avoid future hiring costs, when desired employment returns to normal. Thus hours worked will tend to be more stable over the business cycle when adjustment costs are larger. If real-world data were generated by an economy with labor adjustment costs but an economist ignored the existence of those costs, he would be surprised by how stable observed hours worked were over the business cycle. Measuring the labor wedge with data generated by the economy, he would rationalize this by concluding that the wedge rises during expansions and falls during recessions—exactly the opposite of what we observe in the data.

Search frictions act, at least in part, like a labor adjustment cost, since they imply that it takes unemployed workers time to find a job and it takes firms time to hire workers. If this reduces the volatility of employment, the labor wedge will tend to be positively correlated with employment. Such a model of search frictions will not be useful in explaining the cyclical behavior of labor markets.

The bulk of this book confirms the thrust of this argument. Search frictions do not per se help to explain fluctuations in the labor wedge, but rather they exacerbate the problems of the frictionless model. However, I also argue that subsidiary assumptions, especially alternative assumptions on wage setting, may help to explain why the measured labor wedge is countercyclical and why employment is so volatile.

To understand this last statement, note that in matching models based on Pissarides (1985) and Mortensen and Pissarides (1994), search frictions create a gap between the marginal product of labor and the marginal rate of substitution. This is because workers and firms engage in a time-consuming search for partners before negotiating a wage. Once they have sunk this cost, there is a range of wages at which both prefer to match rather than break up. Loosely speaking, any wage that is larger than the marginal rate of substitution between consumption and leisure but smaller than the marginal product of labor will be mutually preferable to breaking up.

A critical question is how wages are determined. A common assumption in the search-and-matching literature is that the worker and the firm bargain over the gains from trade, splitting the surplus according to the Nash bargaining solution (Nash 1953). In chapter 2, I prove that under this wage-setting assumption, the wage, the marginal rate of substitution, and the marginal product of labor are all proportional to current

productivity under particular assumptions on preferences (balanced growth and additive separability between consumption and leisure) and under the assumption that output is produced using only labor. Productivity shocks affect neither the labor wedge nor the (un)employment rate. This neutrality result is inspired by Blanchard and Galí (2006), who reach a similar conclusion in a model where firms face a labor adjustment cost.

In chapter 3, I break this neutrality result in several ways. First, I allow for more general preferences, although I maintain the balanced-growth restriction. The resulting fluctuations are minuscule. Second, I introduce capital into the model. While the resulting framework generates cyclical movement in employment and the labor wedge, it is inconsistent with the data. In particular, I verify that employment is positively correlated with the measured labor wedge in the model, for the reason described above: search frictions dampen fluctuations in employment, which, viewed through the lens of the frictionless model, suggests that expansions are periods when labor tax rates are higher. Third, I consider other shocks, especially reallocation shocks that change the probability of an employed worker becoming unemployed. This has little effect on the results. I conclude that the (counterfactual) positive comovement of the labor wedge and employment is a robust feature of search models when wages are set via Nash bargaining.

Chapter 4 considers an alternative wage-setting procedure that is no less plausible than the Nash bargaining solution and has qualitatively different implications for the behavior of the model. I assume that wages are backward looking. I find that this form of wage rigidity can potentially explain why employment is so volatile even if the elasticity of labor supply is relatively small. If wages do not fall following a negative productivity shock, firms will be reluctant to hire workers, pushing up unemployment duration and the unemployment rate.

This type of wage rigidity is based on ideas first developed in Hall (2005).<sup>2</sup> In a framework similar to Shimer (2005), Hall shows that if wages are rigid, unemployment is extremely sensitive to underlying shocks. He stresses that this type of wage rigidity is not susceptible to the Barro (1977) critique. That is, no matched worker-firm combination would mutually prefer to renegotiate their wage. Similarly, Blanchard and Galí (2006) consider a real-wage rigidity that makes the wage move less than one-for-one with the shock. Firms respond to relatively low wages during booms by creating many new jobs, driving down the unemployment rate. However, this also implies that part of the productivity increase is spent

---

<sup>2</sup>One may also think of this as a modern attempt to integrate search theory with disequilibrium macroeconomics (Barro and Grossman 1971; Benassy 1982; Malinvaud 1977).

on additional job creation. Consumption then increases by less than productivity, generating a countercyclical labor wedge. Gertler and Trigari (2009) reach a similar conclusion in a model with overlapping wage contracts that are not contingent on the path of productivity shocks.

Chapter 5 briefly concludes by summarizing some recent related research and suggesting the directions that future research may take.

I intend for this book to provide a stand-alone treatment of the business cycle properties of search-and-matching models. It should be suitable for advanced graduate students and other researchers familiar with modern recursive methods, for example at the level of Ljungqvist and Sargent (2004). At the same time, the book is far from exhaustive. In particular, I focus exclusively on business cycle issues, neglecting fascinating and important topics, such as cross-country differences in unemployment rates, that many others have addressed using search models. For these issues and others, the textbook treatment in Pissarides (2000) complements this book. Moreover, Pissarides (2000) provides a less technical introduction to search-and-matching models, which may be particularly useful to a reader who is uncomfortable with the history-contingent notation that I use throughout this book.

This book is also not a comprehensive survey of the literature on business cycles and unemployment. I develop one particular model of unemployment, integrating the search-and-matching model with a standard RBC model. I abstract from important, but difficult and controversial, issues like the role of incomplete markets in search models with aggregate fluctuations (Bils et al. 2007; Krusell et al. 2007; Nakajima 2008). Perhaps most importantly, I do not attempt to review the burgeoning literature on the business cycle properties of search models, mentioning only a few papers from which I knowingly borrow ideas.<sup>3</sup> My excuse is that the scope of this project, originally conceived to accompany three lectures at the Centre de Recerca en Economia Internacional (CREI) in June 2008, prevents me from doing so.

I am grateful for the comments I received at CREI during and after those lectures and for CREI's hospitality during the week I spent there. Comments by Jordi Galí, Jaume Ventura, and others had a significant influence on the shape and emphasis of this manuscript. Thijs van Rens, in addition to providing comments during the lectures at CREI,

---

<sup>3</sup> An inexhaustive reading list would certainly include Yashiv (2006), Krause and Lubik (2007), Mortensen and Nagypál (2007), Rudanko (2009), Farmer and Hollenhorst (2006), Kennan (2006), Rotemberg (2006), Rudanko (2008), and the papers collected in a special issue of the *Scandinavian Journal of Economics* entitled "Macroeconomic Fluctuations and the Labor Market" (2007, volume 107, issue 4).

subsequently used a draft of this book as part of a course and provided me with detailed feedback on the near-final manuscript.

I have taught short series of lectures based on this book at the Massachusetts Institute of Technology, Osaka University, and Study Center Gerzensee. I found that four ninety-minute lectures, one devoted to each of the first four chapters, were sufficient for a thorough overview of the material. Covering all the variants of the models and the related literature takes considerably longer. I appreciate the comments that I received from students at each of those institutions.

I also received detailed feedback from my colleagues at the University of Chicago. Fernando Alvarez's and Robert Lucas's comments were particularly important in revising this book. More broadly, my thinking about the issues in this book was informed by numerous discussions with colleagues at other universities, including Dale Mortensen, Christopher Pissarides, Richard Rogerson, Iván Werning, Randall Wright, and especially Robert Hall.

Katarina Borovickova provided me with fantastic research assistance, replicating all of the algebra and code in this book, thus significantly reducing the number of mistakes in the final manuscript. I am also grateful for the financial support of the National Science Foundation. Finally, I would like to thank Alicia Menendez for her extraordinary patience with me through the research and writing process.

# Contents

<b>Introduction</b>	<b>vii</b>
<b>Preface</b>	<b>ix</b>
<b>1 The Labor Wedge</b>	<b>1</b>
1.1 A Representative-Agent Model	1
1.2 Deriving the Labor Wedge	5
1.3 Measurement	7
1.4 Alternative Specification of Preferences	13
1.5 Preference Shocks	16
1.6 From Hours to Unemployment	18
<b>2 Benchmark Search Model: Neutrality</b>	<b>20</b>
2.1 Steady State	21
2.2 Productivity Shocks	32
2.3 The Planner's Problem	41
2.4 Extensions	44
2.5 Discussion	56
<b>3 Real Effects of Productivity Shocks</b>	<b>58</b>
3.1 General Preferences	59
3.2 Capital	75
3.3 Shocks to the Employment Exit Probability	104
3.4 Other Shocks	112
<b>4 Rigid Wages</b>	<b>113</b>
4.1 Wage Indeterminacy	114
4.2 No Capital	116
4.3 Capital	124
4.4 Using Hours Data to Test the Model	144
<b>5 Looking Ahead</b>	<b>155</b>
5.1 Theories of Rigid Wages	155
5.2 Empirical Evidence on Rigid Wages	157
5.3 Alternatives to the Matching Function	158
5.4 Relevance to Other Markets	159

<b>Appendix A. Data</b>	<b>161</b>
<b>References</b>	<b>165</b>
<b>Author Index</b>	<b>171</b>

# 1

## The Labor Wedge

Throughout this book, I study the interaction of optimizing households and firms in a closed economy. I begin in this chapter by developing a competitive, representative-agent version of the model. The chapter has two objectives. First, I introduce much of the notation that I rely on throughout the book. Because of this, I include details in this chapter that are not really necessary for the second, more substantive objective: I use the model to measure and analyze the behavior of the *labor wedge*, the wedge between the marginal rate of substitution of consumption for leisure and the marginal product of labor. I confirm the well-known result that the labor wedge tends to rise during recessions, so the economy behaves as if there is a countercyclical tax on labor. The remainder of the book explores whether extending the model to incorporate labor market search frictions can explain the behavior of the labor wedge.

I start the chapter by laying out the essential features of the model: optimizing households, optimizing firms, a government that sets taxes and spending, and equilibrium conditions that link the various agents. In section 1.2, I use pieces of the model to derive a static equation that relates hours worked, the consumption-output ratio, and the labor wedge. Section 1.3 discusses how I measure the first two concepts and uses these measures to calculate the implied behavior of the labor wedge in the United States. I establish the main substantive result: that the labor wedge rose strongly during every recession since 1970. I show the robustness of my results to alternative specifications of preferences in section 1.4 and discuss the possibility that the results are driven by preference shocks in section 1.5. I finish the chapter with a brief discussion in section 1.6 on the empirical relationship between the fluctuations in hours, which I analyze here, and fluctuations in employment and unemployment, which are the main topic of subsequent chapters.

### 1.1 A Representative-Agent Model

I denote time by  $t = 0, 1, 2, \dots$  and the state of the economy at time  $t$  by  $s_t$ . Let  $s^t = \{s_0, s_1, \dots, s_t\}$  denote the history of the economy and  $\Pi(s^t)$

denote the time-0 belief about the probability of observing an arbitrary history  $s^t$  through time  $t$ . Exogenous variables like aggregate productivity, government spending, and distortionary tax rates may depend on the history  $s^t$ . At date 0, there is an initial capital stock  $k_0 \equiv k(s^0)$  and an initial stock of government debt  $b_0 \equiv b(s^0)$ . The capital stock is owned by firms, while households hold the debt and own the firms.

## Households

A representative household is infinitely lived and has preferences over history- $s^t$  consumption  $c(s^t)$  and history- $s^t$  hours of work  $h(s^t)$ . To start, I assume that preferences are ordered by the utility function

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \left( \log c(s^t) - \frac{\gamma \varepsilon}{1 + \varepsilon} h(s^t)^{(1+\varepsilon)/\varepsilon} \right), \quad (1.1)$$

where  $\beta \in (0, 1)$  is the discount factor,  $\gamma > 0$  measures the disutility of working, and, as I show below,  $\varepsilon > 0$  is the Frisch (constant marginal utility of wealth) elasticity of labor supply.

This formulation implies that preferences are additively separable over time and across states of the world. It also implies that preferences are consistent with balanced growth—doubling a household's initial assets and its income in every state of the world doubles its consumption but does not affect its labor supply. This is consistent with the absence of a secular trend in hours worked per household, at least in the United States (Aguiar and Hurst 2007; Ramey and Francis 2009). I maintain both of these assumptions throughout this book. The formulation also imposes that the marginal utility of consumption is independent of the worker's leisure. This restriction is more questionable and so I relax it in section 1.4 below.

The household chooses a sequence for consumption and hours of work to maximize utility subject to a single lifetime budget constraint,

$$a_0 = \sum_{t=0}^{\infty} \sum_{s^t} q_0(s^t) (c(s^t) - (1 - \tau(s^t))w(s^t)h(s^t) - T(s^t)). \quad (1.2)$$

The household has initial assets  $a_0 = a(s^0)$ . In addition,  $\tau(s^t)$  is the labor income tax rate,  $w(s^t)$  is the hourly wage rate, and  $T(s^t)$  is a lump-sum transfer in history  $s^t$ , all denominated in contemporaneous units of consumption.<sup>1</sup> Thus  $c - (1 - \tau)wh - T$  represents consumption in excess of after-tax labor income and transfers, which is discounted back to time 0

---

<sup>1</sup> One can easily extend the model to include a consumption tax. Then  $\tau(s^t)$  measures the total tax wedge: the cost to an employer of providing its worker with one unit of the consumption good.

according to the intertemporal price  $q_0(s^t)$ . That is,  $q_0(s^t)$  represents the cost in history  $s^0$  of purchasing one unit of consumption in history  $s^t$ , denominated in units of history- $s^0$  consumption. Put differently,  $q_0(s^t)$  is the history- $s^0$  price of an Arrow-Debreu security that pays one unit of consumption in history  $s^t$  and nothing otherwise. Equation (1.2) states that the household's net purchase of Arrow-Debreu securities in history  $s^0$  must be equal to its initial assets  $a_0$ .

It will be useful to define the assets of the household, following history  $s^t$ , as

$$a(s^t) = \sum_{t'=t}^{\infty} \sum_{s^t|s^t} q_t(s^{t'}) (c(s^{t'}) - (1 - \tau(s^{t'}))w(s^{t'})h(s^{t'}) - T(s^{t'})),$$

where the notation  $s^{t'}|s^t$  indicates that the summation is taken over histories  $s^{t'}$  that are continuation histories of  $s^t$ , i.e.,  $s^{t'} \equiv \{s^t, s_{t+1}, s_{t+2}, \dots, s_{t'}\}$  for some states  $\{s_{t+1}, s_{t+2}, \dots, s_{t'}\}$ . Then  $q_t(s^{t'})$  is the price of a unit of consumption in history  $s^{t'} = \{s^t, s_{t+1}, s_{t+2}, \dots, s_{t'}\}$  paid in units of history- $s^t$  consumption. The absence of arbitrage opportunities requires that  $q_0(s^t)q_t(s^{t+1}) = q_0(s^{t+1})$  for all  $s^t$  and for all  $s^{t+1} \equiv \{s^t, s_{t+1}\}$ . Equivalently, the lifetime budget constraint implies a sequence of intertemporal budget constraints,

$$a(s^t) + (1 - \tau(s^t))w(s^t)h(s^t) + T(s^t) = c(s^t) + \sum_{s^{t+1}|s^t} q_t(s^{t+1})a(s^{t+1}), \quad (1.3)$$

so assets plus labor income plus transfers in history  $s^t$  is equal to consumption plus purchases of assets in continuation histories  $s^{t+1}$ .

## Firms

The representative firm owns the capital stock  $k_0 = k(s^0)$  and has access to a Cobb-Douglas production function, producing gross output  $z(s^t)k(s^t)^\alpha h^d(s^t)^{1-\alpha}$  in history  $s^t$ , where  $z(s^t)$  is history-contingent total factor productivity,<sup>2</sup>  $k(s^t)$  is its capital stock,  $h^d(s^t)$  is the labor it demands, and  $\alpha \in [0, 1)$  is the capital share of income. A fraction  $\delta$  of the capital depreciates in production each period, while at the end of period  $t$ , the firm purchases any capital that it plans to employ in period  $t + 1$ . That is, history- $s^{t+1} \equiv \{s^t, s_{t+1}\}$  capital  $k(s^{t+1})$  is purchased in history  $s^t$  and so must be measurable with respect to  $s^t$ . The present value

<sup>2</sup> Although I do not place explicit restrictions on the productivity process, I do assume that a worker's expected utility is finite so her optimization problem is well-behaved. This is ensured if productivity is bounded but is true under substantially weaker conditions, if productivity does not grow too fast.

of the firm's profits is then given by

$$J(s^0, k_0) = \sum_{t=0}^{\infty} \sum_{s^t} q_0(s^t) (z(s^t)k(s^t)^\alpha h^d(s^t)^{1-\alpha} + (1 - \delta)k(s^t) - k(s^{t+1}) - w(s^t)h^d(s^t)). \quad (1.4)$$

Note that this expression presumes that the firm does not pay any taxes. I do this for notational simplicity alone. In particular, any payroll taxes are rolled into the labor income tax rate  $\tau$ . The firm chooses the sequences  $h^d(s^t)$  and  $k(s^{t+1})$  to maximize  $J$ .

I can also write the value of the firm's profits from history  $s^t$  on as

$$J(s^t, k(s^t)) = \sum_{t'=t}^{\infty} \sum_{s^{t'}|s^t} q_t(s^{t'}) (z(s^{t'})k(s^{t'})^\alpha h^d(s^{t'})^{1-\alpha} + (1 - \delta)k(s^{t'}) - k(s^{t'+1}) - w(s^{t'})h^d(s^{t'})).$$

This implies the recursive equation

$$J(s^t, k(s^t)) = z(s^t)k(s^t)^\alpha h^d(s^t)^{1-\alpha} + (1 - \delta)k(s^t) - k(s^{t+1}) - w(s^t)h^d(s^t) + \sum_{s^{t+1}|s^t} q_t(s^{t+1})J(s^{t+1}, k(s^{t+1})). \quad (1.5)$$

The value of a firm that starts history  $s^t$  with capital  $k(s^t)$  comes from current production  $z(s^t)k(s^t)^\alpha h^d(s^t)^{1-\alpha}$  minus the cost of investment  $k(s^{t+1}) - (1 - \delta)k(s^t)$  minus labor costs  $w(s^t)h^d(s^t)$  plus the value of starting the following period in history  $s^{t+1} \equiv \{s^t, s_{t+1}\}$  with  $k(\{s^{t+1}\})$  units of capital.

## Government

A government sets the path of taxes, transfers, and government debt to fund some spending  $g(s^t)$ . I assume government spending is wasteful or at least is separable from consumption and leisure in preferences. The government faces a budget constraint in any history  $s^t$ ,

$$b(s^t) = \sum_{t'=t}^{\infty} \sum_{s^{t'}|s^t} q_t(s^{t'}) (\tau(s^{t'})w(s^{t'})h(s^{t'}) - g(s^{t'}) - T(s^{t'})), \quad (1.6)$$

so debt  $b(s^t)$  is equal to the present value of future tax receipts in excess of spending and lump-sum transfers. Again, this is equivalent to a sequence of budget constraints of the form

$$b(s^t) + g(s^t) + T(s^t) = \tau(s^t)w(s^t)h(s^t) + \sum_{s^{t+1}|s^t} q_t(s^{t+1})b(s^{t+1}), \quad (1.7)$$

so initial debt plus current spending and transfers is equal to current tax revenue plus new debt issues.

### Market Clearing

There are three markets in this economy: the labor market, the capital market, and the goods market. All of them must clear in equilibrium. Labor market clearing dictates that labor supply equals labor demand in all histories,  $h(s^t) = h^d(s^t)$ . Capital market clearing dictates that household assets are equal to firms' valuation plus government debt,  $a(s^t) = J(s^t, k(s^t)) + b(s^t)$ . Goods market clearing dictates that output plus undepreciated capital is equal to consumption plus government spending plus next period's capital stock:

$$z(s^t)k(s^t)^\alpha h^d(s^t)^{1-\alpha} + (1 - \delta)k(s^t) = c(s^t) + g(s^t) + k(s^{t+1}).$$

One can confirm that goods market clearing is implied by the household budget constraint (equation (1.3)), the firm's value function (equation (1.5)), the government budget constraint (equation (1.7)), and capital and labor market clearing. This is an application of Walras's law.

### Equilibrium

Given arbitrary paths for government spending  $g(s^t)$ , taxes  $\tau(s^t)$ , and government debt  $b(s^t)$ , an equilibrium consists of paths for consumption  $c(s^t)$ , labor supply  $h(s^t)$ , labor demand  $h^d(s^t)$ , capital  $k(s^t)$ , assets  $a(s^t)$ , transfers  $T(s^t)$ , intertemporal prices  $q_0(s^t)$ , and the wage rate  $w(s^t)$  such that:

- $\{c(s^t)\}$ ,  $\{h(s^t)\}$ , and  $\{a(s^t)\}$  solve the household's utility-maximization problem, maximizing equation (1.1) subject to the budget constraint (1.2) given  $\{q(s^t)\}$ ,  $\{w(s^t)\}$ ,  $\{\tau(s^t)\}$ , and  $\{T(s^t)\}$ ;
- $\{h^d(s^t)\}$  and  $\{k(s^t)\}$  maximize firms' profits in (1.4) given  $\{q_0(s^t)\}$  and  $\{w(s^t)\}$ ;
- the government budget is balanced, so equation (1.6) holds; and
- the labor, capital, and goods markets clear.

## 1.2 Deriving the Labor Wedge

To see the implications of this model for the labor wedge, I focus on a subset of the equilibrium conditions. First, consider the household's choice of history- $s^t$  consumption and labor supply. These must satisfy the first-order conditions

$$\beta^t \Pi(s^t) \frac{1}{c(s^t)} = \lambda q_0(s^t) \quad (1.8)$$